

1 Preface

1.1 Brief history

CVM C++ class library¹ encapsulates the concepts of vector and different kinds of matrices including square, band, symmetric and hermitian ones, in Euclidean space of real and complex numbers. The library is distributed under the [Boost Software License, Version 1.0](#). First version of the library was released in 1992. Originally it allowed to simplify a code dealing with matrices and vectors in the following way:

```

rvector a(10), b(20);
rmatrix A(20,10);
.....
b = A * a;
cout << "Norm of b equals " << b.norm() << endl;

```

By overloading of arithmetic operators and parentheses relative readability of source code was reached.

In 1995² Russian office of Intel Corporation began to distribute a CD with last software products from Intel. Among others there was a freely (at that time) distributed library called “Intel BLAS Library”. More than 20 years [BLAS library](#) (Basic Linear Algebra Subprograms) is known to experts in numerical algorithms programming in FORTRAN language. This library features common vector-matrix operations for data types REAL and DOUBLE PRECISION. Both implementations of the BLAS are utilized since version 2.0 of CVM library. For example, operator of addition of two vectors utilizes subroutine DAXPY. Later Intel changed the name “BLAS” to “Intel Math Kernel Library”. Since version 3.0 [LAPACK](#) subroutines were added to the Intel MKL library. This functionality was encapsulated in third release of the CVM library. Version 4.0 featured complex numbers. And since version 5.0 band, symmetric and hermitian matrices are implemented as well. Version 6.0 has many additional algorithms implemented, performance enhancements and bug fixes.

1.2 Libraries Supported

The [last version](#) of the Intel MKL library at the moment of writing of this document for Win32 and Linux is 10.2.6.037. Version 6.0 of the CVM library was built and tested with that MKL library implementation.

Besides that [AMD ACML library](#) is supported as well. Current CVM Library was built and tested with AMD ACML 4.4.0.

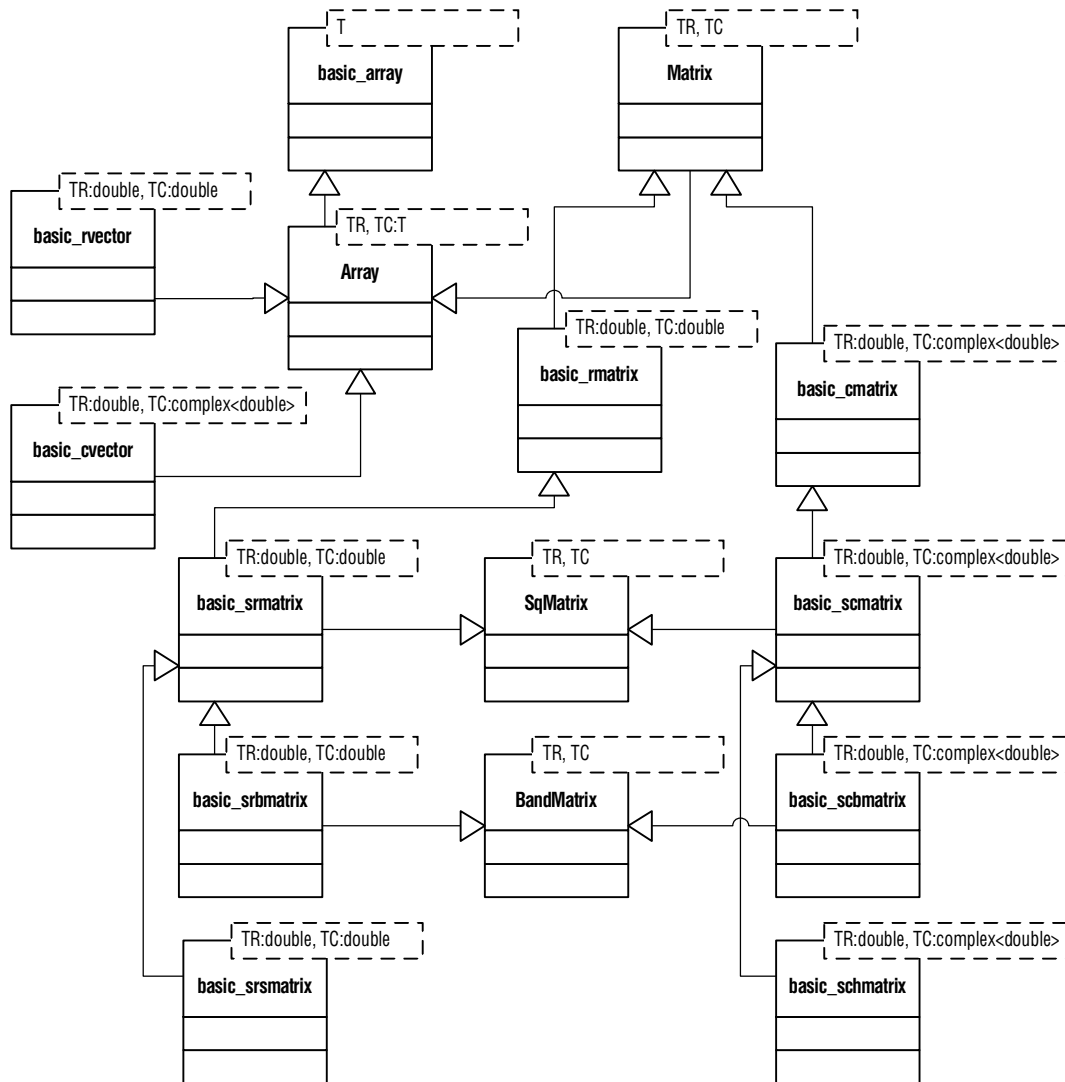
Finally, [native BLAS and LAPACK](#) libraries are supported and starting from CVM 5.7 they are part of binary packages as well.

¹ This document describes version 6.0. Copyright © Sergei Nikolaev, 1992–2010, <http://cvmlib.com>

² I’m not sure actually, may be that was 1994

1.3 Object Model

Object model of the CVM library is shown on the following picture



Base class's names are beginning with capital letters. They implement common interfaces and are *not* designed to be instantiated. This is the list of end-user classes:

- `basic_array` is an abstract array of elements of any type. It's used mostly as an array of integers like `basic_array<int>`, but you can use it in your projects within any other types as well.
- `basic_rvector` is a class encapsulating the concept of vector in the space of real numbers.
- `basic_cvector` is a class encapsulating the concept of vector in the space of complex

numbers.

- `basic_rmatrix` is a class encapsulating the concept of matrix in the space of real numbers.
- `basic_cmatrix` is a class encapsulating the concept of matrix in the space of complex numbers.
- `basic_srmatrix` is a class encapsulating the concept of square matrix in the space of real numbers.
- `basic_scmatrix` is a class encapsulating the concept of square matrix in the space of complex numbers.
- `basic_srbmatrix` is a class encapsulating the concept of square band matrix in the space of real numbers. Packed storage is used.
- `basic_scbmatrix` is a class encapsulating the concept of square band matrix in the space of complex numbers. Packed storage is used.
- `basic_srsmatrix` is a class encapsulating the concept of symmetric matrix in the space of real numbers.
- `basic_schmatrix` is a class encapsulating the concept of hermitian matrix in the space of complex numbers.

You don't need to use those class names directly unless you want to typedef your own ones. Otherwise you should use the following pre-defined classes:

```
typedef basic_array    <int>          iarray;
typedef basic_rvector <treal>         rvector;
typedef basic_rmatrix <treal>         rmatrix;
typedef basic_srmatrix <treal>        srmatrix;
typedef basic_cvector <treal, tcomplex> cvector;
typedef basic_cmatrix <treal, tcomplex> cmatrix;
typedef basic_scmatrix <treal, tcomplex> scmatrix;
typedef basic_srbmatrix<treal>        srbmatrix;
typedef basic_scbmatrix<treal, tcomplex> scbmatrix;
typedef basic_srsmatrix<treal>        srsmatrix;
typedef basic_schmatrix<treal, tcomplex> schmatrix;
```

The rest of this manual describes them in details.

1.4 Installation

1.4.1 Directory Structure

The CVM library distribution has the following directory structure.

- `.*2008.sln` in MS Visual Studio 2008 solution files.
- `ftn` contains FORTRAN and project files for Intel Fortran 9.1 compiler and GNU `gfortran`. This source code is the part of CVM library, it contains some numerical algorithms implementation.
- `lib` is the place for 32-bit libraries.

- `lib64` is the place for 64-bit libraries.
- `test` contains regression test code and projects.
- `src` contains source code of the library along with `cvm.h` header file.

1.4.2 Usage Notes

Here are definitions and data types used in the library.

<code>CVM_ZERO_BASED</code>	define this macro to build the library with 0-based indexing
<code>CVM_ACML</code>	define this macro to link against AMD ACML and/or ACML MP library
<code>CVM_FLOAT</code>	define this macro in order to build a float version
<code>CVM_NO_NAMESPACE</code>	define this macro if you don't want to use namespace
<code>tint</code>	is typedef'ed as <code>long long int</code> (or as <code>__int64</code> for MSVC) if <code>ILP64</code> is defined and as <code>int</code> otherwise (by default)
<code>treal</code>	is typedef'ed as <code>float</code> if <code>CVM_FLOAT</code> is defined and as <code>double</code> otherwise (by default)
<code>tcomplex</code>	is typedef'ed as <code>std::complex<treal></code>

In order to use the library just include its header file:

```
#include <cvm.h>
```

You should also link your project with one of `cvm*.lib` for Microsoft's C++ compilers and `cvm*.so` for GNU C++ compilers (debug versions are `*_debug.lib` and `*_debug.so` respectively).

1.4.3 Installation – Win32

If you don't want to rebuild the library just download an appropriate version of `cvm*.dll` and `cvm*.lib` files from binaries section. If you want to rebuild the whole library you'll need Intel Fortran 11.1 and Intel C++ 11.1 compilers (or higher) along with MS Visual Studio 2008. You'll also need the Intel MKL 10.2 (or higher) library. You will also need STL library coming with MS VC++ or the [STLport](#) library. Open `.\cvmlib*.sln` solution and choose the library version you want to build.

1.4.4 Installation – Unix and MinGW

Use the Makefile provided in the root directory:

```
make [release|debug] [IFORT=1] [ICC=1] [MKL=1] [MKL_PATH=/opt/...]
      [ACML=1] [ACML_MP=1] [ACML_PATH=/opt/...] [EM64T=1] [ILP64=1]
      [CVM_FLOAT=1] [ICCT=1] [STATIC_ONLY=1] [IFORT_PATH=/opt/...]
```

Here

- `release|debug` is a target (by default it builds both)

- `IFORT=1` instructs to use `ifort` compiler (by default it's `gfortran`)
- `ICC=1` instructs to use `icpc` (by default it's `g++`)
- `MKL=1` instructs to use Intel MKL library (by default it uses native BLAS and LAPACK libraries)
- `MKL_PATH=path` specifies the directory where the MKL is installed to. By default it's equal to `/opt/intel/mkl/10.2.3.029/lib` (please make sure that this path contains `32` and `em64t` subdirectories inside).
- `ACML=1` instructs to use AMD ACML library (overrides `MKL=1` and `ACML_MP=1`).
- `ACML_MP=1` instructs to use AMD ACML MP library (overrides `MKL=1`).
- `ACML_PATH=path` specifies the directory where the ACML is installed to. By default it's equal to `/opt/acml4.3.0/` (overrides `MKL_PATH=path`).
- `EM64T=1` instructs to build EM64T version of the library.
- `ILP64=1` instructs to build INT64-based version of the library (EM64T must be turned on).
- `CVM_FLOAT=1` instructs to build `float` version of the library.
- `ICCT=1` instructs to use Intel C++ compiler for building the regression test utility (by default it's `g++`)
- `STATIC_ONLY=1` instructs to build static libraries only. Both `.so` and `.a` will be built otherwise.
- `IFORT_PATH=path` specifies the directory where the Intel Fortran is installed to. It's required only when you build ACML version within Intel Fortran and Intel C++ compilers. By default it's equal to `/opt/intel/Compiler/11.1/064`.

On Unix platforms Intel MKL, AMD ACML and native BLAS/LAPACK libraries as well as both Intel's and GNU compilers are supported. On MinGW platform only the native libraries and compilers are supported.

1.5 Storage

The way of storage of matrices units is the same as the FORTRAN's one. Units are stored by columns, see the following example:

```
cvm::rmatrix m(2,2);
m(1,1) = 1.;      // first row, first column
m(1,2) = 2.;      // first row, second column
m(2,1) = 3.;      // second row, first column
m(2,2) = 4.;      // second row, second column
double* p = m;
cout << p[0] << " " << p[1] << " " << p[2] << " " << p[3] << endl;
```

Output will be the following:

```
1 3 2 4
```

Since version 5.0 band matrices are introduced. Band storage is utilized for such matrices; it can be described as follows (cited from MKL manual): *an m by n band matrix with*

k_l sub-diagonals and k_u super-diagonals is stored compactly in a two-dimensional array with $k_l + k_u + 1$ rows and n columns. Columns of the matrix are stored in the corresponding columns of the array, and diagonals of the matrix are stored in rows of the array. This way of storage can be illustrated as follows (referenced elements are shown as “*”, not referenced as “—”, zeros are not stored):

$$\begin{array}{l}
 m = n = 3, k_l = 0, k_u = 0 : \begin{bmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{bmatrix} \\
 \\
 m = n = 4, k_l = 1, k_u = 0 : \begin{bmatrix} * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ 0 & * & * & 0 \\ 0 & 0 & * & * \\ & & & - \end{bmatrix} \\
 \\
 m = n = 6, k_l = 1, k_u = 2 : \begin{bmatrix} - & & & & & \\ - & - & & & & \\ & * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & * & * & * & * & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \\ & & & & & - \end{bmatrix}
 \end{array}$$

CVM library implements square band matrices only, therefore $m = n$ is satisfied for them.

1.6 Indexing

By default index numbering in CVM library corresponds to the FORTRAN's one: index of the first unit is equal to 1:

```

cvm::vector v(2);
cvm::matrix m (2,3);
v[1] = 1.3;      // first vector unit
v(2) = 2.1;     // second vector unit
m(1) = v;       // first column of matrix
m(1,2) = 3.7;   // element located on the first row
                // and the second column of matrix

```

Starting from version 5.7 0-based indexing is supported as well. In order to use it you'd need to re-build the library with CVM_ZERO_BASED macro defined. To distinguish this fact, *ℓ*-based notation will be used below in this document, which means 1-based by default and 0-based if the macro is used.

1.7 Polymorphism

Major number of CVM Class Library member functions is not declared as virtual, but it doesn't mean that the classes are not polymorphic. Those member functions just wrap virtual ones. For example, the following code

```
void print_solution (const srmatrix& a, const rvector& b)
{
    std::cout << a.solve(b);
}
...
rvector b(3);
srmatrix m(3);
srsrmatrix ms(3);
...
print_solution(m, b);
print_solution(ms, b);
```

will use symmetric solver for symmetric matrix ms.

1.8 Multi-threading

The library fully supports multi-threading environments. Its binary files are linked with multi-threaded versions of appropriate run-time library.

1.9 Regression test utility

The library is shipped with regression utility utilizing almost all its functions and operators. It's strongly recommended to build it upon installation and verify (see test directory for workspace and make files). It has the following syntax:

```
regtest_* [-t<Number of threads to run>] [-r<Number of executions>]
```

Example:

```
D:\cvmlib\lib>regtest_cvm_ia32.exe -t2 -r2
TESTS STARTED
TESTS STARTED
ALL TESTS SUCEEDED
ALL TESTS SUCEEDED
TESTS STARTED
TESTS STARTED
ALL TESTS SUCEEDED
ALL TESTS SUCEEDED
TOTAL TIME 1.83e+000 sec.
```

2 CVM Class Library Reference

2.1 basic_array

This class contains array-specific member functions inherited in other classes. It can be utilized as a standalone class too. It also provides STL-compatible functions and type definitions, so itself and derived classes can be used in the same way as `std::vector<T>`. Since version 5.0 the `iarray` class is defined as

```
typedef basic_array<int> iarray;

template <typename T>
class basic_array {
public:
    int basic_array ();
    explicit basic_array (int nSize, bool bZeroMemory = true);
    basic_array (const T* p, int nSize);
    basic_array (const T* first, const T* last);
    basic_array (const basic_array& a);
    int size () const;
    T* get ();
    const T* get () const;
    operator T* ();
    operator const T* () const;
    T& operator () (int i) throw (cvmexception);
    T operator () (int i) const throw (cvmexception);
    T& operator [] (size_type i) throw (cvmexception);
    T operator [] (size_type i) const throw (cvmexception);
    T& operator [] (int i) throw (cvmexception);
    T operator [] (int i) const throw (cvmexception);
    basic_array& operator = (const basic_array& a) throw (cvmexception);
    basic_array& assign (const T* p);
    basic_array& set (T x);
    basic_array& resize (int nNewSize) throw (cvmexception);
    // STL-specific type definitions
    typedef T value_type;
    typedef value_type* pointer;
    typedef const value_type* const_pointer;
    typedef value_type* iterator;
    typedef const value_type* const_iterator;
    typedef value_type& reference;
    typedef const value_type& const_reference;
    typedef size_t size_type;
    typedef ptrdiff_t difference_type;
```

```
typedef std::reverse_iterator<const_iterator> const_reverse_iterator;
typedef std::reverse_iterator<iterator> reverse_iterator;
// STL-specific functions
iterator begin ();
const_iterator begin () const;
iterator end ();
const_iterator end () const;
reverse_iterator rbegin ();
const_reverse_iterator rbegin () const;
reverse_iterator rend ();
const_reverse_iterator rend () const;
size_type max_size () const;
size_type capacity () const;
bool empty () const;
reference front ();
const_reference front ();
reference back ();
const_reference back () const;
void assign (size_type n, const T& val) throw (cvmexception);
void assign (const_iterator first,
             const_iterator last) throw (cvmexception);
void resize (size_type nNewSize) throw (cvmexception);
void clear ();
void swap (basic_array& v);
reference at (size_type n) throw (cvmexception);
const_reference at (size_type n) const throw (cvmexception);
void push_back (const T& x) throw (cvmexception);
void pop_back () throw (cvmexception);
iterator insert (iterator position, const T& x) throw (cvmexception);
iterator erase (iterator position) throw (cvmexception);

template <typename T>
friend std::istream& operator >> <> (const std::istream& is,
                                     basic_array<T>& aIn);

template <typename T>
friend std::ostream& operator << <> (std::ostream& os,
                                     const basic_array<T>& aOut);
};
```

2.1.1 `basic_array()`

Default constructor

```
basic_array<T>::basic_array();
```

creates an empty `basic_array` object. See also [basic_array](#). Example:

```
using namespace cvm;
```

```
iarray a;  
std::cout << a.size() << std::endl;  
a.resize(10);  
std::cout << a.size() << std::endl;
```

prints

```
0  
10
```

2.1.2 `basic_array(int, bool)`

Constructor

```
explicit basic_array<T>::basic_array(int nSize, bool bZeroMemory = true);
```

creates a `basic_array` object of size equal to `nSize`. Allocated memory is initialized with zero values by default (you can pass `false` in second argument in order to avoid this initialization). The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;
```

```
iarray a(5);  
std::cout << a.size() << " " << a[1] << std::endl;
```

prints

```
5 0
```

2.1.3 `basic_array(const T*, int)`

Constructor

```
basic_array<T>::basic_array (const T* p, int nSize);
```

creates a `basic_array` object of size equal to `nSize` and copies `nSize` elements of an array `p` to it. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;
```

```
const int a[] = {1, 2, 3, 4};  
iarray v (a, 3);  
std::cout << v;
```

prints

```
1 2 3
```

2.1.4 `basic_array(const T*, const T*)`

Constructor

```
basic_array<T>::basic_array (const T* first, const T* last);
```

creates a `basic_array` object of size equal to `last-first` and copies all elements in the range of `[first, last)` to it. The constructor throws an exception of type `cvmexception` in case of wrong range passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;
```

```
const int a[] = {1, 2, 3, 4};  
const iarray v (a+1, a+3);  
std::cout << v << std::endl;
```

prints

```
2 3
```

2.1.5 `basic_array(const basic_array&)`

Copy constructor

```
basic_array<T>::basic_array (const basic_array& a);
```

creates a `basic_array` object of size equal to size of vector `a` and sets every element of created array to a value of appropriate element of an `a`. See also [basic_array](#). Example:

```
using namespace cvm;
```

```
iarray a(5);  
a.set(3);  
iarray b(a);  
std::cout << b;
```

prints

```
3 3 3 3 3
```

2.1.6 `size()`

Function

```
int basic_array<T>::size () const;
```

returns a number of elements of an array. This function is *inherited* in all classes of the library: `rvector`, `cvector`, `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `basic_array`. Example:

```
using namespace cvm;
```

```
rvector v(3);  
cmatrix m(10,20);  
cout << v.size() << " " << m.size() << endl;
```

prints

```
3 200
```

2.1.7 `get()`, `operator T*()`

Functions and operators

```
T* basic_array<T>::get ();
const T* basic_array<T>::get () const;
basic_array<T>::operator T* ();
basic_array<T>::operator const T* () const;
```

return a pointer to the beginning (first element) of an array. These functions and operators are *inherited* in all classes of the library: `rvector`, `cvector`, `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `basic_array`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

void cprint (const std::complex<double>* p, int size)
{
    for (int i = 0; i < size; ++i)
        std::cout << p[i] << " ";
    std::cout << std::endl;
}

...
iarray a(10);
scmatrix m(3);
a[2] = 1;
m(3,1) = std::complex<double>(1., 2.);
std::cout << a.get()[1] << std::endl;
cprint(m, 3);

prints

1
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (1.00e+00,2.00e+00)
```

2.1.8 Indexing operators

Indexing operators

```
T& basic_array<T>::operator () (int i) throw (cvmexception);
T basic_array<T>::operator () (int i) const throw (cvmexception);
T& basic_array<T>::operator [] (size_type i) throw (cvmexception);
T basic_array<T>::operator [] (size_type i) const throw (cvmexception);
T& basic_array<T>::operator [] (int i) throw (cvmexception);
T basic_array<T>::operator [] (int i) const throw (cvmexception);
```

return a reference (or value for constant versions) to *i*-th element of an array. Please note that *all indexing operators of the library are ℓ-based*. These operators are *inherited* in `rvector` and `cvector` classes of the library but *overridden* in other ones: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `basic_array`. Example:

```
using namespace cvm;
```

```
try {
    rvector v (10);
    v[1] = 1.;
    v(2) = 2.;
    std::cout << v;

    double a[] = {1., 2., 3., 4.};
    const rvector vc (a, 4);
    std::cout << vc(1) << " " << vc[2] << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 0 0 0 0 0 0 0 0
1 2
```

2.1.9 operator = (const basic_array&)

Assignment operator

```
basic_array<T>&  
basic_array<T>::operator = (const basic_array& a) throw (cvmexception);
```

sets every element of a calling array to a value of appropriate element of an array a and returns a reference to the object changed. This operator is *overridden* in all classes of the library: `rvector`, `cvector`, `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `basic_array`. The operator throws an exception of type `cvmexception` in case of different sizes of the arrays. Example:

```
using namespace cvm;
```

```
iarray a(5), b(5);  
a.set(3);  
b = a;  
std::cout << b;
```

prints

```
3 3 3 3 3
```

2.1.10 assign(const T*)

Function

```
basic_array<T>& basic_array<T>::assign (const T* p);
```

sets every element of a calling array to a value of appropriate element of an array pointed to by parameter *p* and returns a reference to the object changed. This function is *overridden* in all classes of the library: *rvector*, *cvector*, *rmatrix*, *cmatrix*, *srmatrix*, *scmatrix*, *srbmatrix*, *schmatrix*, *srsmatrix* and *schmatrix*. See also *basic_array*. Example:

```
using namespace cvm;
```

```
const int a[] = {1, 2, 3, 4, 5, 6, 7};  
iarray v (5);
```

```
v.assign(a);  
std::cout << v;
```

prints

```
1 2 3 4 5
```

2.1.11 `set(T)`

Function

```
basic_array<T>& basic_array<T>::set (T x);
```

sets every element of a calling array to a value of parameter `x` and returns a reference to the object changed. This function is *overridden* in all classes of the library: `rvector`, `cvector`, `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `basic_array`. Example:

```
using namespace cvm;
```

```
iarray a(5);  
a.set(3);  
std::cout << a;
```

prints

```
3 3 3 3 3
```

2.1.12 `resize`

Function

```
basic_array<T>&
basic_array<T>::resize (int nNewSize) throw (cvmexception);
```

changes a size of a calling array to be equal to `nNewSize` and returns a reference to the array changed. The array will be filled with zeroes in case of increasing of its size. This function is *overridden* in all classes of the library: `rvector`, `cvector`, `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. The function throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;

try {
    const int a[] = {1, 2, 3, 4};
    iarray v (a, 3);
    std::cout << v;
    v.resize(2);
    std::cout << v;
    v.resize(4);
    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

1 2 3
1 2
1 2 0 0
```

2.1.13 STL-specific type definitions

Type definitions

```
typedef T value_type;
typedef value_type* pointer;
typedef const value_type* const_pointer;
typedef value_type* iterator;
typedef const value_type* const_iterator;
typedef value_type& reference;
typedef const value_type& const_reference;
typedef size_t size_type;
typedef ptrdiff_t difference_type;
typedef std::reverse_iterator<const_iterator> const_reverse_iterator;
typedef std::reverse_iterator<iterator> reverse_iterator;
```

are provided for every class of the library to be compatible with STL algorithms and methods. See also [basic_array](#). Example:

```
using namespace cvm;

rvector vs1(5);
vs1[1] = 1.; vs1[2] = 2.; vs1[3] = 3.; vs1[4] = 4.; vs1[5] = 5.;
std::cout << vs1;

rvector::iterator it = vs1.begin() + 1;
rvector::iterator ite = vs1.erase(it);
std::cout << vs1;
std::cout << *ite << std::endl;

ite = vs1.insert(ite, 10.);
std::cout << vs1;
std::cout << *ite << std::endl;

vs1.push_back(11.);
std::cout << vs1;

vs1.randomize(0., 2.);
std::cout << vs1;
std::sort(vs1.begin(), vs1.end());
std::cout << vs1;
std::cout << *std::max_element(vs1.begin(), vs1.end()) << std::endl;

std::reverse(vs1.begin(), vs1.end());
std::cout << vs1;
```

prints

```
1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000
1.00e+000 3.00e+000 4.00e+000 5.00e+000
3.00e+000
1.00e+000 1.00e+001 3.00e+000 4.00e+000 5.00e+000
1.00e+001
1.00e+000 1.00e+001 3.00e+000 4.00e+000 5.00e+000 1.10e+001
1.11e-001 4.96e-001 1.70e+000 1.91e+000 1.19e-001 1.11e+000
1.11e-001 1.19e-001 4.96e-001 1.11e+000 1.70e+000 1.91e+000
1.91e+000
1.91e+000 1.70e+000 1.11e+000 4.96e-001 1.19e-001 1.11e-001
```

2.1.14 STL-specific functions:

**begin(), end(), rbegin(), rend(),
max_size(), capacity(), empty(), front(), back(),
assign(), resize(), clear(), swap()**

Functions

```
basic_array<T>::iterator basic_array<T>::begin();
basic_array<T>::const_iterator basic_array<T>::begin() const;
basic_array<T>::iterator basic_array<T>::end();
basic_array<T>::const_iterator basic_array<T>::end() const;
basic_array<T>::reverse_iterator basic_array<T>::rbegin();
basic_array<T>::const_reverse_iterator basic_array<T>::rbegin() const;
basic_array<T>::reverse_iterator basic_array<T>::rend();
basic_array<T>::const_reverse_iterator basic_array<T>::rend();
basic_array<T>::size_type basic_array<T>::max_size() const;
basic_array<T>::size_type basic_array<T>::capacity() const;
bool basic_array<T>::empty() const;
basic_array<T>::reference basic_array<T>::front();
basic_array<T>::const_reference basic_array<T>::front();
basic_array<T>::reference basic_array<T>::back();
basic_array<T>::const_reference basic_array<T>::back();
void basic_array<T>::assign (size_type n,
                           const T& val) throw (cvmexception);
void basic_array<T>::assign (const_iterator first,
                           const_iterator last) throw (cvmexception);
void basic_array<T>::resize (size_type nNewSize) throw (cvmexception);
void basic_array<T>::clear();
void basic_array<T>::swap (basic_array& v) throw (cvmexception);
```

are provided for every class of the library to be compatible with STL algorithms and methods. See also [basic_array](#) and [STL documentation](#) for further details. Example:

```
using namespace cvm;

iarray a(5);
a[1] = 1; a[2] = 2; a[3] = 3; a[4] = 4; a[5] = 5;

for (iarray::reverse_iterator it = a.rbegin(); it != a.rend(); ++it)
{
    std::cout << *it << " ";
}
std::cout << std::endl;
std::cout << a.front() << std::endl;
std::cout << a.back() << std::endl;
```

prints

5 4 3 2 1

1

5

2.1.15 `at()`

Functions

```
basic_array<T>::reference  
basic_array<T>::at(size_type n) throw (cvmexception);
```

```
basic_array<T>::const_reference  
basic_array<T>::at(size_type n) throw (cvmexception);
```

return a reference to an (n-1)-th element of an array, i.e., unlike [indexing operators](#), these functions are *0-based*. They are provided for every class of the library to be compatible with STL algorithms and methods. The functions throw an exception of type `cvmexception` in case of negative parameter passed. See also [basic_array](#) and [STL documentation](#) for further details. Example:

```
using namespace cvm;  
  
iarray a(5);  
a[1] = 1; a[2] = 2; a[3] = 3; a[4] = 4; a[5] = 5;  
std::cout << a.at(0) << " " << a.at(1) << std::endl;
```

prints

```
1 2
```

2.1.16 `push_back(const T&)`, `pop_back()`

Functions

```
void basic_array<T>::push_back (const T& x) throw (cvmexception);  
void basic_array<T>::pop_back () throw (cvmexception);
```

add and remove an element to/from an array. They are provided for every class of the library to be compatible with STL algorithms and methods. Since CVM doesn't pre-allocate a memory for extra storage, these functions *will require memory reallocation every time they are being executed* and may slow down your application. Please consider usage of `std::vector<T>` in such cases. The functions throw an exception of type `cvmexception` in case of memory allocation failure. See also [basic_array](#) and [STL documentation](#) for further details. Example:

```
using namespace cvm;
```

```
iarray a(5);  
a.push_back(88);  
std::cout << a;  
a.pop_back();  
std::cout << a;
```

prints

```
0 0 0 0 0 88  
0 0 0 0 0
```

2.1.17 `insert (iterator, const T&), erase (iterator)`

Functions

```
basic_array<T>::iterator  
basic_array<T>::insert (iterator pos, const T& x) throw (cvmexception);
```

```
basic_array<T>::iterator  
basic_array<T>::erase (iterator pos) throw (cvmexception);
```

`insert` and `remove` an element to/from an array at given position `pos`. They are provided for every class of the library to be compatible with STL algorithms and methods. Since CVM doesn't pre-allocate a memory for extra storage, these functions *will require memory reallocation every time they are being executed* and may slow down your application. Please consider usage of `std::vector<T>` in such cases. The functions throw an exception of type `cvmexception` in case of memory allocation failure. See also [basic_array](#) and [STL documentation](#) for further details. Example:

```
using namespace cvm;
```

```
iarray a(5);  
iarray::iterator pos = a.begin() + 2;  
a.insert(pos, 88);  
std::cout << a;  
pos = a.begin() + 1;  
a.erase(pos);  
std::cout << a;
```

prints

```
0 0 88 0 0 0  
0 88 0 0 0
```

2.1.18 operator >> <> (std::istream& is, basic_array<T>& aIn)

Friend template operator

```
template <typename T>
friend std::istream& operator >> <> (std::istream& is,
                                     basic_array<T>& aIn);
```

fills an object referenced by aIn with numbers from is stream. The operator is **redefined** in the class **Array**. See also **basic_array** Example:

```
using namespace cvm;
```

```
try {
    std::ofstream os;
    os.open ("in.txt");
    os << 1 << " " << 2 << std::endl << 3;
    os.close ();

    std::ifstream is("in.txt");
    iarray v(5);
    is >> v;

    std::cout << v;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 2 3 0 0
```

2.1.19 operator << <> (std::ostream& os, const basic_array<T>& aOut)

Friend template operator

```
template <typename T>
friend std::ostream& operator << <> (std::ostream& os,
                                       const basic_array<T>& aOut);
```

outputs an object referenced by aOut into os stream. The operator is **redefined** in the class **Array**. See also **basic_array** Example:

```
using namespace cvm;
```

```
iarray v(5);
v(1) = 1;
v(2) = 2;
```

```
std::cout << v;
```

prints

```
1 2 0 0 0
```

2.2 Array

This class contains a couple of common for arrays member functions inherited in user classes. This class is not designed to be instantiated.

```
template <typename TR, typename TC>
class Array : public basic_array<TC> {
public:
    int incr () const;
    int indofmax () const;
    int indofmin () const;
    virtual TR norm () const;
    virtual TR norminf () const;
    virtual TR norm1 () const;
    virtual TR norm2 () const;

    <typename TR, typename TC>
    friend std::istream& operator >> <> (std::istream& is,
                                         Array<TR,TC>& aIn);

    <typename TR, typename TC>
    friend std::ostream& operator << <> (std::ostream& os,
                                         const Array<TR,TC>& aOut);
};
```

2.2.1 incr

Function

```
int Array<TR,TC>::incr () const;
```

returns an increment between elements of an array. This function is *inherited* in all classes of the library: `rvector`, `cvector`, `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. It always returns 1 for matrices. See also `Array`.
Example:

```
using namespace cvm;  
  
double a[] = {1., 2., 3., 4., 5., 6.};  
rvector v1 (a, 3, 2);  
rvector v2(10);  
  
std::cout << v1 << v1.incr () << std::endl;  
std::cout << v2.incr () << std::endl;
```

prints

```
1 3 5  
2  
1
```

2.2.2 indofmax

Function

```
int Array<TR,TC>::indofmax () const;
```

returns *l*-based index of an array's element with maximum absolute value. The function is *inherited*³ in all classes of the library: `rvector`, `cvector`, `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Array`. Example:

```
using namespace cvm;
```

```
double a[] = {3., 2., -5., -4., 5., -6.};  
const rvector v (a, 4);  
const rmatrix m (a, 2, 3);
```

```
std::cout << v << v.indofmax () << std::endl << std::endl;  
std::cout << m << m.indofmax () << std::endl;
```

prints

```
3 2 -5 -4  
3
```

```
3 -5 5  
2 -4 -6  
6
```

³Calls *virtual function* inside

2.2.3 `indofmin`

Function

```
int Array<TR,TC>::indofmin () const;
```

returns *l*-based index of an array's element with minimum absolute value. The function is *inherited*⁴ in all classes of the library: `rvector`, `cvector`, `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Array`. Example:

```
using namespace cvm;
```

```
double a[] = {3., 2., -5., 0., 0., -6.};  
const rvector v (a, 4);  
const rmatrix m (a, 2, 3);
```

```
std::cout << v << v.indofmin () << std::endl << std::endl;  
std::cout << m << m.indofmin () << std::endl;
```

prints

```
3 2 -5 0  
4
```

```
3 -5 0  
2 0 -6  
4
```

⁴Calls *virtual function* inside

2.2.4 norm

Virtual function

```
virtual TR Array<TR,TC>::norm() const;
```

returns Euclidean norm of an array that for vectors is defined as

$$\|x\|_E = \left(\sum_{i=1}^n |x_i|^2 \right)^{1/2}$$

and for matrices as

$$\|A\|_E = \left(\sum_{i=1}^m \sum_{j=1}^n |a_{ij}|^2 \right)^{1/2},$$

where A is $m \times n$ matrix. The function is *inherited* in the following classes of the library: `rvector`, `cvector`, `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srsmatrix` and `schmatrix`. It's *redefined* in `srbmatrix` and `scbmatrix`. See also `rvector::normalize`, `cvector::normalize`, `rmatrix::normalize`, `cmatrix::normalize` and `Array`. Example:

```
using namespace cvm;
std::cout.setf (ios::scientific | ios::showpos);
std::cout.precision (12);

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norm () << std::endl << std::endl;
std::cout << m << m.norm () << std::endl;
```

prints

```
+1.000000000000e+000 +2.000000000000e+000 +3.000000000000e+000
+3.741657386774e+000

+1.000000000000e+000 +3.000000000000e+000 +5.000000000000e+000
+2.000000000000e+000 -4.000000000000e+000 -6.000000000000e+000
+9.539392014169e+000
```

2.2.5 norminf

Virtual function

```
virtual TR Array<TR,TC>::norminf () const;
```

returns an infinity norm of an array that for vectors is defined as

$$\|x\|_{\infty} = \max_{i=1,\dots,n} |x_i|$$

and for matrices as

$$\|A\|_{\infty} = \max_{i=1,\dots,m} \sum_{j=1}^n |a_{ij}|,$$

where A is $m \times n$ matrix. The function is *inherited* in `rvector` and `cvector` classes of the library. It's *redefined* in `Matrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix::norm1`, `Array`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norminf () << std::endl;
std::cout << m << m.norminf () << std::endl;
```

prints

```
1 2 3
3

1 3 5
2 -4 -6
12
```

2.2.6 norm1

Virtual function

```
virtual TR Array<TR,TC>::norm1 () const;
```

returns a 1-norm of an array that for vectors is defined as

$$\|x\|_1 = \sum_{i=1}^n |x_i|$$

and for matrices as

$$\|A\|_1 = \max_{j=1,\dots,n} \sum_{i=1}^m |a_{ij}|,$$

where x is a vector of size n and A is an $m \times n$ matrix. The function is *inherited* in **rvector** and **cvector** classes. It's *redefined* in **rmatrix** and **cmatrix** and inherited thereafter. See also **Array**. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norm1 () << std::endl << std::endl;
std::cout << m << m.norm1 () << std::endl;

prints

+1.000000000000e+000 +2.000000000000e+000 +3.000000000000e+000
+6.000000000000e+000

+1.000000000000e+000 +3.000000000000e+000 +5.000000000000e+000
+2.000000000000e+000 -4.000000000000e+000 -6.000000000000e+000
+1.100000000000e+001
```

2.2.7 norm2

Virtual function

```
virtual TR Array<TR,TC>::norm2 () const;
```

returns a 2-norm of an array that for vectors is defined as

$$\|x\|_2 = \|x\|_E = \left(\sum_{i=1}^n |x_i|^2 \right)^{1/2}$$

and for matrices as

$$\|A\|_2 = \max_i \sigma_i = \left(\max_{|x|=1} (Ax \cdot Ax) \right)^{1/2},$$

where σ_i is an i -th singular value of $m \times n$ matrix A , $i = 1, \dots, \min(m, n)$. The function is *inherited* in `rvector` and `cvector` classes. It's *redefined* in `rmatrix` and `cmatrix` and inherited thereafter. See also `Array`. Example:

```
using namespace cvm;
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norm2 () << std::endl << std::endl;
std::cout << m << m.norm2 () << std::endl;

prints

+1.000000000000e+000 +2.000000000000e+000 +3.000000000000e+000
+3.741657386774e+000

+1.000000000000e+000 +3.000000000000e+000 +5.000000000000e+000
+2.000000000000e+000 -4.000000000000e+000 -6.000000000000e+000
+9.319612060784e+000
```

2.2.8 operator >> <> (std::istream& is, Array<TR,TC>& aIn)

Friend template operator

```
template <typename TR, typename TC>
friend std::istream& operator >> <> (std::istream& is,
                                       Array<TR,TC>& aIn);
```

fills an object referenced by parameter aIn with numbers from is stream. See also [basic_array::operator >>](#) , [Array](#). Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    std::ofstream os;
    os.open ("in.txt");
    os << 1.2 << " " << 2.3 << std::endl << 3.4;
    os.close ();

    std::ifstream is("in.txt");
    rvector v(5);
    is >> v;

    std::cout << v;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

1.20e+000 2.30e+000 3.40e+000 0.00e+000 0.00e+000
```

2.2.9 operator << <> (std::ostream& os, const Array<TR,TC>& aOut)

Friend template operator

```
template <typename TR, typename TC>
friend std::ostream& operator << <> (std::ostream& os,
                                     const Array<TR,TC>& aOut);
```

outputs an object referenced by aOut into os stream. The operator is **redefined** in the class **Matrix**. See also **basic_array::operator << , Array**. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v(1) = tcomplex (1., 2.);
v(2) = tcomplex (3., 4.);

std::cout << v;

prints

(1.00e+000,2.00e+000) (3.00e+000,4.00e+000) (0.00e+000,0.00e+000)
```

2.3 rvector

This is end-user class encapsulating a vector in Euclidean space of real numbers.

```

template <typename TR>
class rvector : public Array<TR,TR> {
public:
    rvector ();
    explicit rvector (int nSize);
    rvector (int nSize, TR d);
    rvector (TR* pD, int nSize, int nIncr = 1);
    rvector (const TR* pD, int nSize, int nIncr = 1);
    rvector (const rvector& v);
    rvector& operator = (const rvector& v) throw (cvmexception);
    rvector& assign (const TR* p, int nIncr = 1);
    rvector& assign (int n, const TR* p, int nIncr = 1);
    rvector& assign (int n, const rvector& v) throw (cvmexception);
    rvector& set (TR x);
    rvector& resize (int nNewSize) throw (cvmexception);
    bool operator == (const rvector& v) const;
    bool operator != (const rvector& v) const;
    rvector& operator << (const rvector& v) throw (cvmexception);
    rvector operator + (const rvector& v) const throw (cvmexception);
    rvector operator - (const rvector& v) const throw (cvmexception);
    rvector& sum (const rvector& v1,
                 const rvector& v2) const throw (cvmexception);
    rvector& diff (const rvector& v1,
                 const rvector& v2) const throw (cvmexception);
    rvector& operator += (const rvector& v) throw (cvmexception);
    rvector& operator -= (const rvector& v) throw (cvmexception);
    rvector operator - () const;
    rvector operator * (TR d) const throw (cvmexception);
    rvector operator / (TR d) const throw (cvmexception);
    rvector& operator *= (TR d);
    rvector& operator /= (TR d) throw (cvmexception);
    rvector& normalize ();
    TR operator * (const rvector& v) const throw (cvmexception);
    rvector operator * (const rmatrix& m) const
        throw (cvmexception);
    rvector& mult (const rvector& v, const rmatrix& m)
        throw (cvmexception);
    rvector& mult (const rmatrix& m, const rvector& v)
        throw (cvmexception);

```

```

rmatrix ranklupdate (const rvector& v) const;
rvector& solve (const srmatrix& mA,
               const rvector& vB, TR& dErr)
               throw (cvmexception);
rvector& solve (const srmatrix& mA,
               const rvector& vB) throw (cvmexception);
rvector& solve_lu (const srmatrix& mA, const srmatrix& mLU,
                 const int* pPivots, const rvector& vB, TR& dErr)
                 throw (cvmexception);
rvector& solve_lu (const srmatrix& mA, const srmatrix& mLU,
                 const int* pPivots, const rvector& vB)
                 throw (cvmexception);
rvector& gels (bool transpose, const rmatrix& mA, const rvector& mB,
              TR& dErr) throw (cvmexception);
rvector& gelsy (const rmatrix& mA, const rvector& mB,
               int& rank, TR tol = cvmMachSp())
               throw (cvmexception);
rvector& gelss (const rmatrix& mA, const rvector& mB,
               rvector& sv, int& rank, TR tol = cvmMachSp())
               throw (cvmexception);
rvector& gelsd (const rmatrix& mA, const rvector& mB,
               rvector& sv, int& rank, TR tol = cvmMachSp())
               throw (cvmexception);
rvector& svd (const rmatrix& mArg) throw (cvmexception);
rvector& svd (const cmatrix& mArg) throw (cvmexception);
rvector& svd (const rmatrix& mArg,
             srmatrix& mU, srmatrix& mVH) throw (cvmexception);
rvector& svd (const cmatrix& mArg,
             scmatrix& mU, scmatrix& mVH) throw (cvmexception);
rvector& eig (const srmatrix& mArg) throw (cvmexception);
rvector& eig (const srmatrix& mArg,
             srmatrix& mEigVect) throw (cvmexception);
rvector& eig (const schmatrix& mArg) throw (cvmexception);
rvector& eig (const schmatrix& mArg,
             scmatrix& mEigVect) throw (cvmexception);
rvector& gemv (bool bLeft, const rmatrix& m, TR dAlpha,
              const rvector& v, TR dBeta) throw (cvmexception);
rvector& gbmv (bool bLeft, const srbmatrix& m, TR dAlpha,
              const rvector& v, TR dBeta) throw (cvmexception);
rvector& randomize (TR dFrom, TR dTo);
};

```

2.3.1 rvector ()

Constructor

```
rvector::rvector ();
```

creates an empty rvector object. See also [rvector](#). Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
rvector v;  
std::cout << v.size() << std::endl;
```

```
v.resize (5);  
v(1) = 1.5;  
std::cout << v;
```

prints

```
0  
1.50e+000 0.00e+000 0.00e+000 0.00e+000 0.00e+0000
```

2.3.2 *rvector* (int)

Constructor

```
explicit rvector::rvector (int nSize);
```

creates a *rvector* object of size equal to nSize. The constructor throws an exception of type *cvmexception* in case of non-positive size passed or memory allocation failure. See also *rvector*. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
rvector v (5);  
std::cout << v;
```

prints

```
0.00e+000 0.00e+000 0.00e+000 0.00e+000 0.00e+000
```

2.3.3 *rvector* (int, TR)

Constructor

```
rvector::rvector (int nSize, TR d);
```

creates a *rvector* object of size equal to *nSize* and fills it with value of *d*. The constructor throws an exception of type *cvmexception* in case of non-positive size passed or memory allocation failure. See also *rvector*. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
rvector v (5, 1.5);  
std::cout << v;
```

prints

```
1.50e+00 1.50e+00 1.50e+00 1.50e+00 1.50e+00
```

2.3.4 rvector (TR*,int,int)

Constructor

```
rvector::rvector (TR* pD, int nSize, int nIncr = 1);
```

creates a rvector object of size equal to nSize. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by pD using distance between elements equal to nIncr. The constructor throws an exception of type `cvmexception` in case of non-positive size passed. See also `rvector`, `rvector (const TR*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
double a[] = {7., 7., 7., 7., 7., 7., 7.,};
```

```
rvector v1 (a, 4, 2);
```

```
std::cout << v1;
v1(2) = 1.23;
std::cout << v1 << std::endl;
```

```
for (int i = 0; i < 3; i++) {
    std::cout << a[i] << " ";
}
std::cout << std::endl;
```

```
rvector v2 (a, 5);
std::cout << v2;
```

prints

```
7.00e+000 7.00e+000 7.00e+000 7.00e+000
7.00e+000 1.23e+000 7.00e+000 7.00e+000

7.00e+000 7.00e+000 1.23e+000
7.00e+000 7.00e+000 1.23e+000 7.00e+000 7.00e+000
```

2.3.5 rvector (const TR*,int,int)

Constructor

```
rvector::rvector (const TR* pD, int nSize, int nIncr = 1);
```

creates a rvector object of size equal to nSize and copies nSize elements of an array pD to it using increment nIncr (1 by default). The object created has increment set to 1. The constructor throws an exception of type `cvmexception` in case of non-positive size passed. See also `rvector`, `rvector (TR*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
const double a[] = {7., 7., 7., 7., 7., 7., 7.,};
```

```
rvector v1 (a, 4, 2);
```

```
std::cout << v1;
v1(2) = 1.23;
std::cout << v1 << std::endl;
```

```
for (int i = 0; i < 3; i++) {
    std::cout << a[i] << " ";
}
std::cout << std::endl;
```

```
rvector v2 (a, 5);
std::cout << v2;
```

prints

```
7.00e+000 7.00e+000 7.00e+000 7.00e+000
7.00e+000 1.23e+000 7.00e+000 7.00e+000

7.00e+000 7.00e+000 7.00e+000
7.00e+000 7.00e+000 7.00e+000 7.00e+000 7.00e+000
```

2.3.6 rvector (const rvector&)

Copy constructor

```
rvector::rvector (const rvector& v);
```

creates a rvector object of size equal to size of vector v and sets every element of created vector to a value of appropriate element of v. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7.,};  
const rvector v (a, 4, 2);  
rvector vc (v);
```

```
vc(1) = 88.;  
std::cout << vc;  
std::cout << v;
```

prints

```
8.80e+001 3.00e+000 5.00e+000 7.00e+000  
1.00e+000 3.00e+000 5.00e+000 7.00e+000
```

2.3.7 operator = (const rvector&)

Operator

```
rvector& rvector::operator = (const rvector& v) throw (cvmexception);
```

sets every element of a calling vector to a value of appropriate element of a vector v and returns a reference to the vector changed. The operator throws an exception of type `cvmexception` in case of different vector sizes. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5.};
    const rvector v (a, 5);
    rvector vc(5);

    vc = v;
    std::cout << vc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints
1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000
```

2.3.8 assign(const TR*, int)

Function

```
rvector& rvector::assign (const TR* p, int nIncr = 1);
```

sets every element of a calling vector to a value of every nIncr-th element of an array pointed to by parameter p and returns a reference to the vector changed. See also [rvector](#).
Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
const double a[] = {1., 2., 3., 4., 5., 6., 7.};  
rvector v (5);  
rvector v2 (4);
```

```
v.assign(a);  
v2.assign(a, 2);  
std::cout << v;  
std::cout << v2;
```

prints

```
1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000  
1.00e+000 3.00e+000 5.00e+000 7.00e+000
```

2.3.9 assign(int, const TR*, int)

Function

```
rvector& rvector::assign (int n, const TR* p, int nIncr = 1);
```

sets every element of a calling vector, starting from ℓ -based n -th one, to a value of every $nIncr$ -th element of an array pointed to by parameter p and returns a reference to the vector changed. See also [rvector](#). Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
const double a[] = {1., 2., 3., 4., 5.};
rvector v (5);
rvector v2 (5);
```

```
v.assign(2, a);
v2.assign(3, a, 2);
std::cout << v;
std::cout << v2;
```

prints

```
0.00e+000 1.00e+000 2.00e+000 3.00e+000 4.00e+000
0.00e+000 0.00e+000 1.00e+000 3.00e+000 5.00e+000
```

2.3.10 assign (int, const rvector&)

Function

```
rvector& rvector::assign (int n, const rvector& v) throw (cvmexception);
```

sets every element of a calling vector's sub-vector beginning with *l*-based index *n* to a vector *v* and returns a reference to the vector changed. The function throws an exception of type `cvmexception` if *n* is not positive or `v.size()+n-1` is greater than a calling vector's size. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
rvector v1(5);
rvector v2(2);
v1.set(1.);
v2.set(2.);
v1.assign(3, v2);
std::cout << v1;
```

prints

```
1.00e+00 1.00e+00 2.00e+00 2.00e+00 1.00e+00
```

2.3.11 set(TR)

Function

```
rvector& rvector::set (TR x);
```

sets every element of a calling vector to a value of parameter `x` and returns a reference to the vector changed. See also [rvector](#). Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
rvector v(5);  
v.set(3.);  
std::cout << v;
```

prints

```
3.00e+000 3.00e+000 3.00e+000 3.00e+000 3.00e+000
```

2.3.12 `resize`

Function

```
rvector& rvector::resize (int nNewSize) throw (cvmexception);
```

changes a size of a calling vector to be equal to `nNewSize` and returns a reference to the vector changed. In case of increasing of its size, the vector is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. See also `basic_array::resize` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    rvector v (a, 3);
    std::cout << v;
    v.resize(2);
    std::cout << v;
    v.resize(4);
    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 2.00e+00 3.00e+00
1.00e+00 2.00e+00
1.00e+00 2.00e+00 0.00e+00 0.00e+00
```

2.3.13 operator ==

Operator

```
bool rvector::operator == (const rvector& v) const;
```

compares a calling vector with a vector *v* and returns true if they have the same sizes and their appropriate elements differ by not more than the **smallest normalized positive number**. Returns false otherwise. See also **rvector**. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4.};  
rvector v1 (a, 4);  
rvector v2 (4);
```

```
v2 (1) = 1.; v2 (2) = 2.;
```

```
v2 (3) = 3.; v2 (4) = 4.;
```

```
cout << (v1 == v2) << endl;
```

prints

```
1
```

2.3.14 operator !=

Operator

```
bool rvector::operator != (const rvector& v) const;
```

compares a calling vector with a vector `v` and returns `true` if they have different sizes or at least of their appropriate elements differs by more than the **smallest normalized positive number**. Returns `false` otherwise. See also **rvector**. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4.};  
rvector v1 (a, 4);  
rvector v2 (4);
```

```
v2 (1) = 1.; v2 (2) = 2.;
```

```
v2 (3) = 3.; v2 (4) = 4.;
```

```
cout << (v1 != v2) << endl;
```

prints

```
0
```

2.3.15 operator <<

Operator

```
rvector& rvector::operator << (const rvector& v) throw (cvmexception);
```

destroys a calling vector, creates a new one as a copy of `v` and returns a reference to the vector changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v (5);
    rvector vc (3);
    v(1) = 1.;
    v(2) = 2.;
    std::cout << v << vc << std::endl;

    vc << v;
    std::cout << vc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+000 2.00e+000 0.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000

1.00e+000 2.00e+000 0.00e+000 0.00e+000 0.00e+000
```

2.3.16 operator +

Operator

```
rvector rvector::operator + (const rvector& v) const throw (cvmexception);
```

creates an object of type `rvector` as a sum of a calling vector and vector `v`. It throws an exception of type `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `rvector::sum`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    const double b[] = {3., 5., 7., 9.};
    const rvector va (a, 4);
    rvector vb (4);
    vb.assign(b);

    std::cout << va + vb;
    std::cout << va + va;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4.00e+000 7.00e+000 1.00e+001 1.30e+001
2.00e+000 4.00e+000 6.00e+000 8.00e+000
```

2.3.17 operator -

Operator

```
rvector rvector::operator - (const rvector& v) const throw (cvmexception);
```

creates an object of type `rvector` as a difference of a calling vector and vector `v`. It throws an exception of type `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `rvector::diff`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    const double b[] = {3., 5., 7., 9.};
    const rvector va (a, 4);
    rvector vb (4);
    vb.assign(b);

    std::cout << va - vb;
    std::cout << va - va;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-2.00e+000 -3.00e+000 -4.00e+000 -5.00e+000
0.00e+000 0.00e+000 0.00e+000 0.00e+000
```

2.3.18 sum

Function

```
rvector& rvector::sum (const rvector& v1, const rvector& v2)
throw (cvmexception);
```

assigns a result of addition of vectors v1 and v2 to a calling vector and returns a reference to the vector changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `rvector::operator +`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    double b[] = {2., 3., 4., 5.};
    rvector va (a, 4);
    rvector vb (b, 4);
    rvector v (4);

    std::cout << v.sum(va, vb);
    std::cout << v.sum(v, va);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
3.00e+00 5.00e+00 7.00e+00 9.00e+00
4.00e+00 7.00e+00 1.00e+01 1.30e+01
```

2.3.19 diff

Function

```
rvector& rvector::diff (const rvector& v1, const rvector& v2)
throw (cvmexception);
```

assigns a result of subtraction of vectors v1 and v2 to a calling vector and returns a reference to the vector changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `rvector::operator -`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    double b[] = {2., 3., 4., 5.};
    rvector va (a, 4);
    rvector vb (b, 4);
    rvector v (4);

    std::cout << v.diff(va, vb);
    std::cout << v.diff(v, va);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
-1.00e+00 -1.00e+00 -1.00e+00 -1.00e+00
-2.00e+00 -3.00e+00 -4.00e+00 -5.00e+00
```

2.3.20 operator +=

Operator

```
rvector& rvector::operator += (const rvector& v) throw (cvmexception);
```

adds a vector *v* to a calling vector and returns a reference to the vector changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `rvector::operator +`, `rvector::sum`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v1 (4);
    rvector v2 (4);
    v1.set(1.);
    v2.set(2.);

    v1 += v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 += v2;
    std::cout << v2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

3.00e+00 3.00e+00 3.00e+00 3.00e+00
4.00e+00 4.00e+00 4.00e+00 4.00e+00
```

2.3.21 operator -=

Operator

```
rvector& rvector::operator -= (const rvector& v) throw (cvmexception);
```

subtracts a vector *v* from a calling vector and returns a reference to the vector changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `rvector::operator -`, `rvector::diff`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v1 (4);
    rvector v2 (4);
    v1.set(1.);
    v2.set(2.);

    v1 -= v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 -= v2;
    std::cout << v2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

-1.00e+00 -1.00e+00 -1.00e+00 -1.00e+00
0.00e+00 0.00e+00 0.00e+00 0.00e+00
```

2.3.22 operator - ()

Operator

```
rvector rvector::operator - () const throw (cvmexception);
```

creates an object of type `rvector` as a calling vector multiplied by -1 . It throws an exception of type `cvmexception` in case of memory allocation failure. See also `rvector`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};  
const rvector v (a, 4);
```

```
std::cout << - v;
```

prints

```
-1.00e+00 -2.00e+00 -3.00e+00 -4.00e+00
```

2.3.23 operator * (TR)

Operator

```
rvector rvector::operator * (TR d) const throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling vector and a number `d`. It throws an exception of type `cvmexception` in case of memory allocation failure. See also `rvector::operator *=`, `rvector`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};  
const rvector v (a, 4);
```

```
std::cout << v * 5.;
```

prints

```
5.00e+00 1.00e+01 1.50e+01 2.00e+01
```

2.3.24 operator / (TR)

Operator

```
rvector rvector::operator / (TR d) const throw (cvmexception);
```

creates an object of type `rvector` as a quotient of a calling vector and a number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the **smallest normalized positive number**. It also throws the exception in case of memory allocation failure. See also `rvector::operator /=`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    const rvector v (a, 4);

    std::cout << v / 2.;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints
5.00e-01 1.00e+00 1.50e+00 2.00e+00
```

2.3.25 operator *=

Operator

```
rvector& rvector::operator *= (TR d);
```

multiplies a calling vector by number d and returns a reference to the vector changed. See also `rvector::operator *`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
rvector v (4);
v.set(2.);
v *= 2.;
std::cout << v;
```

prints

```
4.00e+00 4.00e+00 4.00e+00 4.00e+00
```

2.3.26 operator /=

Operator

```
rvector& rvector::operator /= (TR d) throw (cvmexception);
```

divides a calling vector by number `d` and returns a reference to the vector changed. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the `smallest normalized positive number`. See also `rvector::operator /` , `rvector`.
Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v (4);
    v.set(3.);
    v /= 2.;
    std::cout << v;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints
1.50e+00 1.50e+00 1.50e+00 1.50e+00
```

2.3.27 normalize

Function

```
rvector& rvector::normalize ();
```

normalizes a calling vector so its **Euclidean norm** becomes equal to 1 if it was greater than the **smallest normalized positive number** before the call (otherwise the function does nothing). See also **rvector**. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
rvector v(4);
v(1) = 1.;
v(2) = 2.;
v(3) = 3.;
v(4) = 4.;
std::cout << v.normalize();
std::cout << v.norm() << std::endl;
```

prints

```
1.83e-01 3.65e-01 5.48e-01 7.30e-01
1.00e+00
```

2.3.28 operator * (const rvector&)

Operator

TR `rvector::operator * (const rvector& v) const throw (cvmexception);`

returns a scalar product of a calling vector and `v`. It throws an exception of type `cvmexception` if the operands have different sizes. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    rvector v1(4);
    rvector v2(4);
    v1.assign(a);
    v2.assign(a);

    std::cout << v1 * v2 << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints
3.00e+01
```

2.3.29 operator * (const rmatrix&)

Operator

```
rvector rvector::operator * (const rmatrix& m) const  
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling vector and a matrix `m`. Use `rvector::mult (const rvector&, const rmatrix&)` in order to avoid creation of the object. This operator throws an exception of type `cvmexception` if the calling vector's size differs from a number of rows in the matrix `m`. See also `rvector`, `rmatrix`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
  
try {  
    rvector v(2);  
    rmatrix m(2, 3);  
    v.set(2.);  
    m.set(1.);  
  
    std::cout << v * m;  
}  
catch (std::exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}  
  
prints  
4.00e+00 4.00e+00 4.00e+00
```

2.3.30 mult (const rvector&, const rmatrix&)

Function

```
rvector& rvector::mult (const rvector& v, const rmatrix& m)
throw (cvmexception);
```

sets a calling vector to be equal to a product of a vector *v* by a matrix *m* and returns a reference to the object changed. See also `rvector::mult (const rmatrix&, const rvector&)`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v2(2), v3(3);
    rmatrix m(2, 3);
    v2.set(2.);
    m.set(1.);

    std::cout << v3.mult(v2, m);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints
4.00e+00 4.00e+00 4.00e+00
```

2.3.31 mult (const rmatrix&, const rvector&)

Function

```
rvector& rvector::mult (const rmatrix& m, const rvector& v)
throw (cvmexception);
```

sets a calling vector to be equal to a product of a matrix `m` by a vector `v` and returns a reference to the vector changed. See also `rvector::mult (const rvector&, const rmatrix&)`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v2(2), v3(3);
    rmatrix m(2, 3);
    v3.set(2.);
    m.set(1.);

    std::cout << v2.mult(m, v3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

6.00e+00 6.00e+00
```

2.3.32 rank1update

Function

```
rmatrix rvector::rank1update (const rvector& v) const;
```

creates an object of type `rmatrix` as a rank-1 update of a calling vector and a vector `v`. The rank-1 update operation of a vector-column x of a size m and a vector-row y of a size n is defined as $m \times n$ matrix

$$\begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ x_m y_1 & x_m y_2 & \cdots & x_m y_n \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} (y_1 \ y_2 \ \cdots \ y_n).$$

See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
rvector v1(3);
rvector v2(2);
v1.set(2.);
v2.set(2.);
std::cout << v1.rank1update (v2);
```

prints

```
4.00e+00 4.00e+00
4.00e+00 4.00e+00
4.00e+00 4.00e+00
```

2.3.33 solve

Functions

```

rvector& rvector::solve (const srmatrix& mA,
                        const rvector& vB, TR& dErr)
                        throw (cvmexception);
rvector& rvector::solve (const srmatrix& mA,
                        const rvector& vB)
                        throw (cvmexception);

```

set a calling vector to be equal to a solution x of a linear equation $Ax = b$ where parameter `mA` is the square matrix A and parameter `vB` is the vector b . Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to a norm of computation error. These functions throw exception of type `cvmexception` in case of inappropriate sizes of the objects or when the matrix A is close to singular. See also `rvector`, `srmatrix`. Example:

```

using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b[] = {1., 2., 3.};
    srmatrix ma(m, 3);
    rvector vb(b, 3);
    rvector vx(3);
    double dErr = 0.;

    std::cout << vx.solve (ma, vb, dErr);
    std::cout << dErr << std::endl;

    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

+8.000000000000e+00 -8.000000000000e+00 +3.000000000000e+00
+6.661338147751e-15
+0.000000000000e+00 +0.000000000000e+00 +0.000000000000e+00

```

2.3.34 solve_lu

Functions

```
rvector&
rvector::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                  const int* pPivots, const rvector& vB, TR& dErr)
                  throw (cvmexception);
```

```
rvector&
rvector::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                  const int* pPivots, const rvector& vB)
                  throw (cvmexception);
```

set a calling vector to be equal to a solution x of a linear equation $Ax = b$ where parameter `mA` is the square matrix A , parameter `mLU` is **LU factorization** of the matrix A , parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and parameter `vB` is the vector b . Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $Ax = b$ with the same matrix A and different vectors b . In such case you save on matrix A factorization since it's needed to be performed just one time. These functions throw exception of type **cvmexception** in case of inappropriate sizes of the objects or when the matrix A is close to singular. See also **rvector**, **srmatrix**. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b1[] = {1., 2., 3.};
    double b2[] = {0., -1., -2.};
    srmatrix ma(m, 3);
    srmatrix mLU(3);
    rvector vb1(b1, 3);
    rvector vb2(b2, 3);
    rvector vx1(3);
    rvector vx2(3);
    iarray nPivots(3);
    double dErr = 0.;

    mLU.low_up(ma, nPivots);
    std::cout << vx1.solve_lu (ma, mLU, nPivots, vb1, dErr);
```

```
    std::cout << dErr << std::endl;
    std::cout << vx2.solve_lu (ma, mLU, nPivots, vb2);
    std::cout << ma * vx1 - vb1 << ma * vx2 - vb2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+8.000000000000000e+00 -8.000000000000000e+00 +3.000000000000000e+00
+6.661338147751e-15
-5.000000000000000e+00 +4.000000000000000e+00 -1.000000000000000e+00
+5.329070518201e-15
+0.000000000000000e+00 +0.000000000000000e+00 +0.000000000000000e+00
+0.000000000000000e+00 +0.000000000000000e+00 +0.000000000000000e+00
```

2.3.35 gels

Function

```
rvector& rvector::gels (bool transpose,
                      const rmatrix& mA, const rvector& mB,
                      TR& dErr) throw (cvmexception);
```

solves overdetermined or underdetermined linear system

$$A * x = b$$

for $m \times n$ matrix A (or its transpose) where b is a vector of length k and $k = m$ in non-transposed case and $k = n$ otherwise. The algorithm uses a QR or LQ factorization of A . It is assumed that A has full rank, infinity returned otherwise. Internally the function uses ?GELS LAPACK routines. If $m > n$ and `transpose=false` or $m < n$ and `transpose=true`, then the system is overdetermined, thus the algorithm tries to find the least squares solution x of the problem

$$\|A * x - b\|_2 \rightarrow \min \quad \text{or} \quad \|A^H * x - b\|_2 \rightarrow \min$$

respectively. Real number `dErr` returns residual sum of squares. The system is underdetermined otherwise, and the algorithm finds its minimum norm solution. In this case `bErr` is set to zero. In both cases the solution computed satisfies $x = \text{pinv}(A) * b$, but this algorithm is faster than pseudo inversion. The function sets calling object to be the solution and returns reference to it. It throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rmatrix::gels`. Example:

```
using namespace cvm;
rmatrix a(7, 5);
rvector b(7), bt(5);
treal dErr;
a.randomize(-10., 10.);
b.randomize(-10., 10.);
bt.randomize(-10., 10.);

rvector x(5);
x.gels(false, a, b, dErr);
cvm::rvector xt(7);
xt.gels(true, a, bt, dErr);
std::cout << (a.pinv()*b - x).norm() << std::endl;
std::cout << (~a.pinv()*bt - xt).norm() << std::endl;
```

prints

```
+5.1347815e-016
+1.1752774e-015
```

2.3.36 gelsy

Function

```
rvector& rvector::gelsy (const rmatrix& mA, const rvector& mB, int& rank,
                       TR tol = cvmMachSp()) throw (cvmexception);
```

computes the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using a complete orthogonal factorization of $m \times n$ matrix A . Here b is a vector of length m . Matrix A may be rank-deficient, the functions return its effective rank in rank output parameter using `tol` tolerance. Internally the function uses ?GELSY LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as an argument `mA`. The function sets calling object to be the solution and returns reference to it. It throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rvector::gelss`, `rvector::gelsd`, `rmatrix::gelsy`. Example:

```
using namespace cvm;
```

```
rmatrix a(4, 5);
rvector b(4);
a.randomize(-10., 10.);
b.randomize(-10., 10.);
rvector x(5);
tint rank;

x.gelsy(a, b, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+2.1326688e-015
+4 +4
```

2.3.37 gelss

Function

```
rvector& rvector::gelss (const rmatrix& mA, const rvector& mB, int& rank,
                        rvector& sv, TR tol = cvmMachSp())
                        throw (cvmexception);
```

computes the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using the singular value decomposition of $m \times n$ matrix A . Here b is a vector of length m . Matrix A may be rank-deficient, the functions return its effective rank in `rank` output parameter using `tol` tolerance. This function also computes singular values of A in decreasing order and returns them in `sv` output parameter having $\min(m, n)$ size. Internally the function uses ?GELSS LAPACK routines, see more details about the algorithm in that routine's documentation. Matrix A is passed as an argument `mA`. The function sets calling object to be the solution and returns reference to it. It throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rvector::gelsy`, `rvector::gelsd`, `rmatrix::gelss`. Example:

```
using namespace cvm;
```

```
rmatrix a(4, 5);
rvector b(4);
a.randomize(-10., 10.);
b.randomize(-10., 10.);
rvector sv(4);
rvector x(5);
tint rank;

x.gelss(a, b, sv, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+7.7937340e-015
+1.2338969e-014
+4 +4
```

2.3.38 gelsd

Function

```
rvector& rvector::gelsd (const rmatrix& mA, const rvector& mB, int& rank,
                        rvector& sv, TR tol = cvmMachSp())
                        throw (cvmexception);
```

computes the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using the singular value decomposition of $m \times n$ matrix A and a divide and conquer method. Here b is a vector of length m . Matrix A may be rank-deficient, the functions return its effective rank in `rank` output parameter using `tol` tolerance. This function also computes singular values of A in decreasing order and returns them in `sv` output parameter having $\min(m, n)$ size. Internally the function uses ?GELSD LAPACK routines, see more details about the algorithm in that routine's documentation. Matrix A is passed as an argument `mA`. The function sets calling object to be the solution and returns reference to it. It throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rvector::gelsy`, `rvector::gelss`, `rmatrix::gelsd`. Example:

```
using namespace cvm;
```

```
rmatrix a(4, 5);
rvector b(4);
a.randomize(-10., 10.);
b.randomize(-10., 10.);
rvector sv(4);
rvector x(5);
tint rank;

x.gelsd(a, b, sv, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+7.8441794e-015
+5.1021966e-015
+4 +4
```

2.3.39 svd

Functions

```

rvector&
rvector::svd (const rmatrix& mArg) throw (cvmexception);
rvector&
rvector::svd (const cmatrix& mArg) throw (cvmexception);

rvector&
rvector::svd (const rmatrix& mArg,
              srmatrix& mU, srmatrix& mVH) throw (cvmexception);
rvector&
rvector::svd (const cmatrix& mArg,
              scmatrix& mU, scmatrix& mVH) throw (cvmexception);

```

set a calling vector to be equal to the singular values

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_{\min(m,n)} \geq 0$$

of $m \times n$ matrix A (parameter `mArg`). These values are the main diagonal of matrix Σ of the singular value decomposition

$$A = U\Sigma V^H$$

where U and V are orthogonal for real A and unitary for complex A . V^H is transposed V for real one and hermitian conjugated V for complex one. First $\min(m, n)$ columns of the matrices U and V are left and right singular vectors of A respectively. Singular values and singular vectors satisfy

$$Av_i = \sigma_i u_i \quad \text{and} \quad A^H u_i = \sigma_i v_i$$

where u_i and v_i are i -th columns of U and V respectively. Third and fourth versions of the functions set output parameter `mU` to be equal to the matrix `mU` of size $m \times m$ and `mVH` to be equal to the matrix V^H of size $n \times n$. All the functions return a reference to the object they change and throw exception of type `cvmexception` in case of inappropriate calling object size (it must be equal to $\min(m, n)$), matrix `mU` size (should be $m \times m$), matrix `mVH` size (should be $n \times n$) or in case of convergence error. See also `rvector`, `rmatrix`, `cmatrix`. Example:

```

using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (10);

try {
    double m[] = {1., -1., 1., 2., -2., 1.,
                  3., -2., 1., 0., -2., 1.};

```

```

rmatrix mA(m,4,3);
rmatrix mSigma(4,3);
rvector v(3);
srmatrix mU(4), mVH(3);

v.svd(mA, mU, mVH);
mSigma.diag(0) = v;

std::cout << mU << std::endl;
std::cout << mVH << std::endl;
std::cout << mSigma << std::endl;

std::cout << (mA * ~mVH - mU * mSigma).norm() << std::endl;
std::cout << (~mA * mU - ~(mSigma * mVH)).norm() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}

prints

-4.8425643615e-01 +1.9516809011e-01 +1.1506232201e-02 -8.5280286542e-01
+2.1685987119e-01 -3.4107922671e-01 -8.8948423927e-01 -2.1320071636e-01
+6.6237057295e-01 +7.1553688692e-01 -6.1787070600e-02 -2.1320071636e-01
-5.2889765022e-01 +5.7756501033e-01 -4.5262319054e-01 +4.2640143271e-01

-2.2124855498e-01 +8.5354150454e-01 -4.7171599183e-01
+9.5937301747e-01 +1.0365951763e-01 -2.6240830353e-01
-1.7507852602e-01 -5.1060905244e-01 -8.4179920723e-01

+4.9561500411e+00 +0.0000000000e+00 +0.0000000000e+00
+0.0000000000e+00 +2.5088408581e+00 +0.0000000000e+00
+0.0000000000e+00 +0.0000000000e+00 +3.7721919242e-01
+0.0000000000e+00 +0.0000000000e+00 +0.0000000000e+00

+1.3710111285e-15
+2.4829995848e-15

```

2.3.40 eig

Functions

```

rvector&
rvector::eig (const srsmatrix& mArg) throw (cvmexception);
rvector&
rvector::eig (const schmatrix& mArg) throw (cvmexception);

rvector&
rvector::eig (const srsmatrix& mArg,
              srmatrix& mEigVect) throw (cvmexception);
rvector&
rvector::eig (const schmatrix& mArg,
              scmatrix& mEigVect) throw (cvmexception);

```

solve a symmetric eigenvalue problem and set a calling vector to be equal to eigenvalues of a square matrix `mArg`. The symmetric eigenvalue problem is defined as follows: given a symmetric or Hermitian matrix A , find the eigenvalues λ and the corresponding eigenvectors z that satisfy the equation

$$Az = \lambda z.$$

All n eigenvalues are real not only for real symmetric but also for complex Hermitian matrices A , and there exists an orthogonal system of n eigenvectors. If A is a symmetric or Hermitian positive-definite matrix, all eigenvalues are positive. See [3] for further details. The third and fourth versions of the functions set an output parameter `mEigVect` to be equal to a square matrix containing eigenvectors as columns. All the functions return a reference to the vector they change and throw an exception of type `cvmexception` in case of inappropriate calling object sizes or in case of convergence error. See also `rvector`, `cvector::eig`, `srsmatrix`, `schmatrix`. Example:

```

using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (3);

try {
    srsmatrix m(3);
    srmatrix me(3);
    rvector v(3);
    m.randomize(1., 3.);

    v.eig (m, me);
    std::cout << v;
}

```

```

std::cout << m * me(1) - me(1) * v(1);
std::cout << m * me(2) - me(2) * v(2);
std::cout << m * me(3) - me(3) * v(3);
std::cout << me(1) * me(2) << std::endl; // orthogonality check

schmatrix mc(3);
scmatrix mce(3);
mc.randomize_real(1., 3.);
mc.randomize_imag(1., 3.);

v.eig (mc, mce);
std::cout << v;

std::cout << mc * mce(1) - mce(1) * v(1);
std::cout << mc * mce(2) - mce(2) * v(2);
std::cout << mc * mce(3) - mce(3) * v(3);
std::cout << mce(1) % mce(2) << std::endl; // orthogonality check
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}

prints

-9.360e-01 +3.535e-01 +6.376e+00
-4.441e-16 -5.551e-16 -6.106e-16
+3.331e-16 +1.145e-16 +1.110e-16
-4.441e-16 +0.000e+00 -4.441e-16
+2.060e-17
-3.274e+00 +9.710e-01 +8.209e+00
(-4.441e-16,-1.221e-15) (-1.443e-15,-4.441e-16) (-8.882e-16,+4.683e-16)
(-5.551e-16,-2.776e-16) (+0.000e+00,-4.025e-16) (+6.661e-16,-2.461e-17)
(-5.551e-16,+0.000e+00) (+4.441e-16,-4.441e-16) (+0.000e+00,+3.896e-16)
(+1.608e-16,-2.261e-17)

```

2.3.41 gemv

Function

```
rvector& rvector::gemv (bool bLeft, const rmatrix& m, TR dAlpha,
                      const rvector& v, TR dBeta) throw (cvmexception);
```

calls one of ?GEMV routines of the [BLAS library](#) performing a matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is a matrix (parameter m) and v and c are vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. The function returns a reference to the vector changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also [rvector](#), [rmatrix](#). Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double alpha = 1.3;
    double beta = -0.7;
    rmatrix m(4,3);
    rvector c(4);
    rvector v(3);
    m.randomize(-1., 2.); v.randomize(-1., 3.); c.randomize(0., 2.);

    std::cout << m * v * alpha + c * beta;
    std::cout << c.gemv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gemv(true, m, alpha, c, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-3.5397829e-02 +3.1990410e-02 +3.2633344e-01 -5.4669713e-01
-3.5397829e-02 +3.1990410e-02 +3.2633344e-01 -5.4669713e-01
-4.7697026e-01 -2.2544922e-01 -5.5204984e-01
-4.7697026e-01 -2.2544922e-01 -5.5204984e-01
```

2.3.42 gbmv

Function

```
rvector& rvector::gbmv (bool bLeft, const srbmatrix& m, TR dAlpha,
                      const rvector& v, TR dBeta) throw (cvmexception);
```

calls one of ?GBMV routines of the [BLAS library](#) performing a matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is a band matrix (parameter m) and v and c are vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. The function returns a reference to the vector changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `srbmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double alpha = 1.3;
    double beta = -0.7;
    srbmatrix m(3, 1, 0);
    rvector c(3);
    rvector v(3);
    m.randomize(-1., 2.); v.randomize(-1., 3.); c.randomize(0., 2.);

    std::cout << m * v * alpha + c * beta;
    std::cout << c.gbmv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gbmv(true, m, alpha, c, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

+1.4551599e+00 -5.1882508e-01 -5.2088503e-02
+1.4551599e+00 -5.1882508e-01 -5.2088503e-02
+7.3471591e-01 -2.6952064e-01 -2.0478054e-01
+7.3471591e-01 -2.6952064e-01 -2.0478054e-01
```

2.3.43 `randomize`

Function

```
rvector& rvector::randomize (TR dFrom, TR dTo);
```

fills a calling vector with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the vector changed. See also `rvector`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (7);
```

```
rvector v(4);  
v.randomize(-2.,3.);  
std::cout << v;
```

prints

```
-1.1160314e+000 2.5649586e+000 8.9345988e-001 -1.1631825e+000
```

2.4 cvector

This is end-user class encapsulating a vector in Euclidean space of complex numbers.

```

template <typename TR, typename TC>
class cvector : public Array<TR,TC> {
public:
    cvector ();
    explicit cvector (int nSize);
    cvector (int nSize, TC c);
    cvector (TC* pD, int nSize, int nIncr = 1);
    cvector (const TC* pD, int nSize, int nIncr = 1);
    cvector (const cvector& v);
    cvector (const TR* pRe, const TR* pIm, int nSize, int nIncr = 1);
    cvector (const rvector& vRe, const rvector& vIm);
    cvector (const TR* pA, int nSize,
            bool bRealPart = true, int nIncr = 1);
    explicit cvector (const rvector& v, bool bRealPart = true);
    rvector real ();
    rvector imag ();
    cvector& operator = (const cvector& v) throw (cvmexception);
    cvector& assign (const TC* p, int nIncr = 1);
    cvector& assign (int n,const TC* p, int nIncr = 1);
    cvector& assign (int n, const cvector& v) throw (cvmexception);
    cvector& set (TC x);
    cvector& assign_real (const rvector& vRe) throw (cvmexception);
    cvector& assign_imag (const rvector& vIm) throw (cvmexception);
    cvector& set_real (TR x);
    cvector& set_imag (TR x);
    cvector& resize (int nNewSize) throw (cvmexception);
    bool operator == (const cvector& v) const;
    bool operator != (const cvector& v) const;
    cvector& operator << (const cvector& v) throw (cvmexception);
    cvector operator + (const cvector& v) const throw (cvmexception);
    cvector operator - (const cvector& v) const throw (cvmexception);
    cvector& sum (const cvector& v1,
                const cvector& v2) const throw (cvmexception);
    cvector& diff (const cvector& v1,
                 const cvector& v2) const throw (cvmexception);
    cvector& operator += (const cvector& v) throw (cvmexception);
    cvector& operator -= (const cvector& v) throw (cvmexception);
    cvector operator - () const throw (cvmexception);
    cvector operator * (TR d) const;

```

```

cvector operator / (TR d) const throw (cvmexception);
cvector operator * (TC c) const;
cvector operator / (TC c) const throw (cvmexception);
cvector& operator *= (TR d);
cvector& operator /= (TR d) throw (cvmexception);
cvector& operator *= (TC c);
cvector& operator /= (TC c) throw (cvmexception);
cvector& normalize ();
cvector operator ~() const throw (cvmexception);
cvector& conj (const cvector& v) throw (cvmexception);
cvector& conj ();
TC operator * (const cvector& v) const throw (cvmexception);
TC operator % (const cvector& v) const throw (cvmexception);
cvector operator * (const cvector& v) const throw (cvmexception);
cvector& mult (const cvector& v, const cmatrix& m)
    throw (cvmexception);
cvector& mult (const cmatrix& m, const cvector& v)
    throw (cvmexception);
cmatrix ranklupdate_u (const cvector& v) const;
cmatrix ranklupdate_c (const cvector& v) const;
cvector& solve (const scmatrix& mA,
    const cvector& vB, TR& dErr) throw (cvmexception);
cvector& solve (const scmatrix& mA,
    const cvector& vB) throw (cvmexception);
cvector& solve_lu (const scmatrix& mA, const scmatrix& mLU,
    const int* pPivots, const cvector& vB, TR& dErr)
    throw (cvmexception);
cvector& solve_lu (const scmatrix& mA, const scmatrix& mLU,
    const int* pPivots, const cvector& vB)
    throw (cvmexception);
cvector& gels (bool transpose, const cmatrix& mA, const cvector& mB,
    TC& cErr) throw (cvmexception);
cvector& gelsy (const cmatrix& mA, const cvector& mB,
    int& rank, TR tol = cvmMachSp())
    throw (cvmexception);
rvector& gelss (const cmatrix& mA, const cvector& mB,
    rvector& sv, int& rank, TR tol = cvmMachSp())
    throw (cvmexception);
rvector& gelsd (const cmatrix& mA, const cvector& mB,
    rvector& sv, int& rank, TR tol = cvmMachSp())
    throw (cvmexception);
cvector& eig (const srmatrix& mArg) throw (cvmexception);
cvector& eig (const scmatrix& mArg) throw (cvmexception);

```

```
cvector& eig (const srmatrix& mArg,  
             scmatrix& mEigVect) throw (cvmexception);  
cvector& eig (const scmatrix& mArg,  
             scmatrix& mEigVect) throw (cvmexception);  
cvector& gemv (bool bLeft, const cmatrix& m, TC cAlpha,  
             const cvector& v, TC dBeta) throw (cvmexception);  
cvector& gbm (bool bLeft, const scbmatrix& m, TC dAlpha,  
            const cvector& v, TC dBeta) throw (cvmexception);  
cvector& randomize_real (TR dFrom, TR dTo);  
cvector& randomize_imag (TR dFrom, TR dTo);  
};
```

2.4.1 `cvector` ()

Constructor

```
cvector::cvector ();
```

creates an empty `cvector` object. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
cvector v;
std::cout << v.size() << std::endl;
```

```
v.resize (3);
v(1) = std::complex<double>(1.5, -1.);
std::cout << v;
```

prints

```
0
(1.50e+00,-1.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
```

2.4.2 *cvector* (int)

Constructor

```
explicit cvector::cvector (int nSize);
```

creates a *cvector* object of size equal to *nSize*. The constructor throws an exception of type *cvmexception* in case of non-positive size passed or memory allocation failure. See also *cvector*. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
std::cout << v.size() << std::endl;
std::cout << v;

prints

3
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
```

2.4.3 `cvector` (`int`, `TC`)

Constructor

```
cvector::cvector (int nSize, TC c);
```

creates a `cvector` object of size equal to `nSize` and fills it with value of `c`. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
cvector v (3, std::complex<double>(1.5, -1.));  
std::cout << v;
```

prints

```
(1.50e+00,-1.00e+00) (1.50e+00,-1.00e+00) (1.50e+00,-1.00e+00)
```

2.4.4 cvector (TC*,int,int)

Constructor

```
cvector::cvector (TC* pD, int nSize, int nIncr = 1);
```

creates a cvector object of size equal to nSize. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by pD with the distance between elements equal to nIncr. The constructor throws an exception of type `cvmexception` in case of non-positive size passed. See also `cvector`, `cvector (const TC*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
cvector v1 ((std::complex<double>*) a, 2, 2);

std::cout << v1;
v1(2) = std::complex<double> (9.99, 9.99);
std::cout << v1 << std::endl;

for (int i = 0; i < 6; i++) {
    std::cout << a[i] << " ";
}
std::cout << std::endl;

cvector v2 ((std::complex<double>*) a, 3);
std::cout << v2;

prints

(1.00e+000,2.00e+000) (5.00e+000,6.00e+000)
(1.00e+000,2.00e+000) (9.99e+000,9.99e+000)

1.00e+000 2.00e+000 3.00e+000 4.00e+000 9.99e+000 9.99e+000
(1.00e+000,2.00e+000) (3.00e+000,4.00e+000) (9.99e+000,9.99e+000)
```

2.4.5 cvector (const TC*,int,int)

Constructor

```
cvector::cvector (const TC* pD, int nSize, int nIncr = 1);
```

creates a cvector object of size equal to nSize and copies nSize elements of an array pD to it using increment nIncr (1 by default). The object created has increment set to 1. The constructor throws an exception of type `cvmexception` in case of non-positive size passed. See also `cvector`, `cvector (TC*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
cvector v1 ((const std::complex<double>*) a, 2, 2);

std::cout << v1;
v1(2) = std::complex<double> (9.99, 9.99);
std::cout << v1 << std::endl;

for (int i = 0; i < 6; i++) {
    std::cout << a[i] << " ";
}
std::cout << std::endl;

cvector v2 ((std::complex<double>*) a, 3);
std::cout << v2;

prints

(1.00e+000,2.00e+000) (5.00e+000,6.00e+000)
(1.00e+000,2.00e+000) (9.99e+000,9.99e+000)

1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000 6.00e+000
(1.00e+000,2.00e+000) (3.00e+000,4.00e+000) (5.00e+000,6.00e+000)
```

2.4.6 cvector (const cvector&)

Copy constructor

```
cvector::cvector (const cvector& v);
```

creates a cvector object of size equal to size of vector v and sets every element of created vector to a value of appropriate element of v. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10.};  
const cvector v ((std::complex<double>*) a, 3, 2);  
cvector vc (v);
```

```
vc(1) = std::complex<double>(7.77,8.88);  
std::cout << vc;  
std::cout << v;
```

prints

```
(7.77e+00,8.88e+00) (5.00e+00,6.00e+00) (9.00e+00,1.00e+01)  
(1.00e+00,2.00e+00) (5.00e+00,6.00e+00) (9.00e+00,1.00e+01)
```

2.4.7 cvector (const TR*,const TR*,int,int)

Constructor

```
cvector::cvector (const TR* pRe, const TR* pIm, int nSize, int nIncr = 1);
```

creates a cvector object of size equal to nSize and copies every nIncr's element of arrays pointed to by pRe and pIm to the real and imaginary part of the object created. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double re[] = {1., 2., 3., 4., 5.};
double im[] = {5., 4., 3., 2., 1.};
cvector v (re, im, 3, 2);

std::cout << v;
re[0] = 7.77;
std::cout << v;

const double rec[] = {1., 2., 3.};
const double imc[] = {5., 4., 3.};
const cvector vc (rec, imc, 3);
std::cout << vc;

prints

(1.00e+00,5.00e+00) (3.00e+00,4.00e+00) (5.00e+00,3.00e+00)
(1.00e+00,5.00e+00) (3.00e+00,4.00e+00) (5.00e+00,3.00e+00)
(1.00e+00,5.00e+00) (2.00e+00,4.00e+00) (3.00e+00,3.00e+00)
```

2.4.8 cvector (const rvector&, const rvector&)

Constructor

```
cvector::cvector (const rvector& vRe, const rvector& vIm);
```

creates a cvector object of size equal to `vRe.size()` and `vIm.size()` and copies vectors `vRe` and `vIm` to the real and imaginary part of the object created. The constructor throws an exception of type `cvmexception` in case of non-equal sizes of the parameters passed or memory allocation failure. See also `cvector` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector vr(3), vi(3);
vr[1] = 1.;
vr[2] = 2.;
vr[3] = 3.;
vi[1] = 5.;
vi[2] = 4.;
vi[3] = 3.;

const cvector vc(vr, vi);
std::cout << vc;

prints

(1.00e+00,5.00e+00) (2.00e+00,4.00e+00) (3.00e+00,3.00e+00)
```

2.4.9 cvector (const TR*,int,bool,int)

Constructor

```
cvector::cvector (const TR* pA, int nSize,  
                 bool bRealPart = true, int nIncr = 1);
```

creates a cvector object of size equal to nSize and copies every nIncr's element of array pointed to by pA to the real (if bRealPart is true) or imaginary (if bRealPart is false) part of the object created. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
  
const double a[] = {1., 2., 3., 4., 5.};  
cvector v1 (a, 3, false, 2);  
cvector v2 (a, 2);  
  
std::cout << v1 << v2;  
  
prints  
  
(0.00e+00,1.00e+00) (0.00e+00,3.00e+00) (0.00e+00,5.00e+00)  
(1.00e+00,0.00e+00) (2.00e+00,0.00e+00)
```

2.4.10 cvector (const rvector&,bool)

Constructor

```
explicit cvector::cvector (const rvector& v, bool bRealPart = true);
```

creates a cvector object of size equal to `v.size()` and copies every element of a vector `v` to the real (if `bRealPart` is true) or imaginary (if `bRealPart` is false) part of the object created. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cvector` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector vr (3);
vr(1) = 1.;
vr(2) = 2.;
vr(3) = 3.;

cvector v1 (vr);
cvector v2 (vr, false);
std::cout << v1 << v2;

prints

(1.00e+00,0.00e+00) (2.00e+00,0.00e+00) (3.00e+00,0.00e+00)
(0.00e+00,1.00e+00) (0.00e+00,2.00e+00) (0.00e+00,3.00e+00)
```

2.4.11 `real`

Function

```
rvector cvector::real ();
```

creates a `rvector` object of size equal to a size of a calling vector sharing a memory with its real part. In other words, the vector returned is an *l-value*. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
cvector vc(3);
vc.set(std::complex<double>(1.,1.));
std::cout << vc << vc.real();
vc.real()(1) = 7.77;
std::cout << vc;
```

prints

```
(1.00e+00,1.00e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
1.00e+00 1.00e+00 1.00e+00
(7.77e+00,1.00e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
```

2.4.12 `imag`

function

```
rvector cvector::imag ();
```

creates a `rvector` object of size equal to a size of a calling vector sharing a memory with its imaginary part. In other words, the vector returned is an *l-value*. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
cvector vc(3);
vc.set(std::complex<double>(1.,1.));
std::cout << vc << vc.imag();
vc.imag()(1) = 7.77;
std::cout << vc;
```

prints

```
(1.00e+00,1.00e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
1.00e+00 1.00e+00 1.00e+00
(1.00e+00,7.77e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
```

2.4.13 operator = (const cvector&)

Operator

```
cvector& cvector::operator = (const cvector& v) throw (cvmexception);
```

sets every element of a calling vector to a value of appropriate element of a vector v and returns a reference to the vector changed. The operator throws an exception of type `cvmexception` in case of different vector sizes. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v(3);
    cvector vc(3);
    v(1) = std::complex<double>(1.,2.);

    vc = v;
    std::cout << vc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

(1.00e+00,2.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
```

2.4.14 assign(const TC*, int)

Function

```
cvector& cvector::assign (const TC* p, int nIncr = 1);
```

sets every element of a calling vector to a value of every nIncr-th element of an array pointed to by parameter p and returns a reference to the vector changed. See also [cvector](#).
Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5., 6., 7.};
cvector v1(3);
cvector v2(2);

v1.assign((const std::complex<double>*) a);
v2.assign((const std::complex<double>*) a, 2);
std::cout << v1;
std::cout << v2;

prints

(1.00e+00,2.00e+00) (3.00e+00,4.00e+00) (5.00e+00,6.00e+00)
(1.00e+00,2.00e+00) (5.00e+00,6.00e+00)
```

2.4.15 assign(int, const TC*, int)

Function

```
cvector& cvector::assign (int n, const TC* p, int nIncr = 1);
```

sets every element of a calling vector, starting from ℓ -based n -th one, to a value of every $nIncr$ -th element of an array pointed to by parameter p and returns a reference to the vector changed. See also [cvector](#). Example:

```
using namespace cvm;
```

```
const double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10.};  
cvector v1(4);  
cvector v2(4);
```

```
v1.assign(3, (const std::complex<double>*) a);  
v2.assign(2, (const std::complex<double>*) a, 2);  
std::cout << v1;  
std::cout << v2;
```

prints

```
(0,0) (0,0) (1,2) (3,4)  
(0,0) (1,2) (5,6) (9,10)
```

2.4.16 assign (int, const cvector&)

Function

```
cvector& cvector::assign (int n, const cvector& v) throw (cvmexception);
```

sets every element of a calling vector's sub-vector beginning with *l*-based index *n* to a vector *v* and returns a reference to the vector changed. The function throws an exception of type `cvmexception` if *n* is not positive or `v.size()+n-1` is greater than a calling vector's size. See also `cvector`. Example:

```
using namespace cvm;
```

```
cvector v1(5);  
cvector v2(2);  
v1.set(std::complex<double>(1.,1.));  
v2.set(std::complex<double>(2.,2.));  
v1.assign(3, v2);  
std::cout << v1;
```

prints

```
(1,1) (1,1) (2,2) (2,2) (1,1)
```

2.4.17 `set(TC)`

Function

```
cvector& cvector::set (TC x);
```

sets every element of a calling vector to a value of parameter `x` and returns a reference to the vector changed. See also `cvector`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
cvector v(3);  
v.set(std::complex<double>(3.,1.));  
std::cout << v;
```

prints

```
(3.00e+00,1.00e+00) (3.00e+00,1.00e+00) (3.00e+00,1.00e+00)
```

2.4.18 assign_real

Function

```
cvector& cvector::assign_real (const rvector& vRe) throw (cvmexception);
```

sets real part of every element of a calling vector to a value of appropriate element of a vector vRe and returns a reference to the vector changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. See also `cvector` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v(3);
cvector vc(3);
v(1) = 1.; v(2) = 2.; v(3) = 3.;

vc.assign_real(v);
std::cout << vc;

prints

(1.00e+00,0.00e+00) (2.00e+00,0.00e+00) (3.00e+00,0.00e+00)
```

2.4.19 assign_imag

Function

```
cvector& cvector::assign_imag (const rvector& vIm) throw (cvmexception);
```

sets imaginary part of every element of a calling vector to a value of appropriate element of a vector vIm and returns a reference to the vector changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. See also `cvector` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v(3);
cvector vc(3);
v(1) = 1.; v(2) = 2.; v(3) = 3.;

vc.assign_imag(v);
std::cout << vc;

prints

(0.00e+00,1.00e+00) (0.00e+00,2.00e+00) (0.00e+00,3.00e+00)
```

2.4.20 `set_real`

Function

```
cvector& cvector::set_real (TR x);
```

sets real part of every element of a calling vector to a value of parameter `x` and returns a reference to the vector changed. See also `cvector`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
cvector v(3);  
v.set_real(1.);  
std::cout << v;
```

prints

```
(1.00e+00,0.00e+00) (1.00e+00,0.00e+00) (1.00e+00,0.00e+00)
```

2.4.21 `set_imag`

Function

```
cvector& cvector::set_imag (TR x);
```

sets imaginary part of every element of a calling vector to a value of parameter `x` and returns a reference to the vector changed. See also `cvector`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
cvector v(3);  
v.set_imag(1.);  
std::cout << v;
```

prints

```
(0.00e+00,1.00e+00) (0.00e+00,1.00e+00) (0.00e+00,1.00e+00)
```

2.4.22 `resize`

Function

```
cvector& cvector::resize (int nNewSize) throw (cvmexception);
```

changes a size of a calling vector to be equal to `nNewSize` and returns a reference to the vector changed. In case of increasing of its size, the vector is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. See also `basic_array::resize` and `cvector`. Example:

```
using namespace cvm;
```

```
try {  
    double a[] = {1., 2., 3., 4.};  
    rvector v (a, 3);  
    std::cout << v;  
    v.resize(2);  
    std::cout << v;  
    v.resize(4);  
    std::cout << v;  
}  
catch (std::exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
(1,2) (3,4) (5,6)  
(1,2) (3,4)  
(1,2) (3,4) (0,0) (0,0)
```

2.4.23 operator ==

Operator

```
bool cvector::operator == (const cvector& v) const;
```

compares a calling vector with a vector *v* and returns true if they have the same sizes and their appropriate elements differ by not more than the **smallest normalized positive number**. Returns false otherwise. See also **cvector**. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4.};  
cvector v1 ((std::complex<double>*)a, 2);  
cvector v2 (2);
```

```
v2(1) = std::complex<double>(1.,2.);  
v2(2) = std::complex<double>(3.,4.);
```

```
std::cout << (v1 == v2) << std::endl;
```

prints

```
1
```

2.4.24 operator !=

Operator

```
bool cvector::operator != (const cvector& v) const;
```

compares a calling vector with a vector `v` and returns true if they have different sizes or some of their appropriate elements differ by more than the **smallest normalized positive number**. Returns false otherwise. See also **cvector**. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4.};  
cvector v1 ((std::complex<double>*)a, 2);  
cvector v2 (2);
```

```
std::cout << (v1 != v2) << std::endl;
```

prints

```
1
```

2.4.25 operator <<

Operator

```
cvector& cvector::operator << (const cvector& v) throw (cvmexception);
```

destroys a calling vector, creates a new one as a copy of `v` and returns a reference to the vector changed. See also `cvector`. The operator throws an exception of type `cvmexception` in case of memory allocation failure. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v(2);
    cvector vc(3);
    v(1) = std::complex<double> (1.,2.);
    v(2) = std::complex<double> (3.,4.);
    std::cout << v << vc << std::endl;

    vc << v;
    std::cout << vc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.00e+000,2.00e+000) (3.00e+000,4.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,0.00e+000)

(1.00e+000,2.00e+000) (3.00e+000,4.00e+000)
```

2.4.26 operator +

Operator

```
cvector cvector::operator + (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a sum of a calling vector and vector `v`. It throws an exception of type `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `cvector::sum`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    va.set(std::complex<double>(1.,1.));
    vb.set(std::complex<double>(2.,2.));

    std::cout << va + vb;
    std::cout << va + va;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.00e+000,3.00e+000) (3.00e+000,3.00e+000) (3.00e+000,3.00e+000)
(2.00e+000,2.00e+000) (2.00e+000,2.00e+000) (2.00e+000,2.00e+000)
```

2.4.27 operator -

Operator

```
cvector cvector::operator - (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a difference of a calling vector and vector `v`. It throws an exception of type `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `cvector::diff`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    va.set(std::complex<double> (1.,1.));
    vb.set(std::complex<double> (2.,2.));

    std::cout << va - vb;
    std::cout << va - va;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

(-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,0.00e+000)
```

2.4.28 sum

Function

```
cvector& cvector::sum (const cvector& v1, const cvector& v2)
throw (cvmexception);
```

assigns a result of addition of vectors v1 and v2 to a calling vector and returns a reference to the vector changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `cvector::operator +`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    cvector v(3);
    va.set(std::complex<double> (1.,1.));
    vb.set(std::complex<double> (2.,2.));

    std::cout << v.sum(va, vb);
    std::cout << v.sum(v, va);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
(3.00e+000,3.00e+000) (3.00e+000,3.00e+000) (3.00e+000,3.00e+000)
(4.00e+000,4.00e+000) (4.00e+000,4.00e+000) (4.00e+000,4.00e+000)
```

2.4.29 diff

Function

```
cvector& cvector::diff (const cvector& v1, const cvector& v2)
throw (cvmexception);
```

assigns a result of subtraction of vectors v1 and v2 to a calling vector and returns a reference to the vector changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `cvector::operator -`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    cvector v(3);
    va.set(std::complex<double> (1.,1.));
    vb.set(std::complex<double> (2.,2.));

    std::cout << v.diff(va, vb);
    std::cout << v.diff(v, va);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
(-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000)
(-2.00e+000,-2.00e+000) (-2.00e+000,-2.00e+000) (-2.00e+000,-2.00e+000)
```

2.4.30 operator +=

Operator

```
cvector& cvector::operator += (const cvector& v) throw (cvmexception);
```

adds to a calling vector a vector *v* and returns a reference to the vector changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `cvector::operator +`, `cvector::sum`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v1(3);
    cvector v2(3);
    v1.set(std::complex<double> (1.,1.));
    v2.set(std::complex<double> (2.,2.));

    v1 += v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 += v2;
    std::cout << v2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

(3.00e+000,3.00e+000) (3.00e+000,3.00e+000) (3.00e+000,3.00e+000)
(4.00e+000,4.00e+000) (4.00e+000,4.00e+000) (4.00e+000,4.00e+000)
```

2.4.31 operator -=

Operator

```
cvector& cvector::operator -= (const cvector& v) throw (cvmexception);
```

subtracts from a calling vector a vector *v* and returns a reference to the vector changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `cvector::operator -`, `cvector::diff`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v1(3);
    cvector v2(3);
    v1.set(std::complex<double> (1.,1.));
    v2.set(std::complex<double> (2.,2.));

    v1 -= v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 -= v2;
    std::cout << v2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

(-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,0.00e+000)
```

2.4.32 operator - ()

Operator

```
cvector cvector::operator - () const throw (cvmexception);
```

creates an object of type `cvector` as a calling vector multiplied by -1 . It throws an exception of type `cvmexception` in case of memory allocation failure. See also `cvector`.
Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};  
const cvector v ((std::complex<double>*) a, 2);
```

```
std::cout << - v;
```

prints

```
(-1.00e+000,-2.00e+000) (-3.00e+000,-4.00e+000)
```

2.4.33 operator * (TR)

Operator

```
cvector cvector::operator * (TR d) const;
```

creates an object of type cvector as a product of a calling vector and a number d. See also [cvector](#). Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};  
const cvector v ((std::complex<double>*) a, 2);
```

```
std::cout << v * 5.;
```

prints

```
(5.00e+000,1.00e+001) (1.50e+001,2.00e+001)
```

2.4.34 operator / (TR)

Operator

```
cvector cvector::operator / (TR d) const throw (cvmexception);
```

creates an object of type `cvector` as a quotient of a calling vector and a number `d`. The operator throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the `smallest normalized positive number`. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);
```

```
std::cout << v / 4.;
```

prints

```
(2.50e-001,5.00e-001) (7.50e-001,1.00e+000)
```

2.4.35 operator * (TC)

Operator

```
cvector cvector::operator * (TC c) const;
```

creates an object of type `cvector` as a product of a calling vector and a complex number `c`. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);
```

```
std::cout << v * std::complex<double>(1.,1.);
```

prints

```
(-1.00e+000,3.00e+000) (-1.00e+000,7.00e+000)
```

2.4.36 operator / (TC)

Operator

```
cvector cvector::operator / (TC c) const  
throw (cvmexception);
```

creates an object of type `cvector` as a quotient of a calling vector and a complex number `c`. The operator throws an exception of type `cvmexception` if `c` has an absolute value equal or less than the `smallest normalized positive number`. See also `cvector`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};  
const cvector v ((std::complex<double>*) a, 2);
```

```
std::cout << v / std::complex<double>(1.,1.);
```

prints

```
(1.50e+000,5.00e-001) (3.50e+000,5.00e-001)
```

2.4.37 operator *= (TR)

Operator

```
cvector& cvector::operator *= (TR d);
```

multiplies a calling vector by real number *d* and returns a reference to the vector changed. See also `cvector::operator *`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);
```

```
std::cout << (v *= 2.);
```

prints

```
(2.00e+000,4.00e+000) (6.00e+000,8.00e+000)
```

2.4.38 operator /= (TR)

Operator

```
cvector& cvector::operator /= (TR d) throw (cvmexception);
```

divides a calling vector by real number *d* and returns a reference to the vector changed. It throws an exception of type `cvmexception` if *d* has an absolute value equal or less than the `smallest normalized positive number`. See also `cvector::operator /`, `cvector`.
Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

std::cout << (v /= 2.);

prints

(5.00e-001,1.00e+000) (1.50e+000,2.00e+000)
```

2.4.39 operator *= (TC)

Operator

```
cvector& cvector::operator *= (TC c);
```

multiplies a calling vector by complex number *c* and returns a reference to the vector changed. See also [cvector::operator *](#), [cvector](#). Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);
```

```
v *= std::complex<double>(1.,1.);
std::cout << v;
```

prints

```
(-1.00e+000,3.00e+000) (-1.00e+000,7.00e+000)
```

2.4.40 operator /= (TC)

Operator

```
cvector& cvector::operator /= (TC c) throw (cvmexception);
```

divides a calling vector by complex number `c` and returns a reference to the vector changed. It throws an exception of type `cvmexception` if `c` has an absolute value equal or less than the `smallest normalized positive number`. See also `cvector::operator /`, `cvector`.
Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);
```

```
v /= std::complex<double>(1.,1.);
std::cout << v;
```

prints

```
(1.50e+000,5.00e-001) (3.50e+000,5.00e-001)
```

2.4.41 normalize

Function

```
cvector& cvector::normalize ();
```

normalizes a calling vector so its **Euclidean norm** becomes equal to 1 if it was greater than the **smallest normalized positive number** before the call (otherwise the function nothing). See also **cvector**. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);
```

```
std::cout << v.normalize();
std::cout << v.norm() << std::endl;
```

prints

```
(1.83e-01,3.65e-01) (5.48e-01,7.30e-01)
1.00e+00
```

2.4.42 conjugation

Operator and functions

```
cvector  cvector::operator ~ () const throw (cvmexception);
cvector& cvector::conj (const cvector& v) throw (cvmexception);
cvector& cvector::conj ();
```

encapsulate complex conjugation. First operator creates an object of type `cvector` as a complex conjugated calling vector (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets calling vector to be equal to vector `v` conjugated (it throws an exception of type `cvmexception` in case of different sizes of the operands), third one makes it to be equal to conjugated itself. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);
cvector vc(2);

std::cout << ~v;
std::cout << vc.conj(v);
std::cout << vc.conj();
```

prints

```
(1.00e+00,-2.00e+00) (3.00e+00,-4.00e+00)
(1.00e+00,-2.00e+00) (3.00e+00,-4.00e+00)
(1.00e+00,2.00e+00) (3.00e+00,4.00e+00)
```

2.4.43 operator * (const cvector&)

Operator

TC `cvector::operator * (const cvector& v) const throw (cvmexception);`

returns a scalar product of a calling vector and `v`. The operator throws an exception of type `cvmexception` if the operands have different sizes. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
double b[] = {1., -1., 1., 2.};
const cvector v1((std::complex<double>*) a, 2);
const cvector v2((std::complex<double>*) b, 2);

std::cout << v1 * v2 << std::endl;

prints

(-2.00e+00,1.10e+01)
```

2.4.44 operator %

Operator

TC `cvector::operator % (const cvector& v) const throw (cvmexception);`

returns a scalar product of a complex conjugated calling vector and v. The operator throws an exception of type `cvmexception` if the operands have different sizes. See also `cvector`.
Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4.};
double b[] = {1., -1., 1., 2.};
const cvector v1((std::complex<double>*) a, 2);
const cvector v2((std::complex<double>*) b, 2);
```

```
std::cout << v1 % v2 << std::endl;
std::cout << ~v1 * v2 << std::endl;
```

prints

```
(1.00e+01,-1.00e+00)
(1.00e+01,-1.00e+00)
```

2.4.45 operator * (const cmatrix&)

Operator

```
cvector cvector::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling vector and a matrix `m`. The operator throws an exception of type `cvmexception` if the calling vector's size is different from the number of rows of matrix `m`. See also `cvector::mult (const cvector&, const cmatrix&)`, `cvector`, `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 3., 2., 1.};
double b[] = {1., -1., 1., 2., -2., 1.,
              3., -2., 1., 2., -1., 3.};
const cvector v((std::complex<double>*) a, 3);
const cmatrix m((std::complex<double>*) b, 3, 2);

std::cout << v << m << std::endl << v * m;

prints

(1.00e+00,2.00e+00) (3.00e+00,3.00e+00) (2.00e+00,1.00e+00)
(1.00e+00,-1.00e+00) (3.00e+00,-2.00e+00)
(1.00e+00,2.00e+00) (1.00e+00,2.00e+00)
(-2.00e+00,1.00e+00) (-1.00e+00,3.00e+00)

(-5.00e+00,1.00e+01) (-1.00e+00,1.80e+01)
```

2.4.46 mult (const cvector&, const cmatrix&)

Function

```
cvector& cvector::mult (const cvector& v, const cmatrix& m)
throw (cvmexception);
```

sets a calling vector to be equal to a product of a vector *v* by a matrix *m* and returns the reference to the object changed. The function throws an exception of type `cvmexception` if case of inappropriate sizes of the operands. See also `cvector::mult (const cmatrix&, const cvector&)`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 1., 2., 3.};
    double b[] = {1., -1., 1., -1., 1., -1.,
                 2., -1., 2., -1., 2., -1.};
    const cvector v ((std::complex<double>*) a, 3);
    const cmatrix m ((std::complex<double>*) b, 3, 2);
    const scmatrix sm ((std::complex<double>*) b, 2);
    cvector vm (2);

    std::cout << vm.mult(v, m) << std::endl;
    std::cout << sm << std::endl;
    std::cout << vm.mult(vm, sm);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.20e+01,0.00e+00) (1.80e+01,6.00e+00)

(1.00e+00,-1.00e+00) (1.00e+00,-1.00e+00)
(1.00e+00,-1.00e+00) (2.00e+00,-1.00e+00)

(3.60e+01,-2.40e+01) (5.40e+01,-1.80e+01)
```

2.4.47 mult (const cmatrix&, const cvector&)

Function

```
cvector& cvector::mult (const cmatrix& m, const cvector& v)
throw (cvmexception);
```

sets a calling vector to be equal to a product of a matrix `m` by a vector `v` and returns a reference to the object changed. The function throws an exception of type `cvmexception` if case of inappropriate sizes of the operands. See also `cvector::mult (const cvector&, const cmatrix&)`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 1., 2., 3.};
    double b[] = {1., -1., 1., -1., 1., -1.,
                  2., -1., 2., -1., 2., -1.};
    const cvector v ((std::complex<double>*) a, 3);
    const cmatrix m ((std::complex<double>*) b, 2, 3);
    const scmatrix sm ((std::complex<double>*) b, 2);
    cvector vm (2);

    std::cout << vm.mult(m, v) << std::endl;
    std::cout << sm << std::endl;
    std::cout << vm.mult(vm, sm);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.40e+01,3.00e+00) (1.70e+01,4.00e+00)

(1.00e+00,-1.00e+00) (1.00e+00,-1.00e+00)
(1.00e+00,-1.00e+00) (2.00e+00,-1.00e+00)

(3.80e+01,-2.40e+01) (5.50e+01,-2.00e+01)
```

2.4.48 ranklupdate_u

Function

```
cmatrix cvector::ranklupdate_u (const cvector& v) const;
```

creates an object of type `cmatrix` as a rank-1 update (unconjugated) of a calling vector and a vector `v`. The rank-1 update (unconjugated) operation of a vector-column x of a size m and a vector-row y of a size n is defined as $m \times n$ matrix

$$\begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ x_m y_1 & x_m y_2 & \cdots & x_m y_n \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} (y_1 \ y_2 \ \cdots \ y_n)$$

See also `cvector`, `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., -2., -1., 1.};
double b[] = {4., 5., 3., 2.};
cvector v1((std::complex<double>*) a, 3);
cvector v2((std::complex<double>*) b, 2);

std::cout << v1.ranklupdate_u (v2);
```

prints

```
(-6.00e+00,1.30e+01) (-1.00e+00,8.00e+00)
(2.20e+01,7.00e+00) (1.30e+01,0.00e+00)
(-9.00e+00,-1.00e+00) (-5.00e+00,1.00e+00)
```

2.4.49 ranklupdate_c

Function

```
cmatrix cvector::ranklupdate_c (const cvector& v) const;
```

creates an object of type `cmatrix` as a rank-1 update (conjugated) of a calling vector and a complex conjugated vector `v`. The rank-1 update (conjugated) operation of a vector-column x of a size m and a complex conjugated vector-column y of a size n is defined as $m \times n$ matrix

$$\begin{pmatrix} x_1 y_1^* & x_1 y_2^* & \cdots & x_1 y_n^* \\ x_2 y_1^* & x_2 y_2^* & \cdots & x_2 y_n^* \\ \dots & \dots & \dots & \dots \\ x_m y_1^* & x_m y_2^* & \cdots & x_m y_n^* \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \text{conj} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} (y_1^* \ y_2^* \ \cdots \ y_n^*),$$

where y_i^* is i -th complex conjugated element of y . See also `cvector`, `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., -2., -1., 1.};
double b[] = {4., 5., 3., 2.};
cvector v1((std::complex<double>*) a, 3);
cvector v2((std::complex<double>*) b, 2);

std::cout << v1.ranklupdate_c (v2) << std::endl;
std::cout << v1.ranklupdate_u (~v2);
```

prints

```
(1.40e+01,3.00e+00) (7.00e+00,4.00e+00)
(2.00e+00,-2.30e+01) (5.00e+00,-1.20e+01)
(1.00e+00,9.00e+00) (-1.00e+00,5.00e+00)

(1.40e+01,3.00e+00) (7.00e+00,4.00e+00)
(2.00e+00,-2.30e+01) (5.00e+00,-1.20e+01)
(1.00e+00,9.00e+00) (-1.00e+00,5.00e+00)
```

2.4.50 solve

Functions

```

cvector&
cvector::solve (const scmatrix& mA,
                const cvector& vB, TR& dErr) throw (cvmexception);

cvector&
cvector::solve (const scmatrix& mA,
                const cvector& vB) throw (cvmexception);

```

set a calling vector to be equal to a solution x of a linear equation $Ax = b$ where parameter `mA` is the square matrix A and parameter `vB` is the vector b . Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to a norm of computation error. These functions throw exception of type `cvmexception` in case of inappropriate sizes of the objects or when the matrix A is close to singular. See also `cvector`, `scmatrix`. Example:

```

using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b[] = {1., 2., 5., -3.};
    scmatrix ma((std::complex<double>*) m, 2);
    cvector vb((std::complex<double>*) b, 2);
    cvector vx(2);
    double dErr = 0.;

    std::cout << vx.solve (ma, vb, dErr);
    std::cout << dErr << std::endl;
    std::cout << ma * vx - vb;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

```

prints

```

(3.5200000e+00,6.4000000e-01) (2.2400000e+00,-1.3200000e+00)
3.2788531e-15
(-7.7715612e-16,4.4408921e-16) (0.0000000e+00,0.0000000e+00)

```

2.4.51 solve_lu

Functions

```
cvector&
cvector::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                  const int* pPivots, const cvector& vB, TR& dErr)
                  throw (cvmexception);
```

```
cvector&
cvector::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                  const int* pPivots, const cvector& vB)
                  throw (cvmexception);
```

set a calling vector to be equal to a solution x of a linear equation $Ax = b$ where parameter `mA` is the square matrix A , parameter `mLU` is [LU factorization](#) of the matrix A , parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and parameter `vB` is the vector b . Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $Ax = b$ with the same matrix A and different vectors b . In such case you save on matrix A factorization since it's needed to be performed just one time. These functions throw exception of type [cvmexception](#) in case of inappropriate sizes of the objects or when the matrix A is close to singular. See also [cvector](#), [scmatrix](#). Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b1[] = {1., 2., 5., -3.};
    double b2[] = {3., -1., 1., 7.};
    scmatrix ma((std::complex<double>*) m, 2);
    scmatrix mLU(2);
    cvector vb1((std::complex<double>*) b1, 2);
    cvector vb2((std::complex<double>*) b2, 2);
    cvector vx1(2);
    cvector vx2(2);
    iarray nPivots(2);
    double dErr = 0.;

    mLU.low_up(ma, nPivots);
    std::cout << vx1.solve_lu (ma, mLU, nPivots, vb1, dErr);
    std::cout << dErr << std::endl;
```

```
    std::cout << vx2.solve_lu (ma, mLU, nPivots, vb2);
    std::cout << ma * vx1 - vb1 << ma * vx2 - vb2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

(3.5200000e+00,6.4000000e-01) (2.2400000e+00,-1.3200000e+00)
3.2788531e-15
(2.2800000e+00,1.9600000e+00) (3.6000000e-01,5.2000000e-01)
(-7.7715612e-16,4.4408921e-16) (0.0000000e+00,0.0000000e+00)
(-8.8817842e-16,0.0000000e+00) (-2.2204460e-16,0.0000000e+00)
```

2.4.52 gels

Function

```
cvector& cvector::gels (bool transpose,
                       const cmatrix& mA, const cvector& mB,
                       TC& cErr) throw (cvmexception);
```

This function solves overdetermined or underdetermined linear system

$$A * x = b$$

for $m \times n$ matrix A (or its conjugate-transpose) where b is a vector of length k and $k = m$ in non-transposed case and $k = n$ otherwise. The algorithm uses a QR or LQ factorization of A . It is assumed that A has full rank, infinity returned otherwise. Internally the function uses ?GELS LAPACK routines. If $m > n$ and `transpose=false` or $m < n$ and `transpose=true`, then the system is overdetermined, thus the algorithm tries to find the least squares solution x of the problem

$$\|A * x - b\|_2 \rightarrow \min \quad \text{or} \quad \|A^H * x - b\|_2 \rightarrow \min$$

respectively. Complex number `cErr` returns residual sum of squares. The system is underdetermined otherwise, and the algorithm finds its minimum norm solution. In this case `cErr` is set to zero. In both cases the solution computed satisfies $x = \text{pinv}(A) * b$, but this algorithm is faster than pseudo inversion. The function sets calling object to be the solution and returns reference to it. It throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cmatrix::gels`. Example:

```
using namespace cvm;
```

```
cvm::cmatrix a(7, 5);
cvm::cvector b(7), bt(5);
tcomplex cErr;
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
bt.randomize_real(-10., 10.);
bt.randomize_imag(-10., 10.);
```

```
cvm::cvector x(5);
x.gels(false, a, b, cErr);
cvm::cvector xt(7);
xt.gels(true, a, bt, cErr);
```

```
std::cout << (a.pinv()*b - x).norm() << std::endl;
std::cout << (~a.pinv()*bt - xt).norm() << std::endl;
```

prints

+1.4506106e-015

+1.1852313e-015

2.4.53 gelsy

Function

```

cvvector& cvvector::gelsy (const cmatrix& mA, const cvvector& mB, int& rank,
                          TR tol = cvmMachSp()) throw (cvmexception);

```

These functions compute the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using a complete orthogonal factorization of $m \times n$ matrix A . Here b is a vector of length m . Matrix A may be rank-deficient, the functions return its effective rank in rank output parameter using tol tolerance. Internally the function uses ?GELSY LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as an argument mA . The function sets calling object to be the solution and returns reference to it. It throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `cvvector`, `cvvector::gelss`, `cvvector::gelsd`, `cmatrix::gelsy`. Example:

```
using namespace cvm;
```

```

cmatrix a(4, 5);
cvvector b(4);
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
cvvector x(5);
tint rank;

x.gelsy(a, b, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;

```

prints

```

+6.0116296e-015
+4 +4

```

2.4.54 gelss

Function

```
cvector& cvector::gelss (const cmatrix& mA, const cvector& mB, int& rank,
                        rvector& sv, TR tol = cvmMachSp())
                        throw (cvmexception);
```

computes the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using the singular value decomposition of $m \times n$ matrix A . Here b is a vector of length m . Matrix A may be rank-deficient, the functions return its effective rank in `rank` output parameter using `tol` tolerance. This function also computes singular values of A in decreasing order and returns them in `sv` output parameter having $\min(m, n)$ size. Internally the function uses ?GELSS LAPACK routines, see more details about the algorithm in that routine's documentation. Matrix A is passed as an argument `mA`. The function sets calling object to be the solution and returns reference to it. It throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cvector::gelsy`, `cvector::gelsd`, `cmatrix::gelss`. Example:

```
using namespace cvm;

cvm::cmatrix a(4, 5);
cvm::cvector b(4);
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
cvm::rvector sv(4);
cvm::cvector x(5);
tint rank;

x.gelss(a, b, sv, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+9.4127010e-015
+4.4408921e-015
+4 +4
```

2.4.55 gelsd

Function

```
cvector& cvector::gelsd (const cmatrix& mA, const cvector& mB, int& rank,
                        rvector& sv, TR tol = cvmMachSp())
                        throw (cvmexception);
```

computes the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using the singular value decomposition of $m \times n$ matrix A and a divide and conquer method. Here b is a vector of length m . Matrix A may be rank-deficient, the functions return its effective rank in `rank` output parameter using `tol` tolerance. This function also computes singular values of A in decreasing order and returns them in `sv` output parameter having $\min(m, n)$ size. Internally the function uses ?GELSD LAPACK routines, see more details about the algorithm in that routine's documentation. Matrix A is passed as an argument `mA`. The function sets calling object to be the solution and returns reference to it. It throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cvector::gelsy`, `cvector::gelss`, `cmatrix::gelsd`. Example:

```
using namespace cvm;

cvm::cmatrix a(4, 5);
cvm::cvector b(4);
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
cvm::rvector sv(4);
cvm::cvector x(5);
tint rank;

x.gelsd(a, b, sv, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;

prints

+1.3491837e-014
+5.3290705e-015
+4 +4
```

2.4.56 eig

Functions

```
cvector&
cvector::eig (const srmatrix& mArg) throw (cvmexception);
cvector&
cvector::eig (const scmatrix& mArg) throw (cvmexception);
```

```
cvector&
cvector::eig (const srmatrix& mArg,
              scmatrix& mEigVect) throw (cvmexception);
cvector&
cvector::eig (const scmatrix& mArg,
              scmatrix& mEigVect) throw (cvmexception);
```

solve a nonsymmetric eigenvalue problem and set a calling vector to be equal to eigenvalues of a square matrix `mArg`. The nonsymmetric eigenvalue problem is defined as follows: given a nonsymmetric (or non-Hermitian) matrix A , find the eigenvalues λ and the corresponding eigenvectors z that satisfy the equation

$$Az = \lambda z.$$

Some eigenvalues may be complex even for real matrix A . Moreover, if a real nonsymmetric matrix has a complex eigenvalue $a + bi$ corresponding to an eigenvector z , then $a - bi$ is also an eigenvalue. The eigenvalue $a - bi$ corresponds to the eigenvector whose elements are complex conjugate to the elements of z . The third and fourth versions of the functions set an output parameter `mEigVect` to be equal to a square matrix containing eigenvectors as columns. All the functions return a reference to the vector they change and throw an exception of type `cvmexception` in case of inappropriate calling object sizes or in case of convergence error. See also `cvector`, `rvector::eig`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    srmatrix m(3);
    scmatrix me(3);
    cvector vl(3);

    m(1,1) = 0.1;  m(1,2) = 0.2;  m(1,3) = 0.1;
    m(2,1) = 0.11; m(2,2) = 2.9; m(2,3) = -8.4;
    m(3,1) = 0.;  m(3,2) = 2.91; m(3,3) = 8.2;
```

```

std::cout << vl.eig (m, me);
std::cout << scmatrix(m) * me(1) - me(1) * vl(1);
std::cout << scmatrix(m) * me(2) - me(2) * vl(2);
std::cout << scmatrix(m) * me(3) - me(3) * vl(3);

scmatrix mc(m);
mc.randomize_imag(-1., 1.);

std::cout << std::endl << vl.eig (mc, me);
std::cout << mc * me(1) - me(1) * vl(1);
std::cout << mc * me(2) - me(2) * vl(2);
std::cout << mc * me(3) - me(3) * vl(3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

(9.69e-02,0.00e+00) (5.55e+00,4.17e+00) (5.55e+00,-4.17e+00)
(0.00e+00,0.00e+00) (8.46e-18,0.00e+00) (7.59e-18,0.00e+00)
(5.55e-17,-2.78e-17) (8.88e-16,1.78e-15) (-1.33e-15,-1.33e-15)
(5.55e-17,2.78e-17) (8.88e-16,-1.78e-15) (-1.33e-15,1.33e-15)

(3.57e-02,-8.47e-01) (6.22e+00,4.76e+00) (4.95e+00,-3.90e+00)
(5.55e-17,0.00e+00) (1.18e-16,1.35e-16) (2.36e-16,6.76e-16)
(3.33e-16,0.00e+00) (-1.55e-15,-3.55e-15) (-4.44e-16,0.00e+00)
(0.00e+00,-3.33e-16) (-1.78e-15,-2.22e-15) (2.22e-16,1.55e-15)

```

2.4.57 gemv

Function

```
cvector& cvector::gemv (bool bLeft, const cmatrix& m, TC dAlpha,
                      const cvector& v, TC dBeta) throw (cvmexception);
```

calls one of ?GEMV routines of the [BLAS library](#) performing a matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are complex numbers (parameters dAlpha and dBeta), M is a complex matrix (parameter m) and v and c are complex vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. The function returns a reference to the vector changed and throws an exception of type `cvmexception` in case of inappropriate calling object sizes. See also `cvector`, `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    std::complex<double> alpha = std::complex<double>(1.3,-0.7);
    std::complex<double> beta  = std::complex<double>(0.15,-1.09);
    cmatrix m(3,2);
    cvector c(3);
    cvector v(2);
    m.randomize_real(-1., 2.); m.randomize_imag(0., 1.);
    v.randomize_real(-1., 3.); v.randomize_imag(2., 4.);
    c.randomize_real(0., 2.); c.randomize_imag(3., 7.);
    std::cout << m * v * alpha + c * beta;
    std::cout << c.gemv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gemv(true, m, alpha, c, beta);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
(2.71e-01,2.44e+00) (2.20e+01,7.16e+00) (-7.89e-01,2.45e+00)
(2.71e-01,2.44e+00) (2.20e+01,7.16e+00) (-7.89e-01,2.45e+00)
(5.92e+01,-1.47e+01) (3.54e+01,-3.14e+00)
(5.92e+01,-1.47e+01) (3.54e+01,-3.14e+00)
```

2.4.58 gbmv

Function

```
cvector& cvector::gbmv (bool bLeft, const scbmatrix& m, TC dAlpha,
                      const cvector& v, TC dBeta) throw (cvmexception);
```

calls one of ?GBMV routines of the [BLAS library](#) performing a matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are complex numbers (parameters dAlpha and dBeta), M is a complex band matrix (parameter m) and v and c are complex vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. The function returns a reference to the vector changed and throws an exception of type `cvmexception` in case of inappropriate calling object sizes. See also [cvector](#), [scbmatrix](#). Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    std::complex<double> alpha = std::complex<double>(1.3,-0.7);
    std::complex<double> beta  = std::complex<double>(0.15,-1.09);
    scbmatrix m(3,1,0);
    cvector c(3);
    cvector v(3);
    m.randomize_real(-1., 2.); m.randomize_imag(0., 1.);
    v.randomize_real(-1., 3.); v.randomize_imag(2., 4.);
    c.randomize_real(0., 2.); c.randomize_imag(3., 7.);
    std::cout << m * v * alpha + c * beta;
    std::cout << c.gbmv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gbmv(true, m, alpha, c, beta);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
(3.73e+00,7.96e-01) (6.89e+00,1.07e+01) (2.16e+00,3.28e+00)
(3.73e+00,7.96e-01) (6.89e+00,1.07e+01) (2.16e+00,3.28e+00)
(3.11e+01,2.51e+01) (-4.93e+00,1.34e+01) (1.70e+00,3.93e+00)
(3.11e+01,2.51e+01) (-4.93e+00,1.34e+01) (1.70e+00,3.93e+00)
```

2.4.59 `randomize_real`

Function

```
cvector& cvector::randomize_real (TR dFrom, TR dTo);
```

fills a real part of a calling vector with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the vector changed. See also `cvector`.

Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.randomize_real(-2.,3.);
std::cout << v;

prints

(-4.93e-01,0.00e+00) (1.37e+00,0.00e+00) (-1.49e-01,0.00e+00)
```

2.4.60 `randomize_imag`

Function

```
cvector& cvector::randomize_imag (TR dFrom, TR dTo);
```

fills an imaginary part of a calling vector with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the vector changed. See also [cvector](#). Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.randomize_imag(-2.,3.);
std::cout << v;

prints

(0.00e+00,-4.37e-01) (0.00e+00,-1.59e+00) (0.00e+00,2.42e+00)
```

2.5 Matrix

This base class contains member functions common for all matrices. This class is not designed to be instantiated.

```
template <typename TR, typename TC>
class Matrix : public Array<TR,TC> {
public:
    int msize () const;
    int nsize () const;
    int ld () const;
    int rowofmax () const;
    int rowofmin () const;
    int colofmax () const;
    int colofmin () const;
    virtual TR norm1 () const;
    <typename TR, typename TC>
    friend std::ostream& operator << <> (std::ostream& os,
                                           const Array<TR,TC>& aOut);
};
```

2.5.1 msize

Function

```
int Matrix<TR,TC>::msize () const;
```

returns a number of rows of a calling matrix. The function is *inherited* in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;
```

```
rmatrix m (100, 200);  
std::cout << m.msize() << std::endl;
```

prints

```
100
```

2.5.2 nsize

Function

```
int Matrix<TR,TC>::nsize () const;
```

returns a number of columns of a calling matrix. The function is *inherited* in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;
```

```
rmatrix m (100, 200);
```

```
std::cout << m.nsize() << std::endl;
```

prints

```
200
```

2.5.3 ld

Function

```
int Matrix<TR,TC>::ld () const;
```

returns a leading dimension of a calling matrix. Leading dimension is equal to a number of rows for every matrix except submatrices. For submatrices it's equal to a number of rows of parent matrix. The function is *inherited* in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;
```

```
rmatrix m (100, 200);  
srmatrix ms (m, 30, 40, 5); // 5x5 submatrix  
std::cout << ms.ld() << std::endl;
```

prints

```
100
```

2.5.4 rowofmax

Function

```
int Matrix<TR,TC>::rowofmax () const;
```

returns a *ℓ*-based number of a row of a calling matrix where the element with the maximum absolute value is located. The function is *inherited*⁵ in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., -3., 1., -1.};  
rmatrix m (a, 2, 3);
```

```
std::cout << m << std::endl << m.rowofmax() << std::endl;
```

prints

```
1 2 1  
0 -3 -1
```

```
2
```

⁵Calls *virtual function* inside

2.5.5 rowofmin

Function

```
int Matrix<TR,TC>::rowofmin () const;
```

returns a *ℓ*-based number of a row of a calling matrix where the element with the minimum absolute value is located. The function is *inherited*⁶ in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., -3., 1., -1.};  
rmatrix m (a, 2, 3);
```

```
std::cout << m << std::endl << m.rowofmin() << std::endl;
```

prints

```
1 2 1  
0 -3 -1
```

```
2
```

⁶Calls `virtual function` inside

2.5.6 colofmax

Function

```
int Matrix<TR,TC>::colofmax () const;
```

returns a *ℓ*-based number of a column of a calling matrix where the element with the maximum absolute value is located. The function is *inherited*⁷ in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., -3., 1., -1.};  
rmatrix m (a, 2, 3);
```

```
std::cout << m << std::endl << m.colofmax() << std::endl;
```

prints

```
1 2 1  
0 -3 -1
```

```
2
```

⁷Calls *virtual function* inside

2.5.7 colofmin

Function

```
int Matrix<TR,TC>::colofmin () const;
```

returns a *ℓ*-based number of a column of a calling matrix where the element with the minimum absolute value is located. The function is *inherited*⁸ in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., -3., 1., -1.};  
rmatrix m (a, 2, 3);
```

```
std::cout << m << std::endl << m.colofmin() << std::endl;
```

prints

```
1 2 1  
0 -3 -1
```

```
1
```

⁸Calls *virtual function* inside

2.5.8 norm1

Virtual function

```
virtual TR Matrix<TR,TC>::norm1 () const;
```

returns a 1-norm of a calling matrix that is defined as

$$\|A\|_1 = \max_{j=1,\dots,n} \sum_{i=1}^m |a_{ij}|,$$

where A is $m \times n$ matrix. The function is *inherited* in the following classes of the library: `rmatrix`, `cmatrix`, `smatrix`, `scmatrix`, `srmatrix` and `schmatrix`. It's *redefined* in `srbmatrix` and `scbmatrix`. See also `Array::norminf` and `Matrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., -3., 1., 0.};
rmatrix m (a, 2, 3);
```

```
std::cout << m << std::endl << m.norm1()
          << std::endl << m.norminf() << std::endl;
```

prints

```
1 2 1
0 -3 0
```

```
5
4
```

2.5.9 operator << >> (std::ostream& os, const Matrix<TR,TC>& aOut)

Friend template operator

```
template <typename TR, typename TC>
friend std::ostream& operator << >> (std::ostream& os,
                                       const Matrix<TR,TC>& aOut);
```

outputs a matrix referenced by parameter aOut into os stream. See also [Array::operator << , Matrix](#). Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
srmatrix m(3);
m(1,1) = 1.;
m(2,3) = 3.;
```

```
std::cout << m;
```

prints

```
1.00e+00 0.00e+00 0.00e+00
0.00e+00 0.00e+00 3.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.6 rmatrix

This is end-user class encapsulating a matrix in Euclidean space of real numbers.

```

template <typename TR>
class rmatrix : public Matrix <TR,TR> {
public:
    rmatrix ();
    rmatrix (int nM, int nN);
    rmatrix (TR* pD, int nM, int nN);
    rmatrix (const TR* pD, int nM, int nN);
    rmatrix (const rmatrix& m);
    explicit rmatrix (const rvector& v, bool bBeColumn = true);
    rmatrix (rmatrix& m, int nRow, int nCol, int nHeight, int nWidth);
    TR& operator () (int im, int in) throw (cvmexception);
    TR operator () (int im, int in) const throw (cvmexception);
    rvector operator () (int i) throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    rvector operator [] (int i) throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    rvector diag (int i) throw (cvmexception);
    const rvector diag (int i) const throw (cvmexception);
    rmatrix& operator = (const rmatrix& m) throw (cvmexception);
    rmatrix& assign (const rvector& v);
    rmatrix& assign (const TR* pD);
    rmatrix& assign (int nRow, int nCol, const rmatrix& m)
        throw (cvmexception);
    rmatrix& set (TR x);
    rmatrix& resize (int nNewM, int nNewN) throw (cvmexception);
    bool operator == (const rmatrix& m) const;
    bool operator != (const rmatrix& m) const;
    rmatrix& operator << (const rmatrix& m) throw (cvmexception);
    rmatrix operator + (const rmatrix& m) const
        throw (cvmexception);
    rmatrix operator - (const rmatrix& m) const
        throw (cvmexception);
    rmatrix& sum (const rmatrix& m1,
        const rmatrix& m2) throw (cvmexception);
    rmatrix& diff (const rmatrix& m1,
        const rmatrix& m2) throw (cvmexception);
    rmatrix& operator += (const rmatrix& m) throw (cvmexception);
    rmatrix& operator -= (const rmatrix& m) throw (cvmexception);
    rmatrix operator - () const;

```

```

rmatrix operator * (TR d) const;
rmatrix operator / (TR d) const
    throw (cvmexception);
rmatrix& operator *= (TR d);
rmatrix& operator /= (TR d) throw (cvmexception);
rmatrix& normalize ();
rmatrix operator ~ () const throw (cvmexception);
rmatrix& transpose (const rmatrix& m) throw (cvmexception);
rmatrix& transpose () throw (cvmexception);
rvector operator * (const rvector& v) const
    throw (cvmexception);
rmatrix operator * (const rmatrix& m) const
    throw (cvmexception);
rmatrix& mult (const rmatrix& m1, const rmatrix& m2)
    throw (cvmexception);
rmatrix& ranklupdate (const rvector& vCol,
    const rvector& vRow)
    throw (cvmexception);
rmatrix& swap_rows (int n1, int n2) throw (cvmexception);
rmatrix& swap_cols (int n1, int n2) throw (cvmexception);
rmatrix& solve (const srmatrix& mA,
    const rmatrix& mB, TR& dErr)
    throw (cvmexception);
rmatrix& solve (const srmatrix& mA,
    const rmatrix& mB) throw (cvmexception);
rmatrix& solve_lu (const srmatrix& mA, const srmatrix& mLU,
    const int* pPivots, const rmatrix& mB, TR& dErr)
    throw (cvmexception);
rmatrix& solve_lu (const srmatrix& mA, const srmatrix& mLU,
    const int* pPivots, const rmatrix& mB)
    throw (cvmexception);
rvector svd () const throw (cvmexception);
rvector svd (srmatrix& mU, srmatrix& mVH) const
    throw (cvmexception);
rmatrix pinv (TR threshold = cvmMachSp()) const
    throw (cvmexception);
rmatrix& pinv (const rmatrix& mA, TR threshold = cvmMachSp())
    throw (cvmexception);
rmatrix gels (bool transpose, const rmatrix& mB,
    rvector& vErr) const throw (cvmexception);
rmatrix& gels (bool transpose, const rmatrix& mA, const rmatrix& mB,
    rvector& vErr) throw (cvmexception);
rvector gels (bool transpose, const rvector& mB,

```

```

        TR& dErr) const throw (cvmexception);
rmatrix gelsy (const rmatrix& mB, int& rank,
              TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix& gelsy (const rmatrix& mA, const rmatrix& mB, int& rank,
              TR tol = cvmMachSp()) throw (cvmexception);
rvector gelsy (const rvector& mB, int& rank,
              TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix gelss (const rmatrix& mB, rvector& sv,
              int& rank, TR tol = cvmMachSp()) const
              throw (cvmexception);
rmatrix& gelss (const rmatrix& mA, const rmatrix& mB, rvector& sv,
              int& rank, TR tol = cvmMachSp())
              throw (cvmexception);
rvector gelss (const rvector& mB, rvector& sv,
              int& rank, TR tol = cvmMachSp()) const
              throw (cvmexception);
rmatrix gelsd (const rmatrix& mB, rvector& sv,
              int& rank, TR tol = cvmMachSp()) const
              throw (cvmexception);
rmatrix& gelsd (const rmatrix& mA, const rmatrix& mB, rvector& sv,
              int& rank, TR tol = cvmMachSp())
              throw (cvmexception);
rvector gelsd (const rvector& mB, rvector& sv,
              int& rank, TR tol = cvmMachSp()) const
              throw (cvmexception);
int rank (TR eps = cvmMachSp ()) const throw (cvmexception);
rmatrix& ger (TR dAlpha, const rvector& vCol,
             const rvector& vRow) throw (cvmexception);
rmatrix& gemm (const rmatrix& m1, bool bTrans1,
             const rmatrix& m2, bool bTrans2,
             TR dAlpha, TR dBeta) throw (cvmexception);
rmatrix& symm (bool bLeft, const srmatrix& ms,
             const rmatrix& m, TR dAlpha, TR dBeta)
             throw (cvmexception);
void qr(rmatrix& mQ, srmatrix& mR) const throw (cvmexception);
void qr(srmatrix& mQ, rmatrix& mR) const throw (cvmexception);
void lq(srmatrix& mL, rmatrix& mQ) const throw (cvmexception);
void lq(rmatrix& mL, srmatrix& mQ) const throw (cvmexception);
void rq(srmatrix& mR, rmatrix& mQ) const throw (cvmexception);
void rq(rmatrix& mR, srmatrix& mQ) const throw (cvmexception);
void ql(rmatrix& mQ, srmatrix& mL) const throw (cvmexception);
void ql(srmatrix& mQ, rmatrix& mL) const throw (cvmexception);
rmatrix& vanish ();

```

```
    rmatrix& randomize (TR dFrom, TR dTo);  
};
```

2.6.1 `rmatrix ()`

Default constructor

```
rmatrix::rmatrix ();
```

creates an empty `rmatrix` object. See also `rmatrix`. Example:

```
using namespace cvm;
```

```
rmatrix m;  
std::cout << m.msize() << std::endl << m.nsize() << std::endl;  
std::cout << m.size() << std::endl;
```

```
m.resize (2, 3);  
std::cout << m;
```

prints

```
0  
0  
0  
0 0 0  
0 0 0
```

2.6.2 `rmatrix (int,int)`

Constructor

```
rmatrix::rmatrix (int nM, int nN);
```

creates an $m \times n$ `rmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). The constructor throws an exception of type `cvmexception` in case of non-positive sizes passed or memory allocation failure. See also `rmatrix`. Example:

```
using namespace cvm;
```

```
rmatrix m (3, 4);  
std::cout << m.msize() << std::endl << m.nsize()  
          << std::endl << m.size() << std::endl << m;
```

prints

```
3  
4  
12  
0 0 0 0  
0 0 0 0  
0 0 0 0
```

2.6.3 `rmatrix (TR*,int,int)`

Constructor

```
rmatrix::rmatrix (TR* pD, int nM, int nN);
```

creates an $m \times n$ `rmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD`. The constructor throws an exception of type `cvmexception` in case of non-positive size passed. See also `rmatrix`, `rmatrix (const TR*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m (a, 2, 3);

m(1,1) = 7.77;
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << std::endl;
```

prints

```
7.77e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000

7.77e+000 2.00e+000 3.00e+000
```

2.6.4 `rmatrix (const TR*,int,int)`

Constructor

```
rmatrix::rmatrix (const TR* pD, int nM, int nN);
```

creates an $m \times n$ `rmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns) and copies $m * n$ elements of an array `pD` to it by `columns`. The constructor throws an exception of type `cvmexception` in case of non-positive size passed. See also `rmatrix`, `rmatrix (TR*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
const double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m (a, 2, 3);

m(1,1) = 7.77;
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << std::endl;
```

prints

```
7.77e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000

1.00e+000 2.00e+000 3.00e+000
```

2.6.5 `rmatrix (const rmatrix&)`

Copy constructor

```
rmatrix::rmatrix (const rmatrix& m);
```

creates a `rmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m (a, 2, 3);
rmatrix mc(m);
```

```
m(1,1) = 7.77;
std::cout << m << std::endl << mc;
```

prints

```
7.77e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000

1.00e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000
```

2.6.6 `rmatrix (const rvector&,bool)`

Constructor

```
explicit rmatrix::rmatrix (const rvector& v, bool bBeColumn = true);
```

creates a `rmatrix` object containing `v.size()` rows and 1 column if `bBeColumn` is `true` or 1 row and `v.size()` columns otherwise. After that it copies the vector `v`'s elements to the matrix created. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `rmatrix`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
rvector v(3);
v(1) = 1.;
v(2) = 2.;
v(3) = 3.;

rmatrix mc (v);
rmatrix mr (v, false);

std::cout << mc << std::endl << mr;
```

prints

```
1.00e+000
2.00e+000
3.00e+000

1.00e+000 2.00e+000 3.00e+000
```

2.6.7 **submatrix**

Submatrix constructor

```
rmatrix::rmatrix (rmatrix& m, int nRow, int nCol,  
                 int nHeight, int nWidth);
```

creates a `rmatrix` object as a *submatrix* of `m`. It means that the matrix object created shares a memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are *l-based*) and its height and width (parameters `nHeight` and `nWidth`). See also `rmatrix`. Example:

```
using namespace cvm;  
rmatrix m(4,5);  
rmatrix subm(m, 2, 2, 2, 2);  
subm.set(1.);
```

```
std::cout << m;
```

prints

```
0 0 0 0 0  
0 1 1 0 0  
0 1 1 0 0  
0 0 0 0 0
```

2.6.8 operator (,)

Indexing operators

```
TR& rmatrix::operator () (int im, int in) throw (cvmexception);
TR rmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if `im` is outside of `[1,msize()]` range or `in` is outside of `[1,nsize()]` range. The operators are *inherited* in the the classes `srmatrix` and `srbmatrix`. The operators are *redefined* in the the class `srsrmatrix`. See also `rmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m (a, 2, 3);
    rmatrix ms(m);

    std::cout << m(1,1) << " " << m(2,3) << std::endl << std::endl;

    ms(2,2) = 7.77;
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 6.00e+00

1.00e+00 3.00e+00 5.00e+00
2.00e+00 7.77e+00 6.00e+00
```

2.6.9 operator ()

Indexing operators

```
rvector rmatrix::operator () (int i) throw (cvmexception);
const rvector rmatrix::operator () (int i) const throw (cvmexception);
```

provide access to an i -th column of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i -th column of the matrix in order to make possible write access to it. The second version creates a *copy* of a column and therefore is *not an l-value*. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if the parameter i is outside of $[1, \text{nsize}()]$ range. The operators are *inherited* in the the class `rmatrix`. The operators are *redefined* in the the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`, `Matrix::nsize()`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m (a, 2, 3);
    srmatrix ms(2);

    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.00e+00 4.00e+00
```

```
0.00e+00 5.00e+00
```

```
0.00e+00 6.00e+00
```

2.6.10 operator []

Indexing operators

```
rvector rmatrix::operator [] (int i) throw (cvmexception);
const rvector rmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to an *i*-th row of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the *i*-th row of the matrix in order to make possible write access to it. The second version creates a *copy* of a row and therefore is *not an l-value*. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if *i* is outside of `[1,msize()]` range. The operators are *inherited* in the the class `srmatrix`. The operators are *redefined* in the the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m (a, 2, 3);
    srmatrix ms(3);

    std::cout << m[1] << std::endl;

    ms[1] = m[2];
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 3.00e+00 5.00e+00

2.00e+00 4.00e+00 6.00e+00
0.00e+00 0.00e+00 0.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.6.11 diag

Functions

```
rvector rmatrix::diag (int i) throw (cvmexception);
const rvector rmatrix::diag (int i) const throw (cvmexception);
```

provide access to an i -th diagonal of a matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. The first version of the function is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i -th diagonal of the matrix in order to make possible write access to it. The second version creates a *copy* of the diagonal and therefore is *not an l-value*. The functions throw an exception of type `cvmexception` if the parameter i is outside of $[-m\text{size}()+1, n\text{size}()-1]$ range. The functions are *inherited* in the the classes `srmatrix` and `srbmatrix`. The functions are *redefined* in the the class `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    rmatrix m(2,3);
    const srmatrix ms(a,3);

    m.diag(-1).set(1.);
    m.diag(0).set(2.);
    m.diag(1).set(3.);
    m.diag(2).set(4.);
    std::cout << m << std::endl;

    std::cout << ms << std::endl;
    std::cout << ms.diag(0) << ms.diag(1);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
2 3 4
1 2 3
```

```
1 4 7
2 5 8
3 6 9
```

```
1 5 9
4 8
```

2.6.12 `operator = (const rmatrix&)`

Operator

```
rmatrix& rmatrix::operator = (const rmatrix& m) throw (cvmexception);
```

sets an every element of a calling matrix to a value of appropriate element of a matrix `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different matrix sizes. The operator is *redefined* in the the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m1(a, 3, 2);
    rmatrix m2(3, 2);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

1.00e+000 4.00e+000
2.00e+000 5.00e+000
3.00e+000 6.00e+000
```

2.6.13 `assign (const TR*)`

Function

```
rmatrix& rmatrix::assign (const rvector& v);  
rmatrix& rmatrix::assign (const TR* pD);
```

sets every element of a calling matrix to a value of appropriate element of a vector `v` or an array pointed to by `pD` and returns a reference to the matrix changed. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
const double a[] = {1., 2., 3., 4., 5., 6.};  
rmatrix m(2, 3);  
  
m.assign(a);  
std::cout << m;
```

prints

```
1.00e+000 3.00e+000 5.00e+000  
2.00e+000 4.00e+000 6.00e+000
```

2.6.14 `assign (int, int, const rmatrix&)`

Function

```
rmatrix& rmatrix::assign (int nRow, int nCol, const rmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with *l*-based row `nRow` and column `nCol` to a matrix `m` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if `nRow` or `nCol` are not positive or matrix `m` doesn't fit. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
```

```
rmatrix m1(4,5);
rmatrix m2(2,2);
m1.set(1.);
m2.set(2.);
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
1 1 1 1 1
1 1 2 2 1
1 1 2 2 1
1 1 1 1 1
```

2.6.15 `set` (TR)

Function

```
rmatrix& rmatrix::set (TR x);
```

sets every element of a calling matrix to a value of parameter `x` and returns a reference to the matrix changed. Use `vanish` to set every element of a calling matrix to be equal to zero. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
rmatrix m(2, 3);
```

```
m.set(3.);
std::cout << m;
```

prints

```
3.00e+000 3.00e+000 3.00e+000
3.00e+000 3.00e+000 3.00e+000
```

2.6.16 *resize*

Function

```
rmatrix& rmatrix::resize (int nNewM, int nNewN)
throw (cvmexception);
```

changes a size of a calling matrix to *nNewM* by *nNewN* and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type *cvmexception* in case of negative size passed or memory allocation failure. The function is *redefined* in the classes *srmatrix*, *srbmatrix* and *srsmatrix*. See also *rmatrix*. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a, 2, 3);
    std::cout << m << std::endl;
    m.resize (2, 2);
    std::cout << m << std::endl;
    m.resize (3, 3);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
1.00e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000
```

```
1.00e+000 3.00e+000
2.00e+000 4.00e+000
```

```
1.00e+000 3.00e+000 0.00e+000
2.00e+000 4.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000
```

2.6.17 operator ==

Operator

```
bool rmatrix::operator == (const rmatrix& m) const;
```

compares a calling matrix with a matrix *m* and returns true if they have the same sizes and their appropriate elements differ by not more than the **smallest normalized positive number**. Returns false otherwise. The operator is *redefined* in the classes **srmatrix**, **srbmatrix** and **srsmatrix**. See also **rmatrix**. Example:

```
using namespace cvm;
double a[] = {1., 2., 3., 4.};
rmatrix m1(a, 2, 2);
rmatrix m2(2, 2);

m2(1,1) = 1.; m2(1,2) = 3.;
m2(2,1) = 2.; m2(2,2) = 4.;

std::cout << (m1 == m2) << std::endl;

prints

1
```

2.6.18 operator !=

Operator

```
bool rmatrix::operator != (const rmatrix& m) const;
```

compares a calling matrix with a matrix *m* and returns true if they have different sizes or at least one of their appropriate elements differs by more than the **smallest normalized positive number**. Returns false otherwise. The operator is *redefined* in the classes **srmatrix**, **srbmatrix** and **srsmatrix**. See also **rmatrix**. Example:

```
using namespace cvm;
double a[] = {1., 2., 3., 4.};
rmatrix m1(a, 2, 2);
rmatrix m2(2, 2);

m2(1,1) = 1.; m2(1,2) = 3.;
m2(2,1) = 2.; m2(2,2) = 4.;

std::cout << (m1 != m2) << std::endl;

prints

0
```

2.6.19 operator <<

Operator

```
rmatrix& rmatrix::operator << (const rmatrix& m)
throw (cvmexception);
```

destroys a calling matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    rmatrix m(3,4);
    rmatrix mc(1,1);
    m(1,2) = 1.;
    m(3,4) = 2.;
    std::cout << m << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.00e+000 1.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 2.00e+000
0.00e+000
```

```
0.00e+000 1.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 2.00e+000
```

2.6.20 operator +

Operator

```
rmatrix rmatrix::operator + (const rmatrix& m) const  
throw (cvmexception);
```

creates an object of type `rmatrix` as a sum of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::sum`, `rmatrix`. Example:

```
using namespace cvm;  
try {  
    double a[] = {1., 2., 3., 4., 5., 6.};  
    const rmatrix ma(a,2,3);  
    rmatrix mb(2,3);  
    mb.set(1.);  
  
    std::cout << ma + mb << std::endl;  
    std::cout << ma + ma;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
2 4 6
```

```
3 5 7
```

```
2 6 10
```

```
4 8 12
```

2.6.21 operator -

Operator

```
rmatrix rmatrix::operator - (const rmatrix& m) const  
throw (cvmexception);
```

creates an object of type `rmatrix` as a difference of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::diff`, `rmatrix`. Example:

```
using namespace cvm;  
try {  
    double a[] = {1., 2., 3., 4., 5., 6.};  
    const rmatrix ma(a,2,3);  
    rmatrix mb(2,3);  
    mb.set(1.);  
  
    std::cout << ma - mb << std::endl;  
    std::cout << ma - ma;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
0 2 4  
1 3 5  
  
0 0 0  
0 0 0
```

2.6.22 `sum`

Function

```
rmatrix& rmatrix::sum (const rmatrix& m1, const rmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator +`, `rmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m1(a, 2, 3);
    rmatrix m2(2, 3);
    rmatrix m(2, 3);
    m2.set(1.);

    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2 4 6
3 5 7
```

```
3 5 7
4 6 8
```

2.6.23 diff

Function

```
rmatrix& rmatrix::diff (const rmatrix& m1, const rmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of matrices *m1* and *m2* to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator -`, `rmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m1(a, 2, 3);
    rmatrix m2(2, 3);
    rmatrix m(2, 3);
    m2.set(1.);

    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0 2 4
1 3 5
```

```
-1 1 3
0 2 4
```

2.6.24 operator +=

Operator

```
rmatrix& rmatrix::operator += (const rmatrix& m) throw (cvmexception);
```

adds a matrix *m* to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator +`, `rmatrix::sum`, `rmatrix`. Example:

```
using namespace cvm;
try {
    rmatrix m1(2, 3);
    rmatrix m2(2, 3);
    m1.set(1.);
    m2.set(2.);

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
3 3 3
```

```
3 3 3
```

```
4 4 4
```

```
4 4 4
```

2.6.25 operator -=

Operator

```
rmatrix& rmatrix::operator -= (const rmatrix& m) throw (cvmexception);
```

subtracts a matrix `m` from a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator -`, `rmatrix::diff`, `rmatrix`. Example:

```
using namespace cvm;
try {
    rmatrix m1(2, 3);
    rmatrix m2(2, 3);
    m1.set(1.);
    m2.set(2.);

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

-1 -1 -1
-1 -1 -1

0 0 0
0 0 0
```

2.6.26 operator - ()

Operator

```
rmatrix rmatrix::operator - () const throw (cvmexception);
```

creates an object of type `rmatrix` as a calling matrix multiplied by -1 . The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.};  
const rmatrix ma(a, 2, 3);
```

```
std::cout << - ma;
```

prints

```
-1 -3 -5  
-2 -4 -6
```

2.6.27 operator * (TR)

Operator

```
rmatrix rmatrix::operator * (TR d) const;
```

creates an object of type `rmatrix` as a product of a calling matrix and a number `d`. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator *= , rmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.};  
rmatrix m(a, 2, 3);
```

```
std::cout << m * 2.;
```

prints

```
2 6 10  
4 8 12
```

2.6.28 operator / (TR)

Operator

```
rmatrix rmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `rmatrix` as a quotient of a calling matrix and a number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the **smallest normalized positive number**. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsrmatrix`. See also `rmatrix::operator /=`, `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a, 2, 3);

    std::cout << m / 2.;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

5.00e-01 1.50e+00 2.50e+00
1.00e+00 2.00e+00 3.00e+00
```

2.6.29 operator *= (TR)

Operator

```
rmatrix& rmatrix::operator *= (TR d);
```

multiplies a calling matrix by a number *d* and returns a reference to the matrix changed. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator *`, `rmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.};  
rmatrix m(a, 2, 3);
```

```
m *= 2.;  
std::cout << m;
```

prints

```
2 6 10  
4 8 12
```

2.6.30 operator /= (TR)

Operator

```
rmatrix& rmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling matrix by a number *d* and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if *d* has an absolute value equal or less than the **smallest normalized positive number**. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator /`, `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a, 2, 3);

    m /= 2.;
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

5.00e-01 1.50e+00 2.50e+00
1.00e+00 2.00e+00 3.00e+00
```

2.6.31 `normalize`

Function

```
rmatrix& rmatrix::normalize ();
```

normalizes a calling matrix so its **Euclidean norm** becomes equal to 1 if it was greater than the **smallest normalized positive number** before the call (otherwise the function does nothing). The function is *redefined* in the classes **srmatrix**, **srbmatrix** and **srsmatrix**. See also **rmatrix**. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(a, 2, 3);
```

```
m.normalize();
std::cout << m;
```

prints

```
1.05e-01 3.14e-01 5.24e-01
2.10e-01 4.19e-01 6.29e-01
```

2.6.32 transposition

Operator and functions

```
rmatrix rmatrix::operator ~ () const throw (cvmexception);
rmatrix& rmatrix::transpose (const rmatrix& m) throw (cvmexception);
rmatrix& rmatrix::transpose () throw (cvmexception);
```

encapsulate matrix transposition. First operator creates an object of type `rmatrix` as a transposed calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `m` transposed (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to transposed itself (it also throws an exception of type `cvmexception` in case of memory allocation failure). The functions are *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a,2,3);
    rmatrix mt(3,2);

    std::cout << m << std::endl << ~m << std::endl ;
    mt.transpose(m);
    std::cout << mt << std::endl;
    mt.transpose();
    std::cout << mt;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 3 5
2 4 6
```

```
1 2
3 4
5 6
```

```
1 2
3 4
5 6
```

1 3 5
2 4 6

2.6.33 `operator * (const rvector&)`

Operator

```
rvector rmatrix::operator * (const rvector& v) const  
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `rvector::mult` in order to get rid of a new object creation. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`, `rvector`. Example:

```
using namespace cvm;  
  
try {  
    rmatrix m(2, 3);  
    rvector v(3);  
    m.set(1.);  
    v.set(1.);  
  
    std::cout << m * v;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}  
  
prints  
3 3
```

2.6.34 `operator * (const rmatrix&)`

Operator

```
rmatrix rmatrix::operator * (const rmatrix& m) const  
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `rmatrix::mult` in order to get rid of a new object creation. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
```

```
try {  
    rmatrix m1(2, 3);  
    rmatrix m2(3, 2);  
    m1.set(1.);  
    m2.set(1.);  
  
    std::cout << m1 * m2;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
3 3  
3 3
```

2.6.35 `mult`

Function

```
rmatrix& rmatrix::mult (const rmatrix& m1, const rmatrix& m2)
throw (cvmexception);
```

sets a calling matrix to be equal to a product of a matrix `m1` by a matrix `m2` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the class `srmatrix` and *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
```

```
try {
    rmatrix m1(2, 3);
    rmatrix m2(3, 2);
    rmatrix m(2, 2);
    m1.set(1.);
    m2.set(1.);

    std::cout << m.mult(m1, m2) << std::endl;
    std::cout << m1.mult(m, m1);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3
3 3
```

```
6 6 6
6 6 6
```

2.6.36 ranklupdate

Function

```

rmatrix&
rmatrix::ranklupdate (const rvector& vCol, const rvector& vRow)
throw (cvmexception);

```

sets a calling matrix to be equal to the **rank-1 update** of vectors `vCol` and `vRow` and returns a reference to the matrix changed. The function throws an exception of type **cvmexception** if the number of rows of the calling matrix is not equal to `vCol.size()` or the number of columns of the calling matrix is not equal to `vRow.size()`. The function is *inherited* in the the class **srmatrix** and *not applicable* to objects of the classes **srbmatrix** and **srsmatrix** (i.e. an exception of type **cvmexception** would be thrown in case of using it for objects of those classes). See also **rvector::ranklupdate**, **rmatrix**. Example:

```

using namespace cvm;

try {
    rvector vc(3), vr(2);
    rmatrix m(3, 2);
    vc(1) = 1.;
    vc(2) = 2.;
    vc(3) = 3.;
    vr(1) = 4.;
    vr(2) = 5.;

    std::cout << m.ranklupdate (vc, vr);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

4 5
8 10
12 15

```

2.6.37 `swap_rows`

Function

```
rmatrix& rmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of rows to be swapped, both are *ℓ-based*). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1,msize()]`. The function is *redefined* in the the class `srmatrix` and *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m (a, 3, 2);

    std::cout << m << std::endl;
    std::cout << m.swap_rows(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4
2 5
3 6
```

```
1 4
3 6
2 5
```

2.6.38 `swap_cols`

Function

```
rmatrix& rmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of columns to be swapped, both are *l-based*). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1, nsize()]`. The function is *redefined* in the the class `srmatrix` and *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m (a, 2, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_cols(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 3 5
```

```
2 4 6
```

```
1 5 3
```

```
2 6 4
```

2.6.39 solve

Functions

```

rmatrix&
rmatrix::solve (const srmatrix& mA,
                const rmatrix& mB, TR& dErr) throw (cvmexception);
rmatrix&
rmatrix::solve (const srmatrix& mA,
                const rmatrix& vB) throw (cvmexception);

```

set a calling matrix to be equal to a solution X of the matrix linear equation $AX = B$ where the parameter `mA` is the square matrix A and the parameter `vB` is the matrix B . Every function returns a reference to the matrix changed. The first version also sets the output parameter `dErr` to be equal to the norm of computation error. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to singular. The functions are *redefined* in the the class `srmatrix` and *inherited* thereafter in the classes `srbmatrix` and `srsrmatrix`. See also `rvector::solve`, `rmatrix`, `srmatrix`. Example:

```

using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix ma(a, 3);
    rmatrix mb(3,4);
    rmatrix mx(3,4);
    double dErr;
    mb.randomize(1., 10.);

    mx.solve (ma, mb, dErr);
    std::cout << mx << std::endl << ma * mx - mb;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

```

prints

```

-4.47e-001 -4.17e+000 -4.21e+000 -8.49e-002
-8.41e+000 2.30e+001 1.57e+001 -7.21e+000
5.70e+000 -1.21e+001 -7.57e+000 5.10e+000

-8.88e-015 5.33e-015 7.99e-015 1.78e-015

```

```
-4.44e-015 1.78e-015 3.55e-015 3.55e-015  
1.78e-015 0.00e+000 -4.44e-015 -8.88e-016
```

2.6.40 solve_lu

Functions

```
rmatrix&
rmatrix::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                  const int* pPivots, const rmatrix& mB, TR& dErr)
                  throw (cvmexception);
```

```
rmatrix&
rmatrix::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                  const int* pPivots, const rmatrix& mB)
                  throw (cvmexception);
```

set a calling matrix to be equal to a solution X of the matrix linear equation $AX = B$ where the parameter `mA` is the square matrix A , parameter `mLU` is **LU factorization** of the matrix A , parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and the parameter `mB` is the matrix B . Every function returns a reference to the matrix changed. The first version also sets output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $AX = B$ with the same matrix A and different matrices B . In such case you save on matrix A factorization since it's needed to be performed just one time. The functions throw exception of type **cvmexception** in case of inappropriate sizes of the operands or when the matrix A is close to singular. See also **rvector::solve**, **rmatrix**, **srmatrix**. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    srmatrix ma(a,3);
    srmatrix mLU(3);
    rmatrix mb1(3,2);
    rmatrix mb2(3,2);
    rmatrix mx1(3,2);
    rmatrix mx2(3,2);
    iarray nPivots(3);
    double dErr = 0.;
    mb1.randomize(-1.,3.);
    mb2.randomize(2.,5.);

    mLU.low_up(ma, nPivots);
    std::cout << mx1.solve_lu (ma, mLU, nPivots, mb1, dErr);
    std::cout << dErr << std::endl;
```

```
    std::cout << mx2.solve_lu (ma, mLU, nPivots, mb2) << std::endl;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.85e+00 5.90e-01
-4.23e+00 -3.67e+00
2.10e+00 2.55e+00
7.04e-15
9.49e+00 8.93e+00
-1.00e+01 -1.42e+01
4.21e+00 7.55e+00

0.00e+00 0.00e+00
0.00e+00 0.00e+00
4.44e-16 -1.11e-16

4.44e-16 0.00e+00
-4.44e-16 0.00e+00
8.88e-16 0.00e+00
```

2.6.41 svd

Functions

rvector

rmatrix::svd () const throw (cvmexception);

rvector

rmatrix::svd (srmatrix& mU, srmatrix& mVH) const throw (cvmexception);

create an object of type rvector as a vector of **singular values** of a calling matrix. The second version of the function sets output parameter mU to be equal to the matrix U of size $m \times m$ and mVH to be equal to the matrix V^H of size $n \times n$. All the functions throw an exception of type **cvmexception** in case of inappropriate sizes of the operands or in case of convergence error. Use **rvector::svd** in order to get rid of a new vector creation. The function is *redefined* in the the classes **srmatrix**, **srbmatrix**, **srsmatrix**. See also **rvector**, **rmatrix**. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double m[] = {1., -1., 1., 2., -2., 1.,
                  3., -2., 1., 0., -2., 1.};
    rmatrix mA(m,4,3);
    rmatrix mSigma(4,3);
    rvector v;
    srmatrix mU(4), mVH(3);

    v << mA.svd(mU, mVH);
    mSigma.diag(0) = v;

    std::cout << mU << std::endl;
    std::cout << mVH << std::endl;
    std::cout << mSigma << std::endl;

    std::cout << (mA * ~mVH - mU * mSigma).norm() << std::endl;
    std::cout << (~mA * mU - ~(mSigma * mVH)).norm() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
prints
-4.84e-01 1.95e-01 1.15e-02 -8.53e-01
2.17e-01 -3.41e-01 -8.89e-01 -2.13e-01
```

```
6.62e-01 7.16e-01 -6.18e-02 -2.13e-01  
-5.29e-01 5.78e-01 -4.53e-01 4.26e-01
```

```
-2.21e-01 8.54e-01 -4.72e-01  
9.59e-01 1.04e-01 -2.62e-01  
-1.75e-01 -5.11e-01 -8.42e-01
```

```
4.96e+00 0.00e+00 0.00e+00  
0.00e+00 2.51e+00 0.00e+00  
0.00e+00 0.00e+00 3.77e-01  
0.00e+00 0.00e+00 0.00e+00
```

```
1.37e-15  
2.48e-15
```

2.6.42 pinv

Functions

```

rmatrix rmatrix::pinv (TR threshold = cvmMachSp()) const
    throw (cvmexception);
rmatrix& rmatrix::pinv (const basic_rmatrix& mA,
    TR threshold = cvmMachSp())
    throw (cvmexception);

```

encapsulate matrix pseudo inversion [6], p. 33 (or Moore-Penrose generalized inversion [5], p. 421). Strictly defined, an $n \times m$ matrix A^+ is a *pseudo inversion* of $m \times n$ matrix A if the following two equations are satisfied:

$$AA^+A = A,$$

$$A^+ = QA^H = A^HP$$

where Q and P are some matrices. To compute the pseudo inversion, we use [Singular Value Decomposition \(SVD\)](#)

$$A = U\Sigma V^H$$

of matrix A , thus

$$A^+ = V\Sigma^{-1}U^H,$$

where Σ^{-1} is a diagonal $n \times m$ matrix having inverted diagonal values of the matrix Σ if they are greater than some threshold, and zeros otherwise.

First version creates an object of type `rmatrix` as a pseudo inverted calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `mA` pseudo inverted (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands). The threshold parameter sets a minimum distinguishable from zero singular value to be used to compute the pseudo inversion. All values equal or less than the threshold are treated as zeros. The functions are *inherited* in the classes `srmatrix` and `srsmatrix` and *redefined* in `srbmatrix`. See also `rmatrix`. Example:

```

using namespace cvm;

try {
    rmatrix mA(3,4);
    mA(1,1) = 1.; mA(1,2) = -1.; mA(1,3) = 2.; mA(1,4) = 0.;
    mA(2,1) = -1.; mA(2,2) = 2.; mA(2,3) = -3.; mA(2,4) = 1.;
    mA(3,1) = 0.; mA(3,2) = 1.; mA(3,3) = -1.; mA(3,4) = 1.;
    rmatrix mX = mA.pinv(1.e-13);
    std::cout << mX << (mA * mX * mA - mA).norm2() << std::endl;
}
catch (exception& e) {

```

```
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+3.3333333e-001 -2.1510571e-016 +3.3333333e-001
+1.1111111e-001 +1.1111111e-001 +2.2222222e-001
+2.2222222e-001 -1.1111111e-001 +1.1111111e-001
+4.4444444e-001 +1.1111111e-001 +5.5555556e-001
+2.5460202e-015
```

Band matrix example:

```
using namespace cvm;
```

```
try {
    srbmatrix mA (40, 1, 0);
    mA.diag(0).randomize(-1.,1.);
    mA.diag(-1).randomize(5.,10.);

    srmatrix mX (40);
    mX.pinv(mA);
    std::cout << (mA * mX * mA - mA).norm2() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+8.1956952e-14
```

2.6.43 gels

Functions

```

rmatrix  rmatrix::gels (bool transpose, const rmatrix& mB,
                       rvector& vErr) const throw (cvmexception);
rmatrix& rmatrix::gels (bool transpose,
                       const rmatrix& mA, const rmatrix& mB,
                       rvector& vErr) throw (cvmexception);
rvector  rmatrix::gels (bool transpose, const rvector& mB,
                       TR& dErr) const throw (cvmexception);

```

solve overdetermined or underdetermined linear systems

$$A * x = b$$

for $m \times n$ matrix A (or its transpose) where b is a vector of length k or systems

$$A * X = B$$

for multiple vectors b stored as columns of $k \times l$ matrix B where $k = m$ in non-transposed case and $k = n$ otherwise. The algorithm uses a QR or LQ factorization of A . It is assumed that A has full rank, infinity returned otherwise. Internally the functions use ?GELS LAPACK routines. If $m > n$ and `transpose=false` or $m < n$ and `transpose=true`, then the system is overdetermined, thus the algorithm tries to find the least squares solution x of the problem

$$\|A * x - b\|_2 \rightarrow \min \quad \text{or} \quad \|A^H * x - b\|_2 \rightarrow \min$$

respectively. Output vector `vErr` of length l (or real number `dErr` for single vector b) returns residual sum of squares. The system is underdetermined otherwise, and the algorithm finds its minimum norm solution. In this case `vErr` (or `bErr`) is set to zero. In both cases the solution computed satisfies $x = \text{pinv}(A) * b$, but this algorithm is faster than pseudo inversion. Matrix A is passed as an argument `mA` or (in second case) it's a calling object. First and third versions return the solution as a new matrix or vector object. Second version sets calling object to be the solution. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in all real matrix classes. See also `rmatrix`, `rvector::gels`, `rmatrix::pinv`. Example:

```
using namespace cvm;
```

```

rmatrix a(7, 5);
rmatrix b(7, 2);
rmatrix bt(5, 2);
rvector bv(5);
rvector vErr(2);

```

```
treal dErr;
a.randomize(-10., 10.);
b.randomize(-10., 10.);
bt.randomize(-10., 10.);
bv.randomize(-10., 10.);

rmatrix x = a.gels(false, b, vErr);
rvector vt = a.gels(true, bv, dErr);
rmatrix xt(7,2);
xt.gels(true, a, bt, vErr);

std::cout << (a.pinv()*b - x).norm() << std::endl;
std::cout << (~a.pinv()*bv - vt).norm() << std::endl;
std::cout << (~a.pinv()*bt - xt).norm() << std::endl;

prints

+2.6295387e-015
+5.8515615e-015
+6.6462869e-015
```

2.6.44 gelsy

Functions

```

rmatrix  rmatrix::gelsy (const rmatrix& mB, int& rank,
                        TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix& rmatrix::gelsy (const rmatrix& mA, const rmatrix& mB, int& rank,
                        TR tol = cvmMachSp()) throw (cvmexception);
rvector  rmatrix::gelsy (const rvector& mB, int& rank,
                        TR tol = cvmMachSp()) const throw (cvmexception);

```

compute the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using a complete orthogonal factorization of $m \times n$ matrix A . Here b is a vector of length m . Multiple vectors b can be stored as columns of $m \times l$ matrix B . Matrix A may be rank-deficient, the functions return its effective rank in `rank` output parameter using `tol` tolerance. Internally the functions use ?GELSY LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as an argument `mA` or (in second case) it's a calling object. First and third versions return the solution as a new matrix or vector object. Second version sets calling object to be the solution. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in all real matrix classes. See also `rmatrix`, `rvector::gelsy`, `rmatrix::gelss`, `rmatrix::gelsd`. Example:

```
using namespace cvm;
```

```

rmatrix a(4, 5);
rmatrix b(4, 2);
rvector bv(4);
a.randomize(-10., 10.);
b.randomize(-10., 10.);
bv.randomize(-10., 10.);
tint rank;

rmatrix x = a.gelsy(b, rank);
rvector xv = a.gelsy(bv, rank);
rmatrix x2(5,2);
x2.gelsy(a, b, rank);

std::cout << (a*x - b).norm() << std::endl;
std::cout << (a*xv - bv).norm() << std::endl;
std::cout << (a*x2 - b).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;

```

prints

+3.3539762e-014

+5.2545344e-015

+3.3539762e-014

+4 +4

2.6.45 gelss

Functions

```

rmatrix  rmatrix::gelss (const rmatrix& mB, int& rank, rvector& sv,
                        TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix& rmatrix::gelss (const rmatrix& mA, const rmatrix& mB,
                        rvector& sv, int& rank, TR tol = cvmMachSp())
                        throw (cvmexception);
rvector  rmatrix::gelss (const rvector& mB, rvector& sv, int& rank,
                        TR tol = cvmMachSp()) const throw (cvmexception);

```

compute the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using the singular value decomposition of $m \times n$ matrix A . Here b is a vector of length m . Multiple vectors b can be stored as columns of $m \times l$ matrix B . Matrix A may be rank-deficient, the functions return its effective rank in `rank` output parameter using `tol` tolerance. These functions also compute singular values of A in decreasing order and return them in `sv` output parameter having $\min(m, n)$ size. Internally the functions use ?GELSS LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as an argument `mA` or (in second case) it's a calling object. First and third versions return the solution as a new matrix or vector object. Second version sets calling object to be the solution. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in all real matrix classes. See also `rmatrix`, `rvector::gelss`, `rmatrix::gelsy`, `rmatrix::gelsd`. Example:

```

using namespace cvm;

rmatrix a(4, 5);
rmatrix b(4, 2);
rvector bv(4);
rvector sv(4);
tint rank;
a.randomize(-10., 10.);
b.randomize(-10., 10.);
bv.randomize(-10., 10.);

rmatrix x = a.gelss(b, sv, rank);
rvector xv = a.gelss(bv, sv, rank);
rmatrix xt(5,2);
xt.gelss(a, b, sv, rank);

```

```
std::cout << (a*x - b).norm() << std::endl;  
std::cout << (a*xv - bv).norm() << std::endl;  
std::cout << (a*xt - b).norm() << std::endl;  
std::cout << (sv - a.svd()).norm() << std::endl;  
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+1.9169279e-014  
+8.9260826e-015  
+1.9169279e-014  
+8.1402897e-015  
+4 +4
```

2.6.46 gelsd

Functions

```

rmatrix  rmatrix::gelsd (const rmatrix& mB, int& rank, rvector& sv,
                        TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix& rmatrix::gelsd (const rmatrix& mA, const rmatrix& mB,
                        rvector& sv, int& rank, TR tol = cvmMachSp())
                        throw (cvmexception);
rvector  rmatrix::gelsd (const rvector& mB, rvector& sv, int& rank,
                        TR tol = cvmMachSp()) const throw (cvmexception);

```

compute the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using the singular value decomposition of $m \times n$ matrix A and a divide and conquer method. Here b is a vector of length m . Multiple vectors b can be stored as columns of $m \times l$ matrix B . Matrix A may be rank-deficient, the functions return its effective rank in `rank` output parameter using `tol` tolerance. These functions also compute singular values of A in decreasing order and return them in `sv` output parameter having $\min(m, n)$ size. Internally the functions use ?GELSD LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as an argument `mA` or (in second case) it's a calling object. First and third versions return the solution as a new matrix or vector object. Second version sets calling object to be the solution. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in all real matrix classes. See also `rmatrix`, `rvector::gelsd`, `rmatrix::gelsy`, `rmatrix::gelss`. Example:

```

using namespace cvm;

rmatrix a(4, 5);
rmatrix b(4, 2);
rvector bv(4);
rvector sv(4);
tint rank;
a.randomize(-10., 10.);
b.randomize(-10., 10.);
bv.randomize(-10., 10.);

rmatrix x = a.gelsd(b, sv, rank);
rvector xv = a.gelsd(bv, sv, rank);
rmatrix xt(5,2);
xt.gelsd(a, b, sv, rank);

```

```
std::cout << (a*x - b).norm() << std::endl;  
std::cout << (a*xv - bv).norm() << std::endl;  
std::cout << (a*xt - b).norm() << std::endl;  
std::cout << (sv - a.svd()).norm() << std::endl;  
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+1.0341178e-014  
+6.0443462e-015  
+1.0341178e-014  
+8.9260826e-015  
+4 +4
```

2.6.47 rank

Function

```
int rmatrix::rank (TR eps = cvmMachSp ()) const throw (cvmexception);
```

returns a rank of a calling matrix as a number of **singular values** with **normalized** absolute value greater than or equal to a parameter eps (this is the **largest relative spacing** by default). The function throws an exception of type **cvmexception** in case of convergence error. The function is *inherited* in the the classes **srmatrix**, **srbmatrix**, **srsmatrix**. See also **rmatrix**. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    rmatrix m(a,3,4);

    std::cout << m << m.rank() << std::endl;
    m(3,4) = 13.;
    std::cout << m.rank() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4 7 10
2 5 8 11
3 6 9 12
2
3
```

2.6.48 ger

Function

```
rmatrix&
rmatrix::ger (TR dAlpha, const rvector& vCol, const rvector& vRow)
throw (cvmexception);
```

calls one of ?GER routines of the [BLAS library](#) performing a **rank-1 update** matrix-vector operation defined as $M = \alpha x \cdot y' + M$, where α is a real number (parameter dAlpha), M is a calling matrix and x and y are real vectors (parameters vCol and vRow respectively). The function returns a reference to the matrix changed and throws an exception of type **cvmexception** in case of inappropriate sizes of the operands. The function is *inherited* in the the class **srmatrix** and *not applicable* to objects of the classes **srbmatrix** and **srsmatrix** (i.e. an exception of type **cvmexception** would be thrown in case of using it for objects of those classes). See also **rvector**, **rmatrix**. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double alpha = 1.3;
    rmatrix m(3,4);
    rvector vc(3);
    rvector vr(4);
    m.randomize(-1., 2.); vc.randomize(-1., 3.); vr.randomize(0., 2.);

    std::cout << m + vc.rank1update (vr) * alpha << std::endl;
    std::cout << m.ger(alpha, vc, vr);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1.7127e-01 2.9410e+00 1.3449e+00 3.6055e+00
1.9057e+00 2.6726e+00 1.7134e+00 2.2154e+00
1.7217e-01 1.3508e+00 8.8949e-01 2.2551e+00

-1.7127e-01 2.9410e+00 1.3449e+00 3.6055e+00
1.9057e+00 2.6726e+00 1.7134e+00 2.2154e+00
1.7217e-01 1.3508e+00 8.8949e-01 2.2551e+00
```

2.6.49 gemm

Function

```
rmatrix& rmatrix::gemm (const rmatrix& m1, bool bTrans1,
                       const rmatrix& m2, bool bTrans2,
                       TR dAlpha, TR dBeta) throw (cvmexception);
```

calls one of ?GEMM routines of the [BLAS library](#) performing a matrix-matrix operation defined as

$$M = \alpha \mathcal{T}(M_1) \cdot \mathcal{T}(M_2) + \beta M,$$

where α and β are real numbers (parameters `dAlpha` and `dBeta`), M is a calling matrix and M_1 and M_2 are matrices (parameters `m1` and `m2` respectively). Function $\mathcal{T}(M_i)$ transposes matrix M_i if appropriate boolean parameter `bTrans*` is equal to `true` and does nothing otherwise. The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the the class `srmatrix` and *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double alpha = 1.3;
    double beta = -0.7;
    rmatrix m1(4,3); rmatrix m2(4,3);
    rmatrix m(3,3);
    m.randomize(-1., 2.); m1.randomize(-1., 3.); m2.randomize(0., 2.);
    std::cout << ~m1 * m2 * alpha + m * beta << std::endl;
    std::cout << m.gemm(m1, true, m2, false, alpha, beta);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
5.0504e+00 6.8736e+00 3.1171e+00
2.3915e+00 2.2544e+00 3.9205e+00
3.4607e+00 3.5351e+00 4.8622e+00

5.0504e+00 6.8736e+00 3.1171e+00
2.3915e+00 2.2544e+00 3.9205e+00
3.4607e+00 3.5351e+00 4.8622e+00
```

2.6.50 symm

Function

```
rmatrix& rmatrix::symm (bool bLeft, const srmatrix& ms,
                       const rmatrix& m, TR dAlpha, TR dBeta)
                       throw (cvmexception);
```

calls one of ?SYMM routines of the [BLAS library](#) performing one of matrix-matrix operations defined as

$$M = \alpha M_s \cdot M_1 + \beta M \quad \text{or} \quad M = \alpha M_1 \cdot M_s + \beta M,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is a calling matrix, M_s is a symmetric matrix and M_1 is a real matrix (parameters ms and m respectively). First operation is performed if bLeft passed is true and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type [cvmexception](#) in case of inappropriate sizes of the operands. The function is *inherited* in the the classes [srmatrix](#) and [srmatrix](#) and *not applicable* to objects of the class [srbmatrix](#) (i.e. an exception of type [cvmexception](#) would be thrown in case of using it for objects of that class). See also [srmatrix](#), [rmatrix](#). Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double alpha = 1.3;
    double beta = -0.7;
    rmatrix m1(3,4);
    rmatrix m2(4,3);
    srmatrix ms(3);
    rmatrix m(3,4);
    m.randomize(-1., 2.); m1.randomize(-1., 3.); m2.randomize(0., 2.);
    ms.randomize(-3., 1.);

    std::cout << ms * m1 * alpha + m * beta << std::endl;
    std::cout << m.symm (true, ms, m1, alpha, beta) << std::endl;

    m.resize(4,3);
    std::cout << m2 * ms * alpha + m * beta << std::endl;
    std::cout << m.symm (false, ms, m2, alpha, beta);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
prints
```

```
-3.3733e+00 -5.0566e+00 -6.3018e+00 -5.4907e+00  
-1.8629e+00 -1.5133e+00 -1.1372e+00 -2.5557e+00  
-3.5695e+00 -1.0012e+01 -1.4239e+00 -6.1786e-01
```

```
-3.3733e+00 -5.0566e+00 -6.3018e+00 -5.4907e+00  
-1.8629e+00 -1.5133e+00 -1.1372e+00 -2.5557e+00  
-3.5695e+00 -1.0012e+01 -1.4239e+00 -6.1786e-01
```

```
-6.4072e+00 7.0534e-01 1.5349e+00  
-4.8219e+00 -6.9891e+00 -5.1766e+00  
6.8503e-01 3.5828e+00 -3.2174e+00  
2.3469e-01 -9.3921e-01 -2.1961e+00
```

```
-6.4072e+00 7.0534e-01 1.5349e+00  
-4.8219e+00 -6.9891e+00 -5.1766e+00  
6.8503e-01 3.5828e+00 -3.2174e+00  
2.3469e-01 -9.3921e-01 -2.1961e+00
```

2.6.51 qr

Functions

```
void rmatrix::qr (rmatrix& mQ, srmatrix& mR) const throw (cvmexception);
void rmatrix::qr (srmatrix& mQ, rmatrix& mR) const throw (cvmexception);
```

compute QR factorization as

$$M = QR$$

where M is a calling matrix, orthogonal matrix Q and upper triangular (trapezoidal) matrix R are mQ and mR respectively. First version encapsulates so-called "economy" algorithm which for a given $m \times n$ matrix M computes $m \times n$ matrix Q and $n \times n$ matrix R . Second version is a "full" mode one computing $m \times m$ matrix Q and $m \times n$ matrix R . The functions throw an exception of type `cvmexception` in case if inappropriate sizes of the operands passed. The functions are *redefined* in the the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6.};
const cvm::rmatrix mh(a, 2, 3);
const cvm::rmatrix mv(a, 3, 2);
cvm::srmatrix s2(2), s3(3);
cvm::rmatrix h(2,3), v(3,2);

mh.qr(h,s3);
std::cout << (eye_real(2) - ~rmatrix(h,1,1,2,2)*rmatrix(h,1,1,2,2)).norm()
           << " " << (mh - h * s3).norm() << std::endl;
mh.qr(s2,h);
std::cout << (eye_real(2) - ~s2 * s2).norm()
           << " " << (mh - s2 * h).norm() << std::endl;
mv.qr(v,s2);
std::cout << (eye_real(2) - ~v * v).norm()
           << " " << (mv - v * s2).norm() << std::endl;
mv.qr(s3,v);
std::cout << (eye_real(3) - ~s3 * s3).norm()
           << " " << (mv - s3 * v).norm() << std::endl;

prints

+4.6933177e-16 +2.2342807e-15
+4.6933177e-16 +2.2342807e-15
+5.1302953e-16 +1.4432899e-15
+5.2889959e-16 +1.4432899e-15
```

2.6.52 lq

Functions

```
void rmatrix::lq (srmatrix& mL, rmatrix& mQ) const throw (cvmexception);
void rmatrix::lq (rmatrix& mL, srmatrix& mQ) const throw (cvmexception);
```

compute LQ factorization as

$$M = LQ$$

where M is a calling matrix, lower triangular (trapezoidal) matrix L and orthogonal matrix Q are mL and mQ respectively. First version encapsulates so-called "economy" algorithm which for a given $m \times n$ matrix M computes $m \times m$ matrix L and $m \times n$ matrix Q . Second version is a "full" mode one computing $m \times n$ matrix L and $n \times n$ matrix Q . The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands passed. The functions are *redefined* in the the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6.};
const cvm::rmatrix mh(a, 2, 3);
const cvm::rmatrix mv(a, 3, 2);
cvm::srmatrix s2(2), s3(3);
cvm::rmatrix h(2,3), v(3,2);

mh.lq(s2,h);
std::cout << (eye_real(2) - h * ~h).norm() << " " << (mh - s2 * h)
           .norm() << std::endl;
mv.lq(s3,v);
std::cout << (eye_real(2) - ~rmatrix(v,1,1,2,2) * rmatrix(v,1,1,2,2))
           .norm() << " " << (mv - s3 * v).norm() << std::endl;
mh.lq(h,s3);
std::cout << (eye_real(3) - s3 * ~s3).norm() << " " << (mh - h * s3)
           .norm() << std::endl;
mv.lq(v,s2);
std::cout << (eye_real(2) - s2 * ~s2).norm() << " " << (mv - v * s2)
           .norm() << std::endl;

prints

+8.6355085e-016 +3.3893638e-015
+1.9229627e-016 +1.6011864e-015
+1.0030483e-015 +3.3893638e-015
+1.9229627e-016 +1.6011864e-015
```

2.6.53 rq

Functions

```
void rmatrix::rq (srmatrix& mR, rmatrix& mQ) const throw (cvmexception);
void rmatrix::rq (rmatrix& mR, srmatrix& mQ) const throw (cvmexception);
```

compute RQ factorization as

$$M = RQ$$

where M is a calling matrix, upper triangular matrix R and orthogonal matrix Q are mR and mQ respectively. First version encapsulates so-called "economy" algorithm which for a given $m \times n$ matrix M computes $m \times m$ matrix R and $m \times n$ matrix Q . Second version is a "full" mode one computing $m \times n$ matrix R and $n \times n$ matrix Q . Following [this definition](#) the implementation assumes that $m \leq n$ and throws `cvmexception` otherwise. The functions also throw an exception of type `cvmexception` in case of inappropriate sizes of the operands passed. The functions are *redefined* in the the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6.};
const cvm::rmatrix mh(a, 2, 3);
const cvm::rmatrix mv(a, 3, 2);
cvm::srmatrix s2(2), s3(3);
cvm::rmatrix h(2,3), v(3,2);

mh.rq(h,s3);
std::cout << (eye_real(2) - rmatrix(s3,2,1,2,3) * ~rmatrix(s3,2,1,2,3))
            .norm() << " " << (mh - h * s3).norm() << std::endl;

mh.rq(s2,h);
std::cout << (eye_real(2) - h * ~h).norm()
            << " " << (mh - s2 * h).norm() << std::endl;

prints

+5.5511151e-016 +2.0471501e-015
+5.5511151e-016 +2.0471501e-015
```

2.6.54 ql

Functions

```
void rmatrix::ql (rmatrix& mQ, srmatrix& mL) const throw (cvmexception);
void rmatrix::ql (srmatrix& mQ, rmatrix& mL) const throw (cvmexception);
```

compute QL factorization as

$$M = QL$$

where M is a calling matrix, orthogonal matrix Q and lower triangular matrix L are mQ and mL respectively. First version encapsulates so-called "economy" algorithm which for a given $m \times n$ matrix M computes $m \times n$ matrix Q and $n \times n$ matrix L . Second version is a "full" mode one computing $m \times m$ matrix Q and $m \times n$ matrix L . Following [this definition](#) the implementation assumes that $m \geq n$ and throws `cvmexception` otherwise. The functions also throw an exception of type `cvmexception` in case of inappropriate sizes of the operands passed. The functions are *redefined* in the the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6.};
const cvm::rmatrix mv(a, 3, 2);
cvm::srmatrix s2(2), s3(3);
cvm::rmatrix v(3,2);

mv.ql(v,s2);
std::cout << (eye_real(2) - ~v * v).norm()
           << " " << (mv - v * s2).norm() << std::endl;

mv.ql(s3,v);
std::cout << (eye_real(2) - ~rmatrix(s3,1,2,3,2) * rmatrix(s3,1,2,3,2))
           .norm() << " " << (mv - s3 * v).norm() << std::endl;

prints

+0.00000000e+000 +2.0947646e-015
+0.00000000e+000 +2.0947646e-015
```

2.6.55 **vanish**

Function

```
rmatrix& rmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than, for example, `rmatrix::set(TR)` with zero operand passed. The function is *redefined* in the classes `srmatrix`, `srsmatrix`, `srbmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
```

```
rmatrix m(3, 4);  
m.randomize(0.,1.);
```

```
std::cout << m << std::endl;  
std::cout << m.vanish ();
```

prints

```
0.856532 0.938261 0.275704 0.186834  
0.651173 0.812159 0.100467 0.536912  
0.0726646 0.695914 0.661824 0.554613
```

```
0 0 0 0  
0 0 0 0  
0 0 0 0
```

2.6.56 `randomize`

Function

```
rmatrix& rmatrix::randomize (TR dFrom, TR dTo);
```

fills a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

rmatrix m(3,4);
m.randomize(-2.,3.);
std::cout << m;

prints

9.6853542e-01 2.7761467e+00 2.3791009e+00 -3.4452345e-01
2.9029511e+00 -9.5519883e-01 -4.9131748e-01 -1.2561113e+00
1.5219886e+00 -1.4494461e+00 2.8193304e+00 4.8817408e-01
```

2.7 *cmatrix*

This is end-user class encapsulating a matrix in Euclidean space of complex numbers.

```
template <typename TR, typename TC>
class cmatrix : public Matrix <TR,TC> {
public:
    cmatrix ();
    cmatrix (int nM, int nN);
    cmatrix (TC* pD, int nM, int nN);
    cmatrix (const TC* pD, int nM, int nN);
    cmatrix (const cmatrix& m);
    explicit cmatrix (const cvector& v, bool bBeColumn = true);
    explicit cmatrix (const rmatrix& m, bool bRealPart = true);
    cmatrix (const TR* pRe, const TR* pIm, int nM, int nN);
    cmatrix (const rmatrix& mRe, const rmatrix& mIm);
    cmatrix (cmatrix& m, int nRow, int nCol, int nHeight, int nWidth);
    TC& operator () (int im, int in) throw (cvmexception);
    TC operator () (int im, int in) const throw (cvmexception);
    cvector operator () (int i) throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    cvector operator [] (int i) throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    cvector diag (int i) throw (cvmexception);
    const cvector diag (int i) const throw (cvmexception);
    const rmatrix real () const;
    const rmatrix imag () const;
    cmatrix& operator = (const cmatrix& m) throw (cvmexception);
    cmatrix& assign (const cvector& v);
    cmatrix& assign (const TC* pD);
    rmatrix& assign (int nRow, int nCol, const cmatrix& m)
        throw (cvmexception);
    cmatrix& set (TC z);
    cmatrix& assign_real (const rmatrix& mRe) throw (cvmexception);
    cmatrix& assign_imag (const rmatrix& mIm) throw (cvmexception);
    cmatrix& set_real (TR d);
    cmatrix& set_imag (TR d);
    cmatrix& resize (int nNewM, int nNewN) throw (cvmexception);
    bool operator == (const cmatrix& v) const;
    bool operator != (const cmatrix& v) const;
    cmatrix& operator << (const cmatrix& m) throw (cvmexception);
    cmatrix operator + (const cmatrix& m) const throw (cvmexception);
    cmatrix operator - (const cmatrix& m) const throw (cvmexception);
```

```

cmatrix& sum (const cmatrix& m1,
             const cmatrix& m2) throw (cvmexception);
cmatrix& diff (const cmatrix& m1,
             const cmatrix& m2) throw (cvmexception);
cmatrix& operator += (const cmatrix& m) throw (cvmexception);
cmatrix& operator -= (const cmatrix& m) throw (cvmexception);
cmatrix operator - () const;
cmatrix operator * (TR d) const;
cmatrix operator / (TR d) const throw (cvmexception);
cmatrix operator * (TC z) const;
cmatrix operator / (TC z) const throw (cvmexception);
cmatrix& operator *= (TR d);
cmatrix& operator /= (TR d) throw (cvmexception);
cmatrix& operator *= (TC z);
cmatrix& operator /= (TC z) throw (cvmexception);
cmatrix& normalize ();
cmatrix operator ~() const throw (cvmexception);
cmatrix& conj (const cmatrix& m) throw (cvmexception);
cmatrix& conj () throw (cvmexception);
cvector operator * (const cvector& v) const
                throw (cvmexception);
cmatrix operator * (const cmatrix& m) const
                throw (cvmexception);
cmatrix& mult (const cmatrix& m1, const cmatrix& m2)
                throw (cvmexception);
cmatrix& ranklupdate_u (const rvector& vCol,
                    const rvector& vRow) throw (cvmexception);
cmatrix& ranklupdate_c (const rvector& vCol,
                    const rvector& vRow) throw (cvmexception);
cmatrix& swap_rows (int n1, int n2) throw (cvmexception);
cmatrix& swap_cols (int n1, int n2) throw (cvmexception);
cmatrix& solve (const scmatrix& mA,
              const cmatrix& mB, TR& dErr)
              throw (cvmexception);
cmatrix& solve (const scmatrix& mA,
              const cmatrix& mB) throw (cvmexception);
cmatrix& solve_lu (const scmatrix& mA, const scmatrix& mLU,
                const int* pPivots, const cmatrix& mB, TR& dErr)
                throw (cvmexception);
cmatrix& solve_lu (const scmatrix& mA, const scmatrix& mLU,
                const int* pPivots, const cmatrix& mB)
                throw (cvmexception);
rvector svd () throw (cvmexception);

```

```

rvector svd (cmatrix& mLSingVect, cmatrix& mRSingVect)
    throw (cvmexception);
cmatrix pinv (TR threshold = cvmMachSp()) const
    throw (cvmexception);
cmatrix& pinv (const cmatrix& mA,
    TR threshold = cvmMachSp())
    throw (cvmexception);
int rank (TR eps = cvmMachSp()) const throw (cvmexception);
cmatrix gels (bool transpose, const cmatrix& mB,
    cvector& vErr) const throw (cvmexception);
cmatrix& gels (bool transpose, const cmatrix& mA, const cmatrix& mB,
    cvector& vErr) throw (cvmexception);
cvector gels (bool transpose, const cvector& mB,
    TC& dErr) const throw (cvmexception);
cmatrix gelsy (const cmatrix& mB, int& rank,
    TR tol = cvmMachSp()) const throw (cvmexception);
cmatrix& gelsy (const cmatrix& mA, const cmatrix& mB, int& rank,
    TR tol = cvmMachSp()) throw (cvmexception);
cvector gelsy (const cvector& mB, int& rank,
    TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix gelss (const cmatrix& mB, rvector& sv,
    int& rank, TR tol = cvmMachSp()) const
    throw (cvmexception);
rmatrix& gelss (const cmatrix& mA, const cmatrix& mB, rvector& sv,
    int& rank, TR tol = cvmMachSp())
    throw (cvmexception);
rvector gelss (const cvector& mB, rvector& sv,
    int& rank, TR tol = cvmMachSp()) const
    throw (cvmexception);
rmatrix gelsd (const cmatrix& mB, rvector& sv,
    int& rank, TR tol = cvmMachSp()) const
    throw (cvmexception);
rmatrix& gelsd (const cmatrix& mA, const cmatrix& mB, rvector& sv,
    int& rank, TR tol = cvmMachSp())
    throw (cvmexception);
rvector gelsd (const cvector& mB, rvector& sv,
    int& rank, TR tol = cvmMachSp()) const
    throw (cvmexception);
void qr(cmatrix& mQ, scmatrix& mR) const throw (cvmexception);
void qr(scmatrix& mQ, cmatrix& mR) const throw (cvmexception);
void lq(scmatrix& mL, cmatrix& mQ) const throw (cvmexception);
void lq(cmatrix& mL, scmatrix& mQ) const throw (cvmexception);
void rq(scmatrix& mR, cmatrix& mQ) const throw (cvmexception);

```

```
void rq(cmatrix& mR, scmatrix& mQ) const throw (cvmexception);
void ql(cmatrix& mQ, scmatrix& mL) const throw (cvmexception);
void ql(scmatrix& mQ, cmatrix& mL) const throw (cvmexception);
cmatrix& vanish ();
cmatrix& geru (TC alpha, const cvector& vCol,
              const cvector& vRow) throw (cvmexception);
cmatrix& gerc (TC alpha, const cvector& vCol,
              const cvector& vRow) throw (cvmexception);
cmatrix& gemm (const cmatrix& m1, bool bTrans1,
              const cmatrix& m2, bool bTrans2,
              TC dAlpha, TC dBeta) throw (cvmexception);
cmatrix& hemm (bool bLeft, const schmatrix& ms, const cmatrix& m,
              TC dAlpha, TC dBeta) throw (cvmexception);
cmatrix& randomize_real (TR dFrom, TR dTo);
cmatrix& randomize_imag (TR dFrom, TR dTo);
};
```

2.7.1 `cmatrix ()`

Constructor

```
cmatrix::cmatrix ();
```

creates an empty `cmatrix` object. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m;  
std::cout << m.msize() << std::endl  
           << m.nsize() << std::endl;
```

```
m << eye_complex(3);  
std::cout << m;
```

prints

```
0  
0  
(1,0) (0,0) (0,0)  
(0,0) (1,0) (0,0)  
(0,0) (0,0) (1,0)
```

2.7.2 `cmatrix (int,int)`

Constructor

```
cmatrix::cmatrix (int nM, int nN);
```

creates an $m \times n$ `cmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). The constructor throws an exception of type `cvmexception` in case of non-positive sizes passed or memory allocation failure. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m (3, 4);  
std::cout << m.msize() << std::endl  
          << m.nsize() << std::endl  
          << m.size() << std::endl << m;
```

prints

```
3  
4  
12  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)
```

2.7.3 *cmatrix* (TC*,int,int)

Constructor

```
cmatrix::cmatrix (TC* pD, int nM, int nN);
```

creates an $m \times n$ *cmatrix* object where m is passed in nM parameter (number of rows) and n is passed in nN (number of columns). Unlike others, this constructor *does not allocate memory*. It just shares a memory with an array pointed to by pD . The constructor throws an exception of type *cvmexception* in case of non-positive size passed. See also *cmatrix*, *cmatrix (const TC*,int,int)*. Example:

```
using namespace cvm;
double a[] = {5., 5., 5., 5., 5., 5.,
              5., 5., 5., 5., 5., 5.};
cmatrix m ((std::complex<double>*) a, 2, 3);

m(1,1) = std::complex<double>(1.,2.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " "
          << a[2] << " " << a[3] << std::endl;
```

prints

```
(1,2) (5,5) (5,5)
(5,5) (5,5) (5,5)
```

```
1 2 5 5
```

2.7.4 *cmatrix* (const TC*,int,int)

Constructor

```
cmatrix::cmatrix (const TC* pD, int nM, int nN);
```

creates an $m \times n$ *cmatrix* object where m is passed in *nM* parameter (number of rows) and n is passed in *nN* (number of columns) and copies $m * n$ elements of an array *pD* to it by *columns*. The constructor throws an exception of type *cvmexception* in case of non-positive size passed. See also *cmatrix*, *cmatrix* (TC*,int,int). Example:

```
using namespace cvm;
const double a[] = {5., 5., 5., 5., 5., 5.,
                   5., 5., 5., 5., 5., 5.};
cmatrix m ((const std::complex<double>*) a, 2, 3);

m(1,1) = std::complex<double>(1.,2.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " "
          << a[2] << " " << a[3] << std::endl;
```

prints

```
(1,2) (5,5) (5,5)
(5,5) (5,5) (5,5)
```

```
5 5 5 5
```

2.7.5 `cmatrix (const cmatrix&)`

Copy constructor

```
cmatrix::cmatrix (const cmatrix& m);
```

creates a `cmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
              7., 8., 9., 10., 11., 12.};  
cmatrix m ((std::complex<double>*) a, 2, 3);  
cmatrix mc(m);
```

```
m(1,1) = std::complex<double>(7.77,7.77);  
std::cout << m << std::endl << mc;
```

prints

```
(7.77,7.77) (5,6) (9,10)  
(3,4) (7,8) (11,12)
```

```
(1,2) (5,6) (9,10)  
(3,4) (7,8) (11,12)
```

2.7.6 `cmatrix (const cvector&,bool)`

Constructor

```
explicit cmatrix::cmatrix (const cvector& v, bool bBeColumn = true);
```

creates a `cmatrix` object containing `v.size()` rows and 1 column if `bBeColumn` is `true` or 1 row and `v.size()` columns otherwise. After that it copies the vector `v`'s elements to the matrix created. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cmatrix`, `cvector`. Example:

```
using namespace cvm;
```

```
cvector v(3);  
v(1) = std::complex<double>(1.,2.);  
v(2) = std::complex<double>(2.,3.);  
v(3) = std::complex<double>(3.,4.);
```

```
cmatrix mc (v);  
cmatrix mr (v, false);
```

```
std::cout << mc << std::endl << mr;
```

prints

```
(1,2)  
(2,3)  
(3,4)
```

```
(1,2) (2,3) (3,4)
```

2.7.7 `cmatrix (const rmatrix&,bool)`

Constructor

```
explicit cmatrix::cmatrix (const rmatrix& m, bool bRealPart = true);
```

creates a `cmatrix` object containing `m.msize()` rows and `m.nsize()` columns and copies the matrix `m` to its real part if `bRealPart` is true or to its imaginary part otherwise. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cmatrix`, `rmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.};  
const rmatrix m (a, 2, 3);  
cmatrix mr(m), mi(m, false);
```

```
std::cout << mr << std::endl << mi;
```

prints

```
(1,0) (3,0) (5,0)  
(2,0) (4,0) (6,0)  
  
(0,1) (0,3) (0,5)  
(0,2) (0,4) (0,6)
```

2.7.8 `cmatrix (const TR*,const TR*,int,int)`

Constructor

```
cmatrix::cmatrix (const TR* pRe, const TR* pIm,
                 const int nM, const int nN);
```

creates a `cmatrix` object of size `nM` by `nN` and copies every element of arrays pointed to by `pRe` and `pIm` to a real and imaginary part of the matrix created respectively. Use `NULL` pointer to fill up appropriate part with zero values. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
double re[] = {1., 2., 3., 4., 5., 6.};
double im[] = {6., 5., 4., 3., 2., 1.};
cmatrix m (re, im, 3, 2);
std::cout << m << std::endl;
re[1] = 7.77;
std::cout << m << std::endl;
```

```
const double re2[] = {1., 2., 3., 4.};
const cmatrix m2 (re2, NULL, 2, 2);
std::cout << m2;
```

prints

```
(1,6) (4,3)
(2,5) (5,2)
(3,4) (6,1)
```

```
(1,6) (4,3)
(2,5) (5,2)
(3,4) (6,1)
```

```
(1,0) (3,0)
(2,0) (4,0)
```

2.7.9 `cmatrix (const rmatrix&, const rmatrix&)`

Constructor

```
cmatrix::cmatrix (const rmatrix& mRe, const rmatrix& mIm);
```

creates a `cmatrix` object of size `mRe.msize()` by `mRe.nsize()` (if one of these sizes differs from appropriate size of matrix `mIm` then the constructor throws an exception of type `cvmexception`) and copies matrices `mRe` and `mIm` to a real and imaginary part of the matrix created respectively. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cmatrix`, `rmatrix`. Example:

```
using namespace cvm;
```

```
rmatrix mr(3,3), mi(3,3);  
mr(1,1) = 1.;  
mr(2,2) = 2.;  
mr(3,3) = 3.;  
mi(1,3) = 6.;  
mi(2,2) = 5.;  
mi(3,1) = 4.;
```

```
const cmatrix mc(mr, mi);  
std::cout << mc;
```

prints

```
(1,0) (0,0) (0,6)  
(0,0) (2,5) (0,0)  
(0,4) (0,0) (3,0)
```

2.7.10 **submatrix**

Submatrix constructor

```
cmatrix::cmatrix (cmatrix& m, int nRow, int nCol,  
                 int nHeight, int nWidth);
```

creates a `cmatrix` object as a *submatrix* of `m`. It means that the matrix object created shares a memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are *l*-based) and its height and width (parameters `nHeight` and `nWidth`). See also `cmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m(4,5);  
cmatrix subm(m, 2, 2, 2, 2);  
subm.set(std::complex<double>(1.,1.));
```

```
std::cout << m;
```

prints

```
(0,0) (0,0) (0,0) (0,0) (0,0)  
(0,0) (1,1) (1,1) (0,0) (0,0)  
(0,0) (1,1) (1,1) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0) (0,0)
```

2.7.11 operator (,)

Indexing operators

```
TC& cmatrix::operator () (int im, int in) throw (cvmexception);
TC cmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if `im` is outside of `[1,msize()]` range or `in` is outside of `[1,nsize()]` range. The operators are *inherited* in the the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m ((std::complex<double>*) a, 2, 3);
    scmatrix ms(2);
    std::cout << m(1,1) << " "
               << m(2,3) << std::endl << std::endl;
    ms(2,2) = std::complex<double>(7.77,7.77);
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

(1,2) (11,12)

(0,0) (0,0)
(0,0) (7.77,7.77)
```

2.7.12 operator ()

Indexing operators

```
cvector cmatrix::operator () (int i) throw (cvmexception);
const cvector cmatrix::operator () (int i) const throw (cvmexception);
```

provide access to an *i*-th column of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the *i*-th column of the matrix in order to make possible write access to it. The second version creates a *copy* of the column and therefore is *not an l-value*. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if the parameter *i* is outside of `[1, nsize()]` range. The operators are *inherited* in the the classes `scmatrix`, `srbmatrix` and `schmatrix`. See also `cmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                 7., 8., 9., 10., 11., 12.};
    const cmatrix m ((std::complex<double>*) a, 2, 3);
    scmatrix ms(2);

    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(5,6) (7,8)

(0,0) (9,10)
(0,0) (11,12)
```

2.7.13 operator []

Indexing operators

```
cvector cmatrix::operator [] (int i) throw (cvmexception);
const cvector cmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to an *i*-th row of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the *i*-th row of the matrix in order to make possible write access to it. The second version creates a *copy* of the row and therefore is *not an l-value*. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if *i* is outside of `[1,msize()]` range. The operators are *inherited* in the the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m ((std::complex<double>*) a, 2, 3);
    scmatrix ms(3);
    std::cout << m[1] << std::endl;
    ms[1] = m[2];
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

(1,2) (5,6) (9,10)

(3,4) (7,8) (11,12)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.14 diag

Functions

```
cvector cmatrix::diag (int i) throw (cvmexception);
const cvector cmatrix::diag (int i) const throw (cvmexception);
```

provide access to an i -th diagonal of a matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. The first version of the function is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i -th diagonal of the matrix in order to make possible write access to it. The second version creates a *copy* of the diagonal and therefore is *not an l-value*. The functions throw an exception of type `cvmexception` if the parameter i is outside of $[-m\text{size}()+1, n\text{size}()-1]$ range. The functions are *inherited* in the the classes `scmatrix` and `scbmatrix`. The functions are *redefined* in the class `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                 10., 11., 12., 13., 14., 15., 16., 17., 18.};
    cmatrix m (2, 3);
    const scmatrix ms((std::complex<double>*)a, 3);
    m.diag(-1).set(std::complex<double>(1.,1.));
    m.diag(0).set(std::complex<double>(2.,2.));
    m.diag(1).set(std::complex<double>(3.,3.));
    m.diag(2).set(std::complex<double>(4.,4.));
    std::cout << m << std::endl;
    std::cout << ms << std::endl;
    std::cout << ms.diag(0) << ms.diag(1);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

(2,2) (3,3) (4,4)
(1,1) (2,2) (3,3)

(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (9,10) (17,18)
(7,8) (15,16)
```

2.7.15 `real`

Function

```
const rmatrix cmatrix::real () const;
```

creates an object of type `const rmatrix` as a real part of a calling matrix. Please note that, unlike `cvector::real`, this function creates new object *not sharing* a memory with a real part of the calling matrix, i.e. the matrix returned is *not an l-value*. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `rmatrix`, `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);

std::cout << m << std::endl << m.real();

prints

(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

1 5 9
3 7 11
```

2.7.16 `imag`

Function

```
const rmatrix cmatrix::imag () const;
```

creates an object of type `const rmatrix` as an imaginary part of a calling matrix. Please note that, unlike `cvvector::imag`, this function creates new object *not sharing* a memory with an imaginary part of the calling matrix, i.e. the matrix returned is *not an l-value*. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `rmatrix`, `cmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
              7., 8., 9., 10., 11., 12.};  
cmatrix m((std::complex<double>*) a, 2, 3);
```

```
std::cout << m << std::endl << m.imag();
```

prints

```
(1,2) (5,6) (9,10)  
(3,4) (7,8) (11,12)
```

```
2 6 10  
4 8 12
```

2.7.17 `operator = (const cmatrix&)`

Operator

```
cmatrix& cmatrix::operator = (const cmatrix& m) throw (cvmexception);
```

sets an every element of a calling *matrix* to a value of appropriate element of a matrix *m* and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different matrix sizes. The operator is *redefined* in the the classes `scmatrix`, `schmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m1((std::complex<double>*) a, 2, 3);
    cmatrix m2(2, 3);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.18 assign (const TC*)

Function

```
cmatrix& cmatrix::assign (const cvector& v);  
cmatrix& cmatrix::assign (const TC* pD);
```

sets every element of a calling matrix to a value of appropriate element of a vector *v* or an array pointed to by *pD* and returns a reference to the matrix changed. The function is *redefined* in the classes *scmatrix*, *scbmatrix* and *schmatrix*. See also *cmatrix*. Example:

```
using namespace cvm;
```

```
const double a[] = {1., 2., 3., 4., 5., 6.,  
                   7., 8., 9., 10., 11., 12.};
```

```
cmatrix m(2, 3);
```

```
m.assign ((const std::complex<double>*) a);  
std::cout << m;
```

prints

```
(1,2) (5,6) (9,10)  
(3,4) (7,8) (11,12)
```

2.7.19 assign (int, int, const cmatrix&)

Function

```
cmatrix& cmatrix::assign (int nRow, int nCol, const cmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with *l*-based row nRow and column nCol to a matrix m and returns a reference to the matrix changed. The function throws an exception of type *cvmexception* if nRow or nCol are not positive or matrix m doesn't fit. The function is *redefined* in the classes *scmatrix*, *scbmatrix* and *schmatrix*. See also *cmatrix*. Example:

```
using namespace cvm;
```

```
rmatrix m1(4,5);
rmatrix m2(2,2);
m1.set(1.);
m2.set(2.);
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
(1,1) (1,1) (1,1) (1,1) (1,1)
(1,1) (1,1) (2,2) (2,2) (1,1)
(1,1) (1,1) (2,2) (2,2) (1,1)
(1,1) (1,1) (1,1) (1,1) (1,1)
```

2.7.20 `set` (TC)

Function

```
cmatrix& cmatrix::set (TC z);
```

sets every element of a calling matrix to a value of parameter `z` and returns a reference to the matrix changed. Use `vanish` to set every element of a calling matrix to be equal to zero. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m(3, 4);  
m.set(std::complex<double>(3.,4.));  
std::cout << m;
```

prints

```
(3,4) (3,4) (3,4) (3,4)  
(3,4) (3,4) (3,4) (3,4)  
(3,4) (3,4) (3,4) (3,4)
```

2.7.21 `assign_real`

Function

```
cmatrix& cmatrix::assign_real (const rmatrix& mRe) throw (cvmexception);
```

sets real part of every element of a calling matrix to a value of appropriate element of a matrix `mRe` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. See also `cmatrix` and `rmatrix`. The function is *redefined* in the classes `scmatrix`, `schmatrix` and `schmatrix`. Example:

```
using namespace cvm;
```

```
rmatrix m (2,3);  
cmatrix mc(2,3);  
m.randomize (0., 1.);
```

```
mc.assign_real(m);  
std::cout << mc;
```

prints

```
(0.126835,0) (0.57271,0) (0.28312,0)  
(0.784417,0) (0.541673,0) (0.663869,0)
```

2.7.22 `assign_imag`

Function

```
cmatrix& cmatrix::assign_imag (const rmatrix& mIm) throw (cvmexception);
```

sets imaginary part of every element of a calling matrix to a value of appropriate element of a matrix `mIm` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix` and `rmatrix`. Example:

```
using namespace cvm;
```

```
rmatrix m (2,3);  
cmatrix mc(2,3);  
m.randomize (0., 1.);
```

```
mc.assign_imag(m);  
std::cout << mc;
```

prints

```
(0,0.13831) (0,0.267373) (0,0.482345)  
(0,0.50618) (0,0.992401) (0,0.444777)
```

2.7.23 `set_real`

Function

```
cmatrix& cmatrix::set_real (TR d);
```

sets real part of every element of a calling matrix to a value of parameter `d` and returns a reference to the matrix changed. See also `cmatrix`. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m(2,3);  
m.set_real(1.);  
std::cout << m;
```

prints

```
(1,0) (1,0) (1,0)  
(1,0) (1,0) (1,0)
```

2.7.24 `set_imag`

Function

```
cmatrix& cmatrix::set_imag (TR d);
```

sets imaginary part of every element of a calling matrix to a value of parameter `d` and returns a reference to the matrix changed. See also `cmatrix`. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m(2,3);  
m.set_imag(1.);  
std::cout << m;
```

prints

```
(0,1) (0,1) (0,1)  
(0,1) (0,1) (0,1)
```

2.7.25 `resize`

Function

```
cmatrix& cmatrix::resize (int nNewM, int nNewN) throw (cvmexception);
```

changes a size of a calling matrix to `nNewM` by `nNewN` and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                 7., 8., 9., 10., 11., 12.};
    cmatrix m((std::complex<double>*) a, 2, 3);
    std::cout << m << std::endl;
    m.resize (2, 2);
    std::cout << m << std::endl;
    m.resize (3, 3);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

```
(1,2) (5,6)
(3,4) (7,8)
```

```
(1,2) (5,6) (0,0)
(3,4) (7,8) (0,0)
(0,0) (0,0) (0,0)
```

2.7.26 `operator ==`

Operator

```
bool cmatrix::operator == (const cmatrix& m) const;
```

compares a calling matrix with a matrix `m` and returns true if they have the same sizes and their appropriate elements differ by not more than the **smallest normalized positive number**. Returns false otherwise. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m1(2, 3);  
cmatrix m2(2, 3);  
m1.set_real(1.);  
m2.set_real(1.);  
std::cout << (m1 == m2) << std::endl;
```

prints

```
1
```

2.7.27 operator !=

Operator

```
bool cmatrix::operator != (const cmatrix& m) const;
```

compares a calling matrix with a matrix *m* and returns true if they have different sizes or at least of their appropriate elements differs by more than the **smallest normalized positive number**. Returns false otherwise. The operator is *redefined* in the classes **scmatrix**, **scbmatrix** and **schmatrix**. See also **cmatrix**. Example:

```
using namespace cvm;
```

```
cmatrix m1(2, 3);  
cmatrix m2(2, 3);  
m1.set_real(1.);  
m2.set_real(1.);  
std::cout << (m1 != m2) << std::endl;
```

prints

```
0
```

2.7.28 operator <<

Operator

```
cmatrix& cmatrix::operator << (const cmatrix& m) throw (cvmexception);
```

destroys a calling matrix, creates a new one as a copy of *m* and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    cmatrix m(2,3);
    cmatrix mc(1,1);
    m(1,2) = std::complex<double>(1.,2.);
    m(2,3) = std::complex<double>(2.,4.);
    std::cout << m << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,0) (1,2) (0,0)
(0,0) (0,0) (2,4)
(0,0)
```

```
(0,0) (1,2) (0,0)
(0,0) (0,0) (2,4)
```

2.7.29 operator +

Operator

```
cmatrix cmatrix::operator + (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as the sum of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cvector::sum`, `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                 7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << ma + mb << std::endl;
    std::cout << ma + ma;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,3) (6,7) (10,11)
(4,5) (8,9) (12,13)

(2,4) (10,12) (18,20)
(6,8) (14,16) (22,24)
```

2.7.30 operator -

Operator

```
cmatrix cmatrix::operator - (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as the difference of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cvector::diff`, `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << ma - mb << std::endl;
    std::cout << ma - ma;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,1) (4,5) (8,9)
(2,3) (6,7) (10,11)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.31 sum

Function

```
cmatrix& cmatrix::sum (const cmatrix& m1, const cmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of matrices *m1* and *m2* to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator +`, `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                 7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    cmatrix m (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << m.sum(ma, mb) << std::endl;
    std::cout << m.sum(m, mb);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,3) (6,7) (10,11)
(4,5) (8,9) (12,13)

(3,4) (7,8) (11,12)
(5,6) (9,10) (13,14)
```

2.7.32 diff

Function

```
cmatrix& cmatrix::diff (const cmatrix& m1, const cmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of matrices *m1* and *m2* to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator -`, `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                 7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    cmatrix m (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << m.diff(ma, mb) << std::endl;
    std::cout << m.diff(m, mb);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,1) (4,5) (8,9)
(2,3) (6,7) (10,11)

(-1,0) (3,4) (7,8)
(1,2) (5,6) (9,10)
```

2.7.33 operator +=

Operator

```
cmatrix& cmatrix::operator += (const cmatrix& m) throw (cvmexception);
```

adds a matrix *m* to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator +`, `cmatrix::sum`, `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    cmatrix m1(2, 3);
    cmatrix m2(2, 3);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(2.,2.));

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3,3) (3,3) (3,3)
(3,3) (3,3) (3,3)

(4,4) (4,4) (4,4)
(4,4) (4,4) (4,4)
```

2.7.34 operator -=

Operator

```
cmatrix& cmatrix::operator -= (const cmatrix& m) throw (cvmexception);
```

subtracts a matrix *m* from a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator -`, `cmatrix::diff`, `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    cmatrix m1(2, 3);
    cmatrix m2(2, 3);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(2.,2.));

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-1,-1) (-1,-1) (-1,-1)
(-1,-1) (-1,-1) (-1,-1)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.35 operator - ()

Operator

```
cmatrix cmatrix::operator - () const throw (cvmexception);
```

creates an object of type `cmatrix` as a calling matrix multiplied by -1 . The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
              7., 8., 9., 10., 11., 12.};  
const cmatrix ma ((std::complex<double>*) a, 2, 3);
```

```
std::cout << - ma;
```

prints

```
(-1,-2) (-5,-6) (-9,-10)  
(-3,-4) (-7,-8) (-11,-12)
```

2.7.36 operator * (TR)

Operator

```
cmatrix cmatrix::operator * (TR d) const;
```

creates an object of type `cmatrix` as a product of a calling matrix and a real number `d`. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator *=`, `cmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
              7., 8., 9., 10., 11., 12.};  
const cmatrix ma ((std::complex<double>*) a, 2, 3);
```

```
std::cout << ma * 5.;
```

prints

```
(5,10) (25,30) (45,50)  
(15,20) (35,40) (55,60)
```

2.7.37 operator / (TR)

Operator

```
cmatrix cmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `cmatrix` as a quotient of a calling matrix and a real number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the **smallest normalized positive number**. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /=`, `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                 7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);

    std::cout << ma / 4.;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.25,0.5) (1.25,1.5) (2.25,2.5)
(0.75,1) (1.75,2) (2.75,3)
```

2.7.38 operator * (TC)

Operator

```
cmatrix cmatrix::operator * (TC z) const;
```

creates an object of type `cmatrix` as a product of a calling matrix and a complex number `z`. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator *=`, `cmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
              7., 8., 9., 10., 11., 12.};  
const cmatrix ma ((std::complex<double>*) a, 2, 3);
```

```
std::cout << ma * std::complex<double>(5.,2.);
```

prints

```
(1,12) (13,40) (25,68)  
(7,26) (19,54) (31,82)
```

2.7.39 operator / (TC)

Operator

```
cmatrix cmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `cmatrix` as a quotient of a calling matrix and a complex number `z`. It throws an exception of type `cvmexception` if `z` has an absolute value equal or less than the **smallest normalized positive number**. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /=`, `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);

    std::cout << ma / std::complex<double>(4.,2.);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.4,0.3) (1.6,0.7) (2.8,1.1)
(1,0.5) (2.2,0.9) (3.4,1.3)
```

2.7.40 operator *= (TR)

Operator

```
cmatrix& cmatrix::operator *= (TR d);
```

multiplies a calling matrix by a real number *d* and returns a reference to the matrix changed. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator *`, `cmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
              7., 8., 9., 10., 11., 12.};  
cmatrix ma ((std::complex<double>*) a, 2, 3);
```

```
ma *= 2.;  
std::cout << ma;
```

prints

```
(2,4) (10,12) (18,20)  
(6,8) (14,16) (22,24)
```

2.7.41 operator /= (TR)

Operator

```
cmatrix& cmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling matrix by a real number *d* and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if *d* has an absolute value equal or less than the `smallest normalized positive number`. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /`, `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    cmatrix ma ((std::complex<double>*) a, 2, 3);

    ma /= 2.;
    std::cout << ma;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.5,1) (2.5,3) (4.5,5)
(1.5,2) (3.5,4) (5.5,6)
```

2.7.42 operator *= (TC)

Operator

```
cmatrix& cmatrix::operator *= (TC z);
```

multiplies a calling matrix by a complex number *z* and returns a reference to the matrix changed. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator *`, `cmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
             7., 8., 9., 10., 11., 12.};  
cmatrix ma ((std::complex<double>*) a, 2, 3);
```

```
ma *= std::complex<double>(2.,1.);  
std::cout << ma;
```

prints

```
(0,5) (4,17) (8,29)  
(2,11) (6,23) (10,35)
```

2.7.43 operator /= (TC)

Operator

```
cmatrix& cmatrix::operator /= (TC z) throw (cvmexception);
```

divides a calling matrix by a complex number *z* and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if *z* has an absolute value equal or less than the `smallest normalized positive number`. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /`, `cmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                 7., 8., 9., 10., 11., 12.};
    cmatrix ma ((std::complex<double>*) a, 2, 3);

    ma /= std::complex<double>(2.,1.);
    std::cout << ma;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.8,0.6) (3.2,1.4) (5.6,2.2)
(2,1) (4.4,1.8) (6.8,2.6)
```

2.7.44 **normalize**

Function

```
cmatrix& cmatrix::normalize ();
```

normalizes a calling matrix so its **Euclidean norm** becomes equal to 1 if it was greater than the **smallest normalized positive number** before the call (otherwise the function does nothing). The function is *redefined* in the classes **scmatrix**, **scbmatrix** and **schmatrix**. See also **cmatrix**. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
              7., 8., 9., 10., 11., 12.};  
cmatrix ma ((std::complex<double>*) a, 2, 3);
```

```
ma.normalize();  
std::cout << ma << ma.norm() << std::endl;
```

prints

```
(0.0392232,0.0784465) (0.196116,0.235339) (0.353009,0.392232)  
(0.11767,0.156893) (0.274563,0.313786) (0.431455,0.470679)  
1
```

2.7.45 conjugation

Operator and functions

```
cmatrix cmatrix::operator ~ () const throw (cvmexception);
cmatrix& cmatrix::conj (const cmatrix& m) throw (cvmexception);
cmatrix& cmatrix::conj () throw (cvmexception);
```

encapsulate complex matrix conjugation. First operator creates an object of type `cmatrix` as a conjugated calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `m` conjugated (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to conjugated itself (it also throws an exception of type `cvmexception` in case of memory allocation failure). The functions are *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);
cmatrix mc(3,2);
std::cout << m << std::endl << ~m << std::endl ;
mc.conj(m);
std::cout << mc << std::endl;
mc.conj();
std::cout << mc;
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

```
(1,-2) (3,-4)
(5,-6) (7,-8)
(9,-10) (11,-12)
```

```
(1,-2) (3,-4)
(5,-6) (7,-8)
(9,-10) (11,-12)
```

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.46 `operator * (const cvector&)`

Operator

```
cvector cmatrix::operator * (const cvector& v) const  
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `cvector::mult` in order to get rid of a new object creation. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`, `cvector`. Example:

```
using namespace cvm;  
  
cmatrix m(2, 3);  
cvector v(3);  
m.set(std::complex<double>(1.,1.));  
v.set(std::complex<double>(1.,1.));  
  
std::cout << m * v;  
  
prints  
  
(0,6) (0,6)
```

2.7.47 operator * (const cmatrix&)

Operator

```
cmatrix cmatrix::operator * (const cmatrix& m) const  
throw (cvmexception);
```

creates an object of type `cmatrix` as a product of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `cmatrix::mult` in order to get rid of a new object creation. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m1(2, 3);  
cmatrix m2(3, 2);  
m1.set(std::complex<double>(1.,1.));  
m2.set(std::complex<double>(1.,1.));
```

```
std::cout << m1 * m2;
```

prints

```
(0,6) (0,6)  
(0,6) (0,6)
```

2.7.48 `mult`

Function

```
cmatrix& cmatrix::mult (const cmatrix& m1, const cmatrix& m2)
throw (cvmexception);
```

sets a calling matrix to be equal to a product of a matrix `m1` by a matrix `m2` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the class `scmatrix` and *redefined* in the classes `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m1(2, 3);
cmatrix m2(3, 2);
scmatrix m(2);
m1.set(std::complex<double>(1.,1.));
m2.set(std::complex<double>(1.,1.));
```

```
m.mult(m1, m2);
std::cout << m;
```

prints

```
(0,6) (0,6)
(0,6) (0,6)
```

2.7.49 ranklupdate_u

Function

```
cmatrix&
cmatrix::ranklupdate_u (const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

sets a calling matrix to be equal to the **rank-1 update** (unconjugated) of vectors `vCol` and `vRow` and returns a reference to the matrix changed. The function throws an exception of type **cvmexception** if the number of rows of the calling matrix is not equal to `vCol.size()` or the number of columns of the calling matrix is not equal to `vRow.size()`. The function is *inherited* in the the class **scmatrix** and *not applicable* to objects of the classes **scbmatrix** and **schmatrix** (i.e. an exception of type **cvmexception** would be thrown in case of using it for objects of those classes). See also **cvector::ranklupdate_u**, **cmatrix**. Example:

```
using namespace cvm;

cvector vc(3), vr(2);
cmatrix m(3, 2);
vc.set(std::complex<double>(1.,1.));
vr.set(std::complex<double>(1.,1.));

std::cout << m.ranklupdate_u (vc, vr);

prints

(0,2) (0,2)
(0,2) (0,2)
(0,2) (0,2)
```

2.7.50 `ranklupdate_c`

Function

```
cmatrix&  
cmatrix::ranklupdate_c (const cvector& vCol, const cvector& vRow)  
throw (cvmexception);
```

sets a calling matrix to be equal to the **rank-1 update** (conjugated) of vectors `vCol` and `vRow` and returns a reference to the matrix changed. The function throws an exception of type **cvmexception** if the number of rows of the calling matrix is not equal to `vCol.size()` or the number of columns of the calling matrix is not equal to `vRow.size()`. The function is *inherited* in the the class **scmatrix** and *not applicable* to objects of the classes **scbmatrix** and **schmatrix** (i.e. an exception of type **cvmexception** would be thrown in case of using it for objects of those classes). See also **cvector::ranklupdate_c**, **cmatrix**. Example:

```
using namespace cvm;  
  
cvector vc(3), vr(2);  
cmatrix m(3, 2);  
vc.set(std::complex<double>(1.,1.));  
vr.set(std::complex<double>(1.,1.));  
  
std::cout << m.ranklupdate_c (vc, vr);  
  
prints  
  
(2,0) (2,0)  
(2,0) (2,0)  
(2,0) (2,0)
```

2.7.51 swap_rows

Function

```
cmatrix& cmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. *n1* and *n2* are the numbers of rows to be swapped, both are *ℓ-based*). The function throws an exception of type *cvmexception* if one of the parameters is outside of the range $[1, \text{msize}()]$. The function is *redefined* in the the class *scmatrix* and *not applicable* to objects of the classes *scbmatrix* and *schmatrix* (i.e. an exception of type *cvmexception* would be thrown in case of using it for objects of those classes). See also *cmatrix*. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m ((std::complex<double>*)a, 3, 2);
```

```
std::cout << m << std::endl;
std::cout << m.swap_rows(2,3);
```

prints

```
(1,2) (7,8)
(3,4) (9,10)
(5,6) (11,12)
```

```
(1,2) (7,8)
(5,6) (11,12)
(3,4) (9,10)
```

2.7.52 `swap_cols`

Function

```
cmatrix& cmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of columns to be swapped, both are *l*-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1, nsize()]`. The function is *redefined* in the the class `scmatrix` and *not applicable* to objects of the classes `schmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `cmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
              7., 8., 9., 10., 11., 12.};  
cmatrix m ((std::complex<double>*)a, 2, 3);
```

```
std::cout << m << std::endl;  
std::cout << m.swap_cols(2,3);
```

prints

```
(1,2) (5,6) (9,10)  
(3,4) (7,8) (11,12)  
  
(1,2) (9,10) (5,6)  
(3,4) (11,12) (7,8)
```

2.7.53 solve

Functions

```

cmatrix&
cmatrix::solve (const scmatrix& mA,
                const cmatrix& mB, TR& dErr) throw (cvmexception);

cmatrix&
cmatrix::solve (const scmatrix& mA,
                const cmatrix& vB) throw (cvmexception);

```

set a calling matrix to be equal to a solution X of the matrix linear equation $AX = B$ where the parameter `mA` is the square matrix A and the parameter `vB` is the matrix B . Every function returns a reference to the matrix changed. The first version also sets the output parameter `dErr` to be equal to the norm of computation error. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to singular. The functions are *redefined* in the the class `scmatrix` and *inherited* thereafter in the classes `scbmatrix` and `schmatrix`. See also `cvvector::solve`, `cmatrix`, `scmatrix`. Example:

```

using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);

scmatrix ma(3);
cmatrix mb(3,2);
cmatrix mx(3,2);
double dErr;
ma.randomize_real(0.,10.); ma.randomize_imag(0.,10.);
mb.randomize_real(0.,10.); mb.randomize_imag(0.,10.);

mx.solve (ma, mb, dErr);
std::cout << mx << std::endl << ma * mx - mb;

prints

(-4.7267e-01,-1.2929e+00) (1.5912e+00,-2.1678e-01)
(1.3961e+00,6.3385e-01) (-9.2872e-01,1.6793e-01)
(7.0048e-01,8.1763e-01) (-1.7369e-01,1.1588e-01)

(0.0000e+00,-8.8818e-16) (1.7764e-15,8.8818e-16)
(-4.4409e-16,-8.8818e-16) (0.0000e+00,-8.8818e-16)
(0.0000e+00,-1.7764e-15) (8.8818e-16,4.4409e-16)

```

2.7.54 solve_lu

Functions

```
cmatrix&
cmatrix::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                  const int* pPivots, const cmatrix& mB, TR& dErr)
                  throw (cvmexception);
```

```
cmatrix&
cmatrix::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                  const int* pPivots, const cmatrix& vB)
                  throw (cvmexception);
```

set a calling matrix to be equal to a solution X of the matrix linear equation $AX = B$ where the parameter `mA` is the square complex matrix A , parameter `mLU` is [LU factorization](#) of the matrix A , parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and the parameter `vB` is the matrix B . Every function returns a reference to the matrix changed. The first version also sets the output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $AX = B$ with the same matrix A and different matrices B . In such case you save on matrix A factorization since it's needed to be performed just one time. The functions throw exception of type [cvmexception](#) in case of inappropriate sizes of the operands or when the matrix A is close to singular. See also [cvector::solve](#), [cmatrix](#), [scmatrix](#). Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    scmatrix ma(3);
    scmatrix mLU(3);
    cmatrix mb1(3,2);
    cmatrix mb2(3,2);
    cmatrix mx1(3,2);
    cmatrix mx2(3,2);
    iarray nPivots(3);
    double dErr = 0.;
    ma.randomize_real(0.,10.); ma.randomize_imag(0.,10.);
    mb1.randomize_real(0.,10.); mb1.randomize_imag(0.,10.);
    mb2.randomize_real(0.,10.); mb2.randomize_imag(0.,10.);

    mLU.low_up(ma, nPivots);
    std::cout << mx1.solve_lu (ma, mLU, nPivots, mb1, dErr);
```

```
    std::cout << dErr << std::endl;
    std::cout << mx2.solve_lu (ma, mLU, nPivots, mb2) << std::endl;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(4.2888e-01,8.2409e-02) (-1.1261e-01,-5.7778e-01)
(5.8052e-01,3.2179e-01) (2.5811e-01,-3.8609e-02)
(-3.1499e-02,-7.0014e-01) (1.2652e+00,4.5309e-01)
5.2931e-15
(3.0153e-01,-5.6606e-01) (-1.6308e-01,1.8217e-01)
(7.4971e-01,-1.1305e-01) (5.2187e-01,2.3441e-01)
(-1.9916e-01,1.4493e+00) (9.1046e-02,3.5242e-01)

(0.0000e+00,-8.8818e-16) (0.0000e+00,-8.8818e-16)
(-4.4409e-16,0.0000e+00) (0.0000e+00,-8.8818e-16)
(0.0000e+00,0.0000e+00) (0.0000e+00,0.0000e+00)

(-8.8818e-16,8.8818e-16) (-8.8818e-16,-1.7764e-15)
(0.0000e+00,0.0000e+00) (2.2204e-16,-8.8818e-16)
(4.4409e-16,-8.8818e-16) (1.3878e-17,-4.4409e-16)
```

2.7.55 svd

Functions

rvector

`cmatrix::svd () throw (cvmexception);`

rvector

`cmatrix::svd (scmatrix& mU, scmatrix& mVH) throw (cvmexception);`

create an object of type `rvector` as a vector of **singular values** of a calling matrix. The second version of the function set the output parameter `mU` to be equal to the matrix `U` of size $m \times m$ (and change the size of the object if it's needed) and `mVH` to be equal to the matrix V^H of size $n \times n$. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. Use `rvector::svd` in order to get rid of a new vector creation. The function is *redefined* in the the classes `scmatrix`, `scbmatrix`, `schmatrix`. See also `rvector`, `cmatrix`. Example:

`using namespace cvm;``std::cout.setf (std::ios::scientific | std::ios::left);``std::cout.precision (3);``try {`

```
double m[] = {1., -1., 1., 2., -2., 1.,
              3., -2., 1., 0., -2., 1.};
```

`cmatrix mA((std::complex<double>*) m, 2, 3);``cmatrix mSigma(2,3);``rvector v(2);``scmatrix mU(2), mVH(3);``v = mA.svd(mU, mVH);``mSigma.diag(0) = cvector(v);``std::cout << mU << std::endl;``std::cout << mVH << std::endl;``std::cout << mSigma << std::endl;``std::cout << (mA * ~mVH - mU * mSigma).norm() << std::endl;``std::cout << (~mA * mU - ~(mSigma * mVH)).norm() << std::endl;``}``catch (exception& e) {``std::cout << "Exception " << e.what () << std::endl;``}`

prints

(-4.861e-01,0.000e+00) (8.739e-01,0.000e+00)
(7.956e-01,-3.616e-01) (4.425e-01,-2.012e-01)

(-7.590e-02,4.474e-01) (7.488e-01,-1.820e-01) (-4.474e-01,1.327e-02)
(8.084e-01,1.878e-01) (-1.576e-02,5.238e-01) (-1.878e-01,3.558e-02)
(1.065e-01,3.065e-01) (3.597e-01,4.669e-02) (8.727e-01,4.012e-02)

(5.452e+00,0.000e+00) (0.000e+00,0.000e+00) (0.000e+00,0.000e+00)
(0.000e+00,0.000e+00) (1.131e+00,0.000e+00) (0.000e+00,0.000e+00)

1.357e-15

1.267e-15

2.7.56 pinv

Functions

```

cmatrix  cmatrix::pinv (TR threshold = cvmMachSp()) const
            throw (cvmexception);
cmatrix& cmatrix::pinv (const basic_rmatrix& mA,
            TR threshold = cvmMachSp())
            throw (cvmexception);

```

encapsulate complex matrix pseudo inversion [6], p. 33 (or Moore-Penrose generalized inversion [5], p. 421). Strictly defined, an $n \times m$ matrix A^+ is a *pseudo inversion* of $m \times n$ matrix A if the following two equations are satisfied:

$$\begin{aligned}
 AA^+A &= A, \\
 A^+ &= QA^H = A^HP
 \end{aligned}$$

where Q and P are some matrices. To compute the pseudo inversion, we use [Singular Value Decomposition \(SVD\)](#)

$$A = U\Sigma V^H$$

of matrix A , thus

$$A^+ = V\Sigma^{-1}U^H,$$

where Σ^{-1} is a diagonal $n \times m$ matrix having inverted diagonal values of the matrix Σ if they are greater than some threshold, and zeros otherwise.

First version creates an object of type `cmatrix` as a pseudo inverted calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `mA` pseudo inverted (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands). The threshold parameter sets a minimum distinguishable from zero singular value to be used to compute the pseudo inversion. All values equal or less than the threshold are treated as zeros. The functions are *inherited* in the classes `scmatrix` and `schmatrix` and *redefined* in `scbmatrix`. See also `cmatrix`. Example:

```

using namespace cvm;

try {
    cmatrix mA(2,3);
    mA(1,1) = tcomplex(1.,2.);  mA(1,2) = tcomplex(-1.,3.);
    mA(1,3) = tcomplex(0.,-1.);
    mA(2,1) = tcomplex(1.,-2.); mA(2,2) = tcomplex(1.,-4.);
    mA(2,3) = tcomplex(1.,0.);

    cmatrix mX = mA.pinv();
    std::cout << mX << (mA * mX * mA - mA).norm2() << std::endl;
}

```

```

}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

(+3.2407407e-01,-1.1111111e-01) (+2.5925926e-01,-1.8518519e-02)
(-1.3888889e-01,-2.7777778e-02) (-5.5555556e-02,+1.6666667e-01)
(+1.4814815e-01,+2.3148148e-01) (+1.6666667e-01,+1.4814815e-01)
+3.3217718e-15

```

Band matrix example:

```

using namespace cvm;

try {
    scbmatrix mA (40, 1, 0);
    mA.diag(0).randomize_real(-1.,1.);
    mA.diag(0).randomize_imag(-3.,2.);
    mA.diag(-1).randomize_real(5.,10.);
    mA.diag(-1).randomize_imag(-3.,7.);

    scmatrix mX (40);
    mX.pinv(mA);
    std::cout << (mA * mX * mA - mA).norm2() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

+6.3113999e-14

```

2.7.57 gels

Functions

```

cmatrix  cmatrix::gels (bool transpose, const cmatrix& mB,
                      cvector& vErr) const throw (cvmexception);
cmatrix& cmatrix::gels (bool transpose,
                      const cmatrix& mA, const cmatrix& mB,
                      cvector& vErr) throw (cvmexception);
cvector  cmatrix::gels (bool transpose, const cvector& mB,
                      TC& cErr) const throw (cvmexception);

```

solve overdetermined or underdetermined linear systems

$$A * x = b$$

for $m \times n$ matrix A (or its conjugate-transpose) where b is a vector of length k or systems

$$A * X = B$$

for multiple vectors b stored as columns of $k \times l$ matrix B where $k = m$ in non-transposed case and $k = n$ otherwise. The algorithm uses a QR or LQ factorization of A . It is assumed that A has full rank, infinity returned otherwise. Internally the functions use ?GELS LAPACK routines. If $m > n$ and `transpose=false` or $m < n$ and `transpose=true`, then the system is overdetermined, thus the algorithm tries to find the least squares solution x of the problem

$$\|A * x - b\|_2 \rightarrow \min \quad \text{or} \quad \|A^H * x - b\|_2 \rightarrow \min$$

respectively. Output vector `vErr` of length l (or complex number `cErr` for single vector b) returns residual sum of squares. The system is underdetermined otherwise, and the algorithm finds its minimum norm solution. In this case `vErr` (or `cErr`) is set to zero. In both cases the solution computed satisfies $x = \text{pinv}(A) * b$, but this algorithm is faster than pseudo inversion. Matrix A is passed as an argument `mA` or (in second case) it's a calling object. First and third versions return the solution as a new matrix or vector object. Second version sets calling object to be the solution. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in all complex matrix classes. See also `cmatrix`, `cvector::gels`, `cmatrix::pinv`. Example:

```
using namespace cvm;
```

```

cmatrix a(7, 5);
cmatrix b(7, 2);
cmatrix bt(5, 2);
cvector bv(5);

```

```
cvector vErr(2);
tcomplex dErr;
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
bt.randomize_real(-10., 10.);
bt.randomize_imag(-10., 10.);
bv.randomize_real(-10., 10.);
bv.randomize_imag(-10., 10.);

cmatrix x = a.gels(false, b, vErr);
cvector vt = a.gels(true, bv, dErr);
cmatrix xt(7,2);
xt.gels(true, a, bt, vErr);

std::cout << (a.pinv()*b - x).norm() << std::endl;
std::cout << (~a.pinv()*bv - vt).norm() << std::endl;
std::cout << (~a.pinv()*bt - xt).norm() << std::endl;

prints

+1.3258216e-015
+1.0696705e-015
+1.1685394e-015
```

2.7.58 gelsy

Functions

```

cmatrix  cmatrix::gelsy (const cmatrix& mB, int& rank,
                        TR tol = cvmMachSp()) const throw (cvmexception);
cmatrix& cmatrix::gelsy (const cmatrix& mA, const cmatrix& mB, int& rank,
                        TR tol = cvmMachSp()) throw (cvmexception);
cvector  cmatrix::gelsy (const cvector& mB, int& rank,
                        TR tol = cvmMachSp()) const throw (cvmexception);

```

compute the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using a complete orthogonal factorization of $m \times n$ matrix A . Here b is a vector of length m . Multiple vectors b can be stored as columns of $m \times l$ matrix B . Matrix A may be rank-deficient, the functions return its effective rank in `rank` output parameter using `tol` tolerance. Internally the functions use ?GELSY LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as an argument `mA` or (in second case) it's a calling object. First and third versions return the solution as a new matrix or vector object. Second version sets calling object to be the solution. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in all complex matrix classes. See also `cmatrix`, `cvector::gelsy`, `cmatrix::gelss`, `cmatrix::gelsd`. Example:

```
using namespace cvm;
```

```

cmatrix a(4, 5);
cmatrix b(4, 2);
cvector bv(4);
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
bv.randomize_real(-10., 10.);
bv.randomize_imag(-10., 10.);
tint rank;

cmatrix x = a.gelsy(b, rank);
cvector xv = a.gelsy(bv, rank);
cmatrix x2(5,2);
x2.gelsy(a, b, rank);

std::cout << (a*x - b).norm() << std::endl;

```

```
std::cout << (a*xv - bv).norm() << std::endl;  
std::cout << (a*x2 - b).norm() << std::endl;  
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+8.9701623e-015  
+6.9342294e-015  
+8.9701623e-015  
+4 +4
```

2.7.59 gelss

Functions

```

cmatrix  cmatrix::gelss (const cmatrix& mB, int& rank, rvector& sv,
                        TR tol = cvmMachSp()) const throw (cvmexception);
cmatrix& cmatrix::gelss (const cmatrix& mA, const cmatrix& mB,
                        rvector& sv, int& rank, TR tol = cvmMachSp())
                        throw (cvmexception);
cvector  cmatrix::gelss (const cvector& mB, rvector& sv, int& rank,
                        TR tol = cvmMachSp()) const throw (cvmexception);

```

compute the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using the singular value decomposition of $m \times n$ matrix A . Here b is a vector of length m . Multiple vectors b can be stored as columns of $m \times l$ matrix B . Matrix A may be rank-deficient, the functions return its effective rank in `rank` output parameter using `tol` tolerance. These functions also compute singular values of A in decreasing order and return them in `sv` output parameter having $\min(m, n)$ size. Internally the functions use ?GELSS LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as an argument `mA` or (in second case) it's a calling object. First and third versions return the solution as a new matrix or vector object. Second version sets calling object to be the solution. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in all complex matrix classes. See also `cmatrix`, `cvector::gelss`, `cmatrix::gelsy`, `cmatrix::gelsd`. Example:

```

using namespace cvm;

cmatrix a(4, 5);
cmatrix b(4, 2);
cvector bv(4);
rvector sv(4);
tint rank;
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
bv.randomize_real(-10., 10.);
bv.randomize_imag(-10., 10.);

cmatrix x = a.gelss(b, sv, rank);
cvector xv = a.gelss(bv, sv, rank);

```

```
cmatrix xt(5,2);
xt.gelss(a, b, sv, rank);

std::cout << (a*x - b).norm() << std::endl;
std::cout << (a*xv - bv).norm() << std::endl;
std::cout << (a*xt - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+2.3444861e-013
+4.6335022e-014
+2.3444861e-013
+1.1234667e-014
+4 +4
```

2.7.60 gelsd

Functions

```

cmatrix  cmatrix::gelsd (const cmatrix& mB, int& rank, rvector& sv,
                        TR tol = cvmMachSp()) const throw (cvmexception);
cmatrix& cmatrix::gelsd (const cmatrix& mA, const cmatrix& mB,
                        rvector& sv, int& rank, TR tol = cvmMachSp())
                        throw (cvmexception);
cvector  cmatrix::gelsd (const cvector& mB, rvector& sv, int& rank,
                        TR tol = cvmMachSp()) const throw (cvmexception);

```

compute the minimum-norm solution to a linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using the singular value decomposition of $m \times n$ matrix A and a divide and conquer method. Here b is a vector of length m . Multiple vectors b can be stored as columns of $m \times l$ matrix B . Matrix A may be rank-deficient, the functions return its effective rank in `rank` output parameter using `tol` tolerance. These functions also compute singular values of A in decreasing order and return them in `sv` output parameter having $\min(m, n)$ size. Internally the functions use ?GELSD LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as an argument `mA` or (in second case) it's a calling object. First and third versions return the solution as a new matrix or vector object. Second version sets calling object to be the solution. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in all complex matrix classes. See also `cmatrix`, `cvector::gelsd`, `cmatrix::gelsy`, `cmatrix::gelss`. Example:

```

using namespace cvm;

cmatrix a(4, 5);
cmatrix b(4, 2);
cvector bv(4);
rvector sv(4);
tint rank;
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
bv.randomize_real(-10., 10.);
bv.randomize_imag(-10., 10.);

cmatrix x = a.gelsd(b, sv, rank);
cvector xv = a.gelsd(bv, sv, rank);

```

```
cmatrix xt(5,2);
xt.gelsd(a, b, sv, rank);

std::cout << (a*x - b).norm() << std::endl;
std::cout << (a*xv - bv).norm() << std::endl;
std::cout << (a*xt - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+1.2865301e-014
+1.2824878e-014
+1.2865301e-014
+1.2809491e-014
+4 +4
```

2.7.61 rank

Function

```
int cmatrix::rank (TR eps = cvmMachSp()) const throw (cvmexception);
```

returns a rank of a calling matrix as a number of **singular values** with **normalized** absolute value greater than or equal to a parameter **eps** (this is the **largest relative spacing** by default). The function throws an exception of type **cvmexception** in case of convergence error. The function is *inherited* in the the classes **scmatrix**, **scbmatrix**, **schmatrix**. See also **cmatrix**. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
              7., 8., 9., 10., 11., 12.};  
cmatrix m(a, NULL, 3, 4);
```

```
std::cout << m << m.rank() << std::endl;  
m(3,4) = std::complex<double>(0.,1.);  
std::cout << m.rank() << std::endl;
```

prints

```
(1,0) (4,0) (7,0) (10,0)  
(2,0) (5,0) (8,0) (11,0)  
(3,0) (6,0) (9,0) (12,0)  
2  
3
```

2.7.62 qr

Functions

```
void cmatrix::qr (cmatrix& mQ, scmatrix& mR) const throw (cvmexception);
void cmatrix::qr (scmatrix& mQ, cmatrix& mR) const throw (cvmexception);
```

compute QR factorization as

$$M = QR$$

where M is a calling matrix, unitary matrix Q and upper triangular matrix R are mQ and mR respectively. First version encapsulates so-called "economy" algorithm which for a given $m \times n$ matrix M computes $m \times n$ matrix Q and $n \times n$ matrix R . Second version is a "full" mode one computing $m \times m$ matrix Q and $m \times n$ matrix R . The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *redefined* in the the class `scmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;
```

```
treal ar[] = {1., 2., 3., 4., 5., 6.};
treal ai[] = {1., -1., 2., -2., 3., -3.};
const cvm::cmatrix mh(ar, ai, 2, 3);
const cvm::cmatrix mv(ar, ai, 3, 2);
cvm::scmatrix s2(2), s3(3);
cvm::cmatrix h(2,3), v(3,2);

mh.qr(h,s3);
std::cout << (eye_complex(2)-~cmatrix(h,1,1,2,2)*cmatrix(h,1,1,2,2)).norm()
  << " " << (mh - h * s3).norm() << std::endl;
mh.qr(s2,h);
std::cout << (eye_complex(2) - ~s2 * s2).norm()
  << " " << (mh - s2 * h).norm() << std::endl;
mv.qr(v,s2);
std::cout << (eye_complex(2) - ~v * v).norm()
  << " " << (mv - v * s2).norm() << std::endl;
mv.qr(s3,v);
std::cout << (eye_complex(3) - ~s3 * s3).norm()
  << " " << (mv - s3 * v).norm() << std::endl;
```

prints

```
+2.5145832e-16 +1.3506446e-15
+2.5145832e-16 +1.3506446e-15
+2.5367068e-16 +1.4432899e-15
+3.8435519e-16 +1.4432899e-15
```

2.7.63 lq

Functions

```
void cmatrix::lq (scmatrix& mL, cmatrix& mQ) const throw (cvmexception);
void cmatrix::lq (cmatrix& mL, scmatrix& mQ) const throw (cvmexception);
```

compute LQ factorization as

$$M = LQ$$

where M is a calling matrix, upper triangular (trapezoidal) matrix L and unitary matrix Q are mL and mQ respectively. First version encapsulates so-called "economy" algorithm which for a given $m \times n$ matrix M computes $m \times m$ matrix L and $m \times n$ matrix Q . Second version is a "full" mode one computing $m \times n$ matrix L and $n \times n$ matrix Q . The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *redefined* in the the class `scmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6.};
treal ai[] = {1., -1., 2., -2., 3., -3.};
const cvm::cmatrix mh(ar, ai, 2, 3);
const cvm::cmatrix mv(ar, ai, 3, 2);
cvm::scmatrix s2(2), s3(3);
cvm::cmatrix h(2,3), v(3,2);

mh.lq(s2,h);
std::cout << (eye_complex(2) - h * ~h).norm() << " " << (mh - s2 * h)
           .norm() << std::endl;
mv.lq(s3,v);
std::cout << (eye_complex(2) - ~cmatrix(v,1,1,2,2) * cmatrix(v,1,1,2,2))
           .norm() << " " << (mv - s3 * v).norm() << std::endl;
mh.lq(h,s3);
std::cout << (eye_complex(3) - s3 * ~s3).norm() << " " << (mh - h * s3)
           .norm() << std::endl;
mv.lq(v,s2);
std::cout << (eye_complex(2) - s2 * ~s2).norm() << " " << (mv - v * s2)
           .norm() << std::endl;
```

prints

```
+1.3374617e-015 +7.4806297e-015
+6.6728929e-016 +2.9205408e-015
+1.4349634e-015 +7.4806297e-015
+6.7217793e-016 +2.9205408e-015
```

2.7.64 rq

Functions

```
void cmatrix::rq (scmatrix& mR, cmatrix& mQ) const throw (cvmexception);
void cmatrix::rq (cmatrix& mR, scmatrix& mQ) const throw (cvmexception);
```

compute RQ factorization as

$$M = RQ$$

where M is a calling matrix, upper triangular matrix R and unitary matrix Q are mR and mQ respectively. First version encapsulates so-called "economy" algorithm which for a given $m \times n$ matrix M computes $m \times m$ matrix R and $m \times n$ matrix Q . Second version is a "full" mode one computing $m \times n$ matrix R and $n \times n$ matrix Q . Following [this definition](#) the implementation assumes that $m \leq n$ and throws `cvmexception` otherwise. The functions also throw an exception of type `cvmexception` in case of inappropriate sizes of the operands passed. The functions are *redefined* in the the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6.};
treal ai[] = {1., -1., 2., -2., 3., -3.};
const cvm::cmatrix mh(ar, ai, 2, 3);
const cvm::cmatrix mv(ar, ai, 3, 2);
cvm::scmatrix s2(2), s3(3);
cvm::cmatrix h(2,3), v(3,2);

mh.rq(h,s3);
std::cout << (eye_complex(2) - cmatrix(s3,2,1,2,3) * ~cmatrix(s3,2,1,2,3))
            .norm() << " " << (mh - h * s3).norm() << std::endl;

mh.rq(s2,h);
std::cout << (eye_complex(2) - h * ~h).norm()
            << " " << (mh - s2 * h).norm() << std::endl;

prints

+3.1898416e-016 +2.2342807e-015
+3.1898416e-016 +2.2342807e-015
```

2.7.65 ql

Functions

```
void cmatrix::ql (cmatrix& mQ, scmatrix& mL) const throw (cvmexception);
void cmatrix::ql (scmatrix& mQ, cmatrix& mL) const throw (cvmexception);
```

compute QL factorization as

$$M = QL$$

where M is a calling matrix, unitary matrix Q and lower triangular matrix L are mQ and mL respectively. First version encapsulates so-called "economy" algorithm which for a given $m \times n$ matrix M computes $m \times n$ matrix Q and $n \times n$ matrix L . Second version is a "full" mode one computing $m \times m$ matrix Q and $m \times n$ matrix L . Following [this definition](#) the implementation assumes that $m \geq n$ and throws `cvmexception` otherwise. The functions also throw an exception of type `cvmexception` in case of inappropriate sizes of the operands passed. The functions are *redefined* in the the class `scmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6.};
treal ai[] = {1., -1., 2., -2., 3., -3.};
const cvm::cmatrix mv(ar, ai, 3, 2);
cvm::scmatrix s2(2), s3(3);
cvm::cmatrix v(3,2);

mv.ql(v,s2);
std::cout << (eye_complex(2) - ~v * v).norm()
           << " " << (mv - v * s2).norm() << std::endl;

mv.ql(s3,v);
std::cout << (eye_complex(2) - ~cmatrix(s3,1,2,3,2) * cmatrix(s3,1,2,3,2))
           .norm() << " " << (mv - s3 * v).norm() << std::endl;

prints

+1.1857187e-016 +9.4857497e-016
+1.1857187e-016 +9.4857497e-016
```

2.7.66 **vanish**

Function

```
cmatrix& cmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than, for example, `cmatrix::set(TC)` with zero operand passed. The function is *redefined* in the classes `scmatrix`, `scbmatrix`, `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m(4, 3);  
m.randomize_real(0.,1.);  
m.randomize_imag(1.,2.);
```

```
std::cout << m << std::endl;  
std::cout << m.vanish ();
```

prints

```
(0.851527,1.16376) (0.557512,1.90188) (0.0343638,1.52068)  
(0.478042,1.29106) (0.561724,1.19764) (0.320994,1.35804)  
(0.264534,1.40986) (0.113468,1.75137) (0.37727,1.54994)  
(0.521409,1.83035) (0.559465,1.35072) (0.809198,1.12537)
```

```
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)
```

2.7.67 geru

Function

```
cmatrix&
cmatrix::geru (TC dAlpha, const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

calls one of ?GERU routines of the [BLAS library](#) performing a **rank-1 update** (unconjugated) matrix-vector operation defined as $M = \alpha x \cdot y + M$, where α is a complex number (parameter `dAlpha`), M is a calling matrix and x and y are complex vectors (parameters `vCol` and `vRow` respectively). The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the the class `scmatrix` and *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `cvector`, `cmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

std::complex<double> alpha = std::complex<double>(1.2,4.11);
cmatrix m(3,2);
cvector vc(3);
cvector vr(2);
m.randomize_real(-1., 2.); m.randomize_imag(-3., 2.);
vc.randomize_real(-1., 3.); vc.randomize_imag(1., 3.);
vr.randomize_real(0., 2.); vr.randomize_imag(-1., 2.);

std::cout << m + vc.rank1update_u (vr) * alpha << std::endl;
std::cout << m.geru(alpha, vc, vr);
```

prints

```
(2.88144e+00,3.54299e+00) (-8.14760e+00,-1.03789e+00)
(6.33361e-01,3.35209e+00) (-4.81787e+00,-8.53964e+00)
(5.44811e-01,1.37156e+00) (-5.97006e+00,-5.00794e+00)

(2.88144e+00,3.54299e+00) (-8.14760e+00,-1.03789e+00)
(6.33361e-01,3.35209e+00) (-4.81787e+00,-8.53964e+00)
(5.44811e-01,1.37156e+00) (-5.97006e+00,-5.00794e+00)
```

2.7.68 gerc

Function

```
cmatrix&
cmatrix::gerc (TC dAlpha, const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

calls one of ?GERC routines of the [BLAS library](#) performing a **rank-1 update** (conjugated) matrix-vector operation defined as $M = \alpha x \cdot y' + M$, where α is a complex number (parameter `dAlpha`), M is a calling matrix and x and y are complex vectors (parameters `vCol` and `vRow` respectively). The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the the class `scmatrix` and *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `cvector`, `cmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

std::complex<double> alpha = std::complex<double>(1.2,4.11);
cmatrix m(3,2);
cvector vc(3);
cvector vr(2);
m.randomize_real(-1., 2.); m.randomize_imag(-3., 2.);
vc.randomize_real(-1., 3.); vc.randomize_imag(1., 3.);
vr.randomize_real(0., 2.); vr.randomize_imag(-1., 2.);

std::cout << m + vc.rank1update_c (vr) * alpha << std::endl;
std::cout << m.gerc(alpha, vc, vr);
```

prints

```
(1.27138e+01,1.58049e+01) (1.00616e+01,2.21197e+01)
(1.93326e+01,1.41763e+01) (1.74769e+01,2.49013e+01)
(8.09961e+00,1.36259e+01) (5.86738e+00,1.97800e+01)

(1.27138e+01,1.58049e+01) (1.00616e+01,2.21197e+01)
(1.93326e+01,1.41763e+01) (1.74769e+01,2.49013e+01)
(8.09961e+00,1.36259e+01) (5.86738e+00,1.97800e+01)
```

2.7.69 gemm

Function

```
cmatrix& cmatrix::gemm (const cmatrix& m1, bool bTrans1,
                      const cmatrix& m2, bool bTrans2,
                      TC dAlpha, TC dBeta) throw (cvmexception);
```

calls one of ?GEMM routines of the [BLAS library](#) performing a matrix-matrix operation defined as

$$M = \alpha \mathcal{C}(M_1) \cdot \mathcal{C}(M_2) + \beta M,$$

where α and β are complex numbers (parameters `dAlpha` and `dBeta`), M is a calling matrix and M_1 and M_2 are matrices (parameters `m1` and `m2` respectively). Function $\mathcal{C}(M_i)$ conjugates matrix M_i if appropriate boolean parameter `bTrans*` is equal to `true` and does nothing otherwise. The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the the class `scmatrix` and *not applicable* to objects of the classes `schmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

try {
    std::complex<double> alpha = std::complex<double>(1.1,2.1);
    std::complex<double> beta = std::complex<double>(0.71,0.12);
    cmatrix m1(4,3); cmatrix m2(4,3);
    cmatrix m(3,3);
    m.randomize_real(-1., 2.); m.randomize_imag(1., 3.);
    m1.randomize_real(-1., 3.); m1.randomize_imag(-2., 4.);
    m2.randomize_real(0., 2.); m2.randomize_imag(-3., 2.);

    std::cout << ~m1 * m2 * alpha + m * beta << std::endl;
    std::cout << m.gemm(m1, true, m2, false, alpha, beta);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints
(3.225e+01,3.611e+01) (2.042e+01,1.206e+01) (5.065e+01,-2.261e+01)
(3.009e+01,3.665e+00) (2.167e+01,-3.327e+00) (4.305e+01,-1.960e+01)
(1.156e+01,-4.966e+00) (4.067e+00,-1.181e+01) (1.121e+01,-2.684e+01)
```

(3.225e+01,3.611e+01) (2.042e+01,1.206e+01) (5.065e+01,-2.261e+01)
(3.009e+01,3.665e+00) (2.167e+01,-3.327e+00) (4.305e+01,-1.960e+01)
(1.156e+01,-4.966e+00) (4.067e+00,-1.181e+01) (1.121e+01,-2.684e+01)

2.7.70 hemm

Function

```
cmatrix& cmatrix::hemm (bool bLeft, const schmatrix& ms,
                      const cmatrix& m, TC dAlpha, TC dBeta)
                      throw (cvmexception);
```

calls one of ?HEMM routines of the [BLAS library](#) performing one of matrix-matrix operations defined as

$$M = \alpha M_h \cdot M_1 + \beta M \quad \text{or} \quad M = \alpha M_1 \cdot M_h + \beta M,$$

where α and β are complex numbers (parameters `dAlpha` and `dBeta`), M is a calling matrix, M_h is a hermitian matrix and M_1 is a complex matrix (parameters `ms` and `m` respectively). First operation is performed if `bLeft` passed is true and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the the classes `scmatrix` and `schmatrix` and *not applicable* to objects of the class `scbmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of that class). See also `schmatrix`, `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

try {
    std::complex<double> alpha = std::complex<double>(1.3,0.21);
    std::complex<double> beta = std::complex<double>(0.5,-0.1);
    cmatrix m1(2,3);
    cmatrix m2(3,2);
    schmatrix ms(2);
    cmatrix m(2,3);
    m.randomize_real(-1., 2.); m.randomize_imag(1., 3.);
    m1.randomize_real(-1., 3.); m1.randomize_imag(1., 2.);
    m2.randomize_real(0., 2.); m2.randomize_imag(-3., -1.);
    ms.randomize_real(-3., 1.); ms.randomize_imag(-1.3, 4.);

    std::cout << ms * m1 * alpha + m * beta << std::endl;
    std::cout << m.hemm (true, ms, m1, alpha, beta) << std::endl;

    m.resize(3,2);
    m.randomize_real(-1.4, 1.3); m.randomize_imag(1.1, 3.);
    std::cout << m2 * ms * alpha + m * beta << std::endl;
    std::cout << m.hemm (false, ms, m2, alpha, beta);
}
```

```
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
(1.096e+00,-7.692e+00) (-7.923e+00,-3.909e+00) (-1.324e+01,-5.264e+00)  
(2.415e+00,1.240e+00) (4.384e-01,-1.771e+00) (7.495e-01,-2.740e+00)
```

```
(1.096e+00,-7.692e+00) (-7.923e+00,-3.909e+00) (-1.324e+01,-5.264e+00)  
(2.415e+00,1.240e+00) (4.384e-01,-1.771e+00) (7.495e-01,-2.740e+00)
```

```
(-5.007e+00,1.010e+01) (2.341e+00,3.248e+00)  
(-8.753e+00,7.854e+00) (3.152e+00,4.491e+00)  
(-9.162e+00,6.401e+00) (-1.168e+00,3.973e+00)
```

```
(-5.007e+00,1.010e+01) (2.341e+00,3.248e+00)  
(-8.753e+00,7.854e+00) (3.152e+00,4.491e+00)  
(-9.162e+00,6.401e+00) (-1.168e+00,3.973e+00)
```

2.7.71 `randomize_real`

Function

```
cmatrix& cmatrix::randomize_real (TR dFrom, TR dTo);
```

fills a real part of a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

cmatrix m(2,3);
m.randomize_real(-1., 2.);
std::cout << m;

prints

(1.090e+00,0.000e+00) (-6.375e-01,0.000e+00) (1.248e+00,0.000e+00)
(-1.272e-01,0.000e+00) (-8.557e-01,0.000e+00) (4.848e-01,0.000e+00)
```

2.7.72 `randomize_imag`

Function

```
cmatrix& cmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills an imaginary part of a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`.
Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (3);
```

```
cmatrix m(2,3);  
m.randomize_imag(-1., 2.);  
std::cout << m;
```

prints

```
(0.000e+00,1.113e+00) (0.000e+00,6.615e-01) (0.000e+00,1.017e+00)  
(0.000e+00,-3.397e-01) (0.000e+00,1.577e+00) (0.000e+00,8.071e-01)
```

2.8 srmatrix

This is end-user class encapsulating a square matrix in Euclidean space of real numbers.

```

template <typename TR>
class srmatrix : public rmatrix <TR>, public SqMatrix <TR,TR> {
public:
    srmatrix ();
    explicit srmatrix (int nMN);
    srmatrix (TR* pD, int nMN);
    srmatrix (const TR* pD, int nMN);
    srmatrix (const srmatrix& m);
    srmatrix (const rmatrix& m);
    explicit srmatrix (const rvector& v);
    srmatrix (rmatrix& m, int nRow, int nCol, int nSize);
    TR& operator () (int im, int in) throw (cvmexception);
    TR operator () (int im, int in) const throw (cvmexception);
    rvector operator () (int i) throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    rvector operator [] (int i) throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    srmatrix& operator = (const srmatrix& m) throw (cvmexception);
    srmatrix& assign (const rvector& v);
    srmatrix& assign (const TR* pD);
    srmatrix& assign (int nRow, int nCol, const rmatrix& m)
        throw (cvmexception);
    srmatrix& set (TR x);
    srmatrix& resize (int nNewMN) throw (cvmexception);
    srmatrix& operator << (const srmatrix& m) throw (cvmexception);
    srmatrix operator + (const srmatrix& m) const
        throw (cvmexception);
    srmatrix operator - (const srmatrix& m) const
        throw (cvmexception);
    srmatrix& sum (const srmatrix& m1,
        const srmatrix& m2) throw (cvmexception);
    srmatrix& diff (const srmatrix& m1,
        const srmatrix& m2) throw (cvmexception);
    srmatrix& operator += (const srmatrix& m) throw (cvmexception);
    srmatrix& operator -= (const srmatrix& m) throw (cvmexception);
    srmatrix operator - () const;
    srmatrix& operator ++ ();
    srmatrix& operator ++ (int);
    srmatrix& operator -- ();

```

```

srmatrix& operator -- (int);
srmatrix operator * (TR d) const;
srmatrix operator / (TR d) const throw (cvmexception);
srmatrix& operator *= (TR d);
srmatrix& operator /= (TR d) throw (cvmexception);
srmatrix& normalize ();
srmatrix operator ~ () const throw (cvmexception);
srmatrix& transpose (const srmatrix& m) throw (cvmexception);
srmatrix& transpose ();
rvector operator * (const rvector& v) const throw (cvmexception);
rmatrix operator * (const rmatrix& m) const throw (cvmexception);
srmatrix operator * (const srmatrix& m) const throw (cvmexception);
srmatrix& operator *= (const srmatrix& m) throw (cvmexception);
srmatrix& swap_rows (int n1, int n2) throw (cvmexception);
srmatrix& swap_cols (int n1, int n2) throw (cvmexception);
rvector solve (const rvector& vB) const throw (cvmexception);
rmatrix solve (const rmatrix& mB) const throw (cvmexception);
rvector solve (const rvector& vB, TR& dErr) const
    throw (cvmexception);
rmatrix solve (const rmatrix& mB, TR& dErr) const
    throw (cvmexception);
rvector solve_lu (const srmatrix& mLU, const int* pPivots,
    const rvector& vB, TR& dErr) throw (cvmexception);
rvector solve_lu (const srmatrix& mLU, const int* pPivots,
    const rvector& vB) throw (cvmexception);
rmatrix solve_lu (const srmatrix& mLU, const int* pPivots,
    const rmatrix& mB, TR& dErr) throw (cvmexception);
rmatrix solve_lu (const srmatrix& mLU, const int* pPivots,
    const rmatrix& mB) throw (cvmexception);
TR det () const throw (cvmexception);
srmatrix& low_up (const srmatrix& m,
    int* nPivots) throw (cvmexception);
srmatrix low_up (int* nPivots) const throw (cvmexception);
TR cond () const throw (cvmexception);
srmatrix& inv (const srmatrix& mArg) throw (cvmexception);
srmatrix inv () const throw (cvmexception);
srmatrix& exp (const srmatrix& m,
    TR tol = cvmMachSp ()) throw (cvmexception);
srmatrix exp (TR tol = cvmMachSp ()) const throw (cvmexception);
srmatrix& polynom (const srmatrix& m, const rvector& v)
    throw (cvmexception);
srmatrix polynom (const rvector& v) const throw (cvmexception);
cvector eig (srmatrix& mEigVect,

```

```
        bool bRightVect = true) const throw (cvmexception);  
cvector eig () const throw (cvmexception);  
srmatrix& cholesky (const srmatrix& m) throw (cvmexception);  
srmatrix& bunch_kaufman (const srmatrix& m,  
                        int* pivots) throw (cvmexception);  
void qr(srmatrix& mQ, srmatrix& mR) const throw (cvmexception);  
void lq(srmatrix& mL, srmatrix& mQ) const throw (cvmexception);  
void rq(srmatrix& mR, srmatrix& mQ) const throw (cvmexception);  
void ql(srmatrix& mQ, srmatrix& mL) const throw (cvmexception);  
srmatrix& identity ();  
srmatrix& vanish ();  
srmatrix& randomize (TR dFrom, TR dTo);  
};
```

2.8.1 `srmatrix ()`

Constructor

```
srmatrix::srmatrix ();
```

creates an empty `srmatrix` object. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
srmatrix m;  
std::cout << m.msize() << std::endl << m.nsize() << std::endl;  
std::cout << m.size() << std::endl;
```

```
m.resize (3);  
std::cout << m;
```

prints

```
0  
0  
0  
0 0 0  
0 0 0  
0 0 0
```

2.8.2 srmatrix (int)

Constructor

```
explicit srmatrix::srmatrix (int nMN);
```

creates an $n \times n$ srmatrix object where n is passed in nMN parameter. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
srmatrix m (4);  
std::cout << m.msize() << std::endl << m.nsize()  
          << std::endl << m.size() << std::endl << m;
```

prints

```
4  
4  
16  
0 0 0 0  
0 0 0 0  
0 0 0 0  
0 0 0 0
```

2.8.3 srmatrix (TR*,int)

Constructor

```
srmatrix::srmatrix (TR* pD, int nMN);
```

creates an $n \times n$ srmatrix object where n is passed in nMN parameter. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by pD. The constructor throws an exception of type `cvmexception` in case of non-positive size passed. See also `srmatrix`, `srmatrix (const TR*,int)`. Example:

```
using namespace cvm;
double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.};
srmatrix m (a, 3);

m(1,1) = 5.;
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << std::endl;
```

prints

```
5 1 1
1 1 1
1 1 1

5 1 1
```

2.8.4 srmatrix (const TR*,int)

Constructor

```
srmatrix::srmatrix (const TR* pD, int nMN);
```

creates an $n \times n$ srmatrix object where n is passed in nMN parameter and copies $n * n$ elements of an array pD to it **by columns**. The constructor throws an exception of type **cvmexception** in case of non-positive size passed. See also **srmatrix**, **srmatrix (TR*,int)**. Example:

```
using namespace cvm;
const double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.};
srmatrix m (a, 3);

m(1,1) = 5.;
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << std::endl;
```

prints

```
5 1 1
1 1 1
1 1 1

1 1 1
```

2.8.5 srmatrix (const srmatrix&)

Copy constructor

```
srmatrix::srmatrix (const srmatrix& m);
```

creates a srmatrix object as a copy of m. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
srmatrix m(a, 3);  
srmatrix mc(m);
```

```
m(1,1) = 7.77;  
std::cout << m << std::endl << mc;
```

prints

```
7.77 4 7  
2 5 8  
3 6 9
```

```
1 4 7  
2 5 8  
3 6 9
```

2.8.6 srmatrix (const rmatrix&)

Constructor

```
srmatrix::srmatrix (const rmatrix& m);
```

creates a srmatrix object as a copy of matrix m. It's assumed that $m \times n$ matrix m must have equal sizes, i.e. $m = n$ is satisfied. The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. Please note that this constructor is *not explicit* anymore. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.};  
rmatrix m(a, 2, 3);  
std::cout << m << std::endl;
```

```
m.resize(3, 3);  
srmatrix ms (m);  
std::cout << ms;
```

prints

```
1 3 5  
2 4 6
```

```
1 3 5  
2 4 6  
0 0 0
```

2.8.7 srmatrix (const rvector&)

Constructor

```
explicit srmatrix::srmatrix (const rvector& v);
```

creates a srmatrix object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srmatrix`, `rvector`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5.};  
rvector v(a, 5);  
srmatrix m(v);  
std::cout << m;
```

prints

```
1 0 0 0 0  
0 2 0 0 0  
0 0 3 0 0  
0 0 0 4 0  
0 0 0 0 5
```

2.8.8 **submatrix**

Submatrix constructor

```
srmatrix::srmatrix (rmatrix& m, int nRow, int nCol, int nSize);
```

creates a *srmatrix* object as a *submatrix* of *m*. It means that the matrix object created shares a memory with some part of *m*. This part is defined by its upper left corner (parameters *nRow* and *nCol*, both are *ℓ-based*) and its size (parameter *nSize*). See also *srmatrix*.
Example:

```
using namespace cvm;
```

```
rmatrix m(4,5);  
srmatrix subm(m, 2, 2, 2);  
subm.set(1.);
```

```
std::cout << m;
```

prints

```
0 0 0 0 0  
0 1 1 0 0  
0 1 1 0 0  
0 0 0 0 0
```

2.8.9 operator (,)

Indexing operators

```
TR& srmatrix::operator () (int im, int in) throw (cvmexception);
TR srmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. The operators are *inherited* in the the class `srbmatrix` and *redefined* in the the class `srsmatrix`. See also `srmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m (a, 3);
    srmatrix ms(m);

    std::cout << m(1,1) << " " << m(2,3) << std::endl << std::endl;

    ms(2,2) = 7.77;
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

1 8

1 4 7
2 7.77 8
3 6 9
```

2.8.10 operator ()

Indexing operators

```
rvector srmatrix::operator () (int i) throw (cvmexception);
const rvector srmatrix::operator () (int i) const throw (cvmexception);
```

provide access to an *i*-th column of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the *i*-th column of the matrix in order to make possible write access to it. The second version creates a *copy* of a column and therefore it's *not an l-value*. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if the parameter *i* is outside of `[1, nsize()]` range. The operators are *redefined* in the the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m (a, 3);
    srmatrix ms(3);

    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4.00e+00 5.00e+00 6.00e+00

0.00e+00 7.00e+00 0.00e+00
0.00e+00 8.00e+00 0.00e+00
0.00e+00 9.00e+00 0.00e+00
```

2.8.11 operator []

Indexing operators

```
rvector srmatrix::operator [] (int i) throw (cvmexception);
const rvector srmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to an *i*-th row of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the *i*-th row of the matrix in order to make possible write access to it. The second version creates a *copy* of a row and therefore it's *not an l-value*. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if the parameter *i* is outside of `[1,msize()]` range. The operators are *redefined* in the the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m (a, 3);
    srmatrix ms(3);

    std::cout << m[2] << std::endl;

    ms[2] = m[3];
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2.00e+00 5.00e+00 8.00e+00

0.00e+00 0.00e+00 0.00e+00
3.00e+00 6.00e+00 9.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.8.12 operator = (const srmatrix&)

Operator

```
srmatrix& srmatrix::operator = (const srmatrix& m)
throw (cvmexception);
```

sets an every element of a calling matrix to a value of appropriate element of a matrix `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m1(a, 3);
    srmatrix m2(3);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 4.00e+00 7.00e+00
2.00e+00 5.00e+00 8.00e+00
3.00e+00 6.00e+00 9.00e+00
```

2.8.13 assign (const TR*)

Function

```
srmatrix& srmatrix::assign (const rvector& v);  
srmatrix& srmatrix::assign (const TR* pD);
```

sets every element of a calling matrix to a value of appropriate element of a vector *v* or an array pointed to by *pD* and returns a reference to the matrix changed. The function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
  
const double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
srmatrix m(3);  
  
m.assign(a);  
std::cout << m;
```

prints

```
1.00e+00 4.00e+00 7.00e+00  
2.00e+00 5.00e+00 8.00e+00  
3.00e+00 6.00e+00 9.00e+00
```

2.8.14 assign (int, int, const rmatrix&)

Function

```
srmatrix& srmatrix::assign (int nRow, int nCol, const rmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with *l*-based row nRow and column nCol to a matrix m and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if nRow or nCol are not positive or matrix m doesn't fit. The function is *redefined* in the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;
```

```
srmatrix m1(5);
rmatrix m2(2,3);
m1.set(1.);
m2.set(2.);
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
1 1 1 1 1
1 1 2 2 2
1 1 2 2 2
1 1 1 1 1
1 1 1 1 1
```

2.8.15 `set (TR)`

Function

```
srmatrix& srmatrix::set (TR x);
```

sets every element of a calling matrix to a value of parameter `x` and returns a reference to the matrix changed. Use `vanish` to set every element of a calling matrix to be equal to zero. The function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`.
Example:

```
using namespace cvm;  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
srmatrix m(3);
```

```
m.set(3.);  
std::cout << m;
```

prints

```
3.00e+00 3.00e+00 3.00e+00  
3.00e+00 3.00e+00 3.00e+00  
3.00e+00 3.00e+00 3.00e+00
```

2.8.16 `resize`

Function

```
srmatrix& srmatrix::resize (int nNewMN) throw (cvmexception);
```

changes a size of a calling matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. The function is *redefined* in the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
try {  
    double a[] = {1., 2., 3., 4.};  
    srmatrix m(a, 2);  
  
    std::cout << m << std::endl;  
  
    m.resize (3);  
    std::cout << m;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
1 3  
2 4  
  
1 3 0  
2 4 0  
0 0 0
```

2.8.17 operator <<

Operator

```
srmatrix& srmatrix::operator << (const srmatrix& m)
throw (cvmexception);
```

destroys a calling matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. The operator is *redefined* in the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    srmatrix m(3);
    srmatrix mc(1);
    m(1,2) = 1.;
    m(2,3) = 2.;
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

0.00e+00 1.00e+00 0.00e+00
0.00e+00 0.00e+00 2.00e+00
0.00e+00 0.00e+00 0.00e+00

0.00e+00

0.00e+00 1.00e+00 0.00e+00
0.00e+00 0.00e+00 2.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.8.18 operator +

Operator

```
srmatrix srmatrix::operator + (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a sum of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix::sum`, `srmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80., 90.};
    srmatrix m1(a, 3);
    srmatrix m2(b, 3);

    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
11 44 77
22 55 88
33 66 99
```

```
2 8 14
4 10 16
6 12 18
```

2.8.19 operator -

Operator

```
srmatrix srmatrix::operator - (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a difference of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix::diff`, `srmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80., 90.};
    srmatrix m1(a, 3);
    srmatrix m2(b, 3);

    std::cout << m2 - m1 << std::endl << m1 - m1;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
9 36 63
18 45 72
27 54 81
```

```
0 0 0
0 0 0
0 0 0
```

2.8.20 sum

Function

```
srmatrix& srmatrix::sum (const srmatrix& m1, const srmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of matrices m1 and m2 to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix::operator +`, `srmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m1(a, 3);
    srmatrix m2(3);
    srmatrix m(3);
    m2.set(1.);

    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2 5 8
3 6 9
4 7 10

3 6 9
4 7 10
5 8 11
```

2.8.21 diff

Function

```
srmatrix& srmatrix::diff (const srmatrix& m1, const srmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of matrices m1 and m2 to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix::operator -`, `srmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m1(a, 3);
    srmatrix m2(3);
    srmatrix m(3);
    m2.set(1.);

    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0 3 6
1 4 7
2 5 8
```

```
-1 2 5
0 3 6
1 4 7
```

2.8.22 operator +=

Operator

```
srmatrix& srmatrix::operator += (const srmatrix& m) throw (cvmexception);
```

adds a matrix *m* to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator +`, `srmatrix::sum`, `srmatrix`. Example:

```
using namespace cvm;
try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(2.);

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
3 3 3
3 3 3
```

```
4 4 4
4 4 4
4 4 4
```

2.8.23 operator -=

Operator

```
srmatrix& srmatrix::operator -= (const srmatrix& m) throw (cvmexception);
```

subtracts a matrix `m` from a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator -`, `srmatrix::diff`, `srmatrix`. Example:

```
using namespace cvm;
```

```
try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(2.);

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1 -1 -1
-1 -1 -1
-1 -1 -1
```

```
0 0 0
0 0 0
0 0 0
```

2.8.24 operator - ()

Operator

```
srmatrix srmatrix::operator - () const throw (cvmexception);
```

creates an object of type `srmatrix` as a calling matrix multiplied by -1 . The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
srmatrix m(a, 3);
```

```
std::cout << -m;
```

prints

```
-1 -4 -7  
-2 -5 -8  
-3 -6 -9
```

2.8.25 operator ++

Operator

```
srmatrix& srmatrix::operator ++ ();  
srmatrix& srmatrix::operator ++ (int);
```

adds identity matrix to a calling matrix and returns a reference to the matrix changed. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
srmatrix m (a, 3);
```

```
m++;  
std::cout << m << std::endl;  
std::cout << ++m;
```

prints

```
2 4 7  
2 6 8  
3 6 10
```

```
3 4 7  
2 7 8  
3 6 11
```

2.8.26 operator --

Operator

```
srmatrix& srmatrix::operator -- ();  
srmatrix& srmatrix::operator -- (int);
```

subtracts identity matrix from a calling matrix and returns a reference to the matrix changed. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
srmatrix m (a, 3);
```

```
m--;  
std::cout << m << std::endl;  
std::cout << --m;
```

prints

```
0 4 7  
2 4 8  
3 6 8
```

```
-1 4 7  
2 3 8  
3 6 7
```

2.8.27 operator * (TR)

Operator

```
srmatrix srmatrix::operator * (TR d) const;
```

creates an object of type `srmatrix` as a product of a calling matrix and a number `d`. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator *=`, `srmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
srmatrix m(a, 3);
```

```
std::cout << m * 5.;
```

prints

```
5 20 35  
10 25 40  
15 30 45
```

2.8.28 operator / (TR)

Operator

```
srmatrix srmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `srmatrix` as a quotient of a calling matrix and a number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the **smallest normalized positive number**. The operator is *redefined* in the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix::operator /=` , `srmatrix`. Example:

```
using namespace cvm;
```

```
try {  
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
    srmatrix m(a, 3);  
  
    std::cout << m / 4.;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
0.25 1 1.75  
0.5 1.25 2  
0.75 1.5 2.25
```

2.8.29 operator *= (TR)

Operator

```
srmatrix& srmatrix::operator *= (TR d);
```

multiplies a calling matrix by a number d and returns a reference to the matrix changed. The operator is *redefined* in the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix::operator *`, `srmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
srmatrix m(a, 3);
```

```
m *= 2.;  
std::cout << m;
```

prints

```
2 8 14  
4 10 16  
6 12 18
```

2.8.30 operator /= (TR)

Operator

```
srmatrix& srmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling matrix by a number *d* and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if *d* has an absolute value equal or less than the **smallest normalized positive number**. The operator is *redefined* in the classes `srbmatrix` and `srsrmatrix`. See also `srmatrix::operator /`, `srmatrix`. Example:

```
using namespace cvm;
```

```
try {  
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
    srmatrix m(a, 3);  
  
    m /= 2.;  
    std::cout << m;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
0.5 2 3.5  
1 2.5 4  
1.5 3 4.5
```

2.8.31 normalize

Function

```
srmatrix& srmatrix::normalize ();
```

normalizes a calling matrix so its **Euclidean norm** becomes equal to 1 if it was greater than the **smallest normalized positive number** before the call (otherwise the function does nothing). The function is *redefined* in the classes **srbmatrix** and **srsmatrix**. See also **srmatrix**. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (3);
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
srmatrix m(a, 3);
```

```
m.normalize();  
std::cout << m << m.norm() << std::endl;
```

prints

```
5.923e-02 2.369e-01 4.146e-01  
1.185e-01 2.962e-01 4.739e-01  
1.777e-01 3.554e-01 5.331e-01  
1.000e+00
```

2.8.32 transposition

Operator and functions

```
srmatrix srmatrix::operator ~ () const throw (cvmexception);
srmatrix& srmatrix::transpose (const srmatrix& m) throw (cvmexception);
srmatrix& srmatrix::transpose ();
```

encapsulate matrix transposition. First operator creates an object of type `srmatrix` as a transposed calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `m` transposed (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to transposed itself. The functions are *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m(a,3);
    srmatrix mt(3);
    std::cout << ~m << std::endl ;
    mt.transpose(m);
    std::cout << mt << std::endl;
    mt.transpose();
    std::cout << mt;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
1 2 3
4 5 6
7 8 9
```

```
1 2 3
4 5 6
7 8 9
```

```
1 4 7
2 5 8
3 6 9
```

2.8.33 operator * (const rvector&)

Operator

```
rvector srmatrix::operator * (const rvector& v) const  
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `rvector::mult` in order to get rid of a new object creation. The function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`, `rvector`. Example:

```
using namespace cvm;
```

```
try {  
    srmatrix m(3);  
    rvector v(3);  
    m.set(1.);  
    v.set(1.);  
  
    std::cout << m * v;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
3 3 3
```

2.8.34 operator * (const rmatrix&)

Operator

```
rmatrix srmatrix::operator * (const rmatrix& m) const  
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `rmatrix::mult` in order to get rid of a new object creation. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;
```

```
try {  
    srmatrix ms(3);  
    rmatrix m(3,2);  
    ms.set(1.);  
    m.set(1.);  
  
    std::cout << ms * m;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
3 3  
3 3  
3 3
```

2.8.35 operator * (const srmatrix&)

Operator

```
srmatrix srmatrix::operator * (const srmatrix& m) const  
throw (cvmexception);
```

creates an object of type `srmatrix` as a product of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `srmatrix::mult` in order to get rid of a new object creation. The operator is *inherited* in the class `srbmatrix` and *redefined* in `srsrmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
try {  
    srmatrix m1(3);  
    srmatrix m2(3);  
    m1.set(1.);  
    m2.set(1.);  
  
    std::cout << m1 * m2;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
3 3 3  
3 3 3  
3 3 3
```

2.8.36 operator *= (const srmatrix&)

Operator

```
srmatrix& srmatrix::operator *= (const srmatrix& m)
throw (cvmexception);
```

sets a calling matrix to be equal to a product of itself by a matrix `m` and returns a reference to the object it changes. The operator throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *inherited* in the class `srbmatrix` and *redefined* in `srsrmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(1.);

    m1 *= m2;
    std::cout << m1 << std::endl;
    m1 *= m1;
    std::cout << m1;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
3 3 3
3 3 3
```

```
27 27 27
27 27 27
27 27 27
```

2.8.37 swap_rows

Function

```
srmatrix& srmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of rows to be swapped, both are *l*-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1,msize()]`. The function is *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `srmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m (a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_rows(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4 7
2 5 8
3 6 9
```

```
1 4 7
3 6 9
2 5 8
```

2.8.38 swap_cols

Function

```
srmatrix& srmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of columns to be swapped, both are *l*-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1, nsize())`. The function is *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `srmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m (a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_cols(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4 7
2 5 8
3 6 9
```

```
1 7 4
2 8 5
3 9 6
```

2.8.39 solve

Functions

```

rvector srmatrix::solve (const rvector& vB) const throw (cvmexception);
rmatrix srmatrix::solve (const rmatrix& mB) const throw (cvmexception);
rvector srmatrix::solve (const rvector& vB, TR& dErr) const
throw (cvmexception);
rmatrix srmatrix::solve (const rmatrix& mB, TR& dErr) const
throw (cvmexception);

```

return a solution of a linear equation of kind $Ax = b$ or $AX = B$ where A is a calling matrix. The first and the third versions solve the equation $Ax = b$ where vector b is passed in the parameter vB and the second and fourth versions solve the equation $AX = B$ where matrix B is passed in the parameter mB . The last two versions also set output parameter $dErr$ to be equal to a norm of computation error. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the classes `srbmatrix` and `srsmatrix`. See also `rvector::solve`, `rmatrix::solve`, `srmatrix`. Example:

```

using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix ma(a, 3);
    rmatrix mb(3,4);
    rmatrix mx(3,4);
    double dErr;

    mb(1).set(1.);
    mb(2).set(2.);
    mb(3).set(3.);
    mb(1,4) = 1.; mb(2,4) = 2.; mb(3,4) = 3.;

    mx = ma.solve (mb, dErr);
    std::cout << mx << dErr
              << std::endl << ma * mx - mb << std::endl;

    rvector vb(3), vx(3);
    vb = mb(2);
    vx = ma.solve (vb, dErr);
    std::cout << vx << dErr << std::endl << ma * vx - vb;
}

```

```
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
-3.333e-01 -6.667e-01 -1.000e+00 1.000e+00  
3.333e-01 6.667e-01 1.000e+00 0.000e+00  
6.661e-16 1.332e-15 0.000e+00 0.000e+00  
3.301e-14  
0.000e+00 0.000e+00 0.000e+00 0.000e+00  
-1.110e-16 -2.220e-16 0.000e+00 0.000e+00  
2.220e-16 4.441e-16 0.000e+00 0.000e+00  
  
-6.667e-01 6.667e-01 1.332e-15  
3.301e-14  
0.000e+00 -2.220e-16 4.441e-16
```

2.8.40 solve_lu

Functions

```
rvector
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rvector& vB, TR& dErr) throw (cvmexception);
```

```
rvector
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rvector& vB) throw (cvmexception);
```

```
rmatrix
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rmatrix& mB, TR& dErr) throw (cvmexception);
```

```
rmatrix
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rmatrix& mB) throw (cvmexception);
```

create an object of type `rvector` or `rmatrix` as a solution x or X of the matrix linear equation $Ax = b$ or $AX = B$ respectively. Here A is a calling matrix, parameter `mLU` is **LU factorization** of the matrix A , parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and parameters `vB` and `mB` are the vector b and matrix B respectively. The first and third version also set output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $Ax = b$ or $AX = B$ with the same matrix A and different vectors b or matrices B . In such case you save on matrix A factorization since it's needed to be performed just one time. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to cingular. The function is *inherited* in the classes `srbmatrix` and `srsrmatrix`. See also `rvector::solve`, `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    srmatrix ma(a,3);
    srmatrix mLU(3);
    rmatrix mb1(3,2); rvector vb1(3);
    rmatrix mb2(3,2); rvector vb2(3);
    rmatrix mx1(3,2); rvector vx1(3);
    rmatrix mx2(3,2); rvector vx2(3);
    iarray nPivots(3);
```

```

double   dErr = 0.;
mb1.randomize(-1.,3.); vb1.randomize(-2.,4.);
mb2.randomize(-2.,5.); vb2.randomize(-3.,1.);

mLU.low_up(ma, nPivots);
mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
std::cout << mx1 << dErr << std::endl;
mx2 = ma.solve_lu (mLU, nPivots, mb2);
std::cout << mx2 << std::endl;;
std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;

vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
std::cout << vx1 << dErr << std::endl;
vx2 = ma.solve_lu (mLU, nPivots, vb2);
std::cout << vx2 << std::endl;;
std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

```

prints

```

2.807e+00 1.107e+00
-3.651e-01 -4.843e+00
-5.412e-01 3.095e+00
6.438e-15
-7.639e-01 1.082e+01
-2.869e-01 -1.110e+01
4.890e-01 3.443e+00

0.000e+00 -4.441e-16
1.110e-16 -4.441e-16
-4.441e-16 4.441e-16

0.000e+00 -4.441e-16
0.000e+00 8.882e-16
0.000e+00 -4.441e-16
-1.651e+00 2.361e-01 -6.384e-02
3.828e-15
-5.886e+00 7.038e+00 -3.125e+00

0.000e+00 0.000e+00 0.000e+00

```

0.000e+00 0.000e+00 2.220e-16

2.8.41 det

Function

```
TR srmatrix::det () const throw (cvmexception);
```

returns a determinant of a calling matrix. It uses the [LU factorization](#) inside and may throw the same exceptions as the factorizer. The function is *inherited* in the classes [srbmatrix](#) and [srsrmatrix](#). See also [srmatrix](#). Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix m(a, 3);

    std::cout << m << std::endl << m.det() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.000e+00 4.000e+00 7.000e+00
2.000e+00 5.000e+00 8.000e+00
3.000e+00 6.000e+00 1.000e+01

-3.000e+00
```

2.8.42 low_up

Functions

```

srmatrix&
srmatrix::low_up (const srmatrix& m, int* nPivots) throw (cvmexception);
srmatrix
srmatrix::low_up (int* nPivots) const throw (cvmexception);

```

compute the LU factorization of a calling matrix as

$$A = PLU$$

where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and U is an upper triangular matrix. All the functions store the result as the matrix L without main diagonal combined with U. All the functions return pivot indices as an array of integers (it should support at least `m.size()` elements) pointed to by `nPivots` so *i*-th row was interchanged with `nPivots[i]`-th row. The first version sets a calling matrix to be equal to the *m*'s LU factorization and the second one creates an object of type `srmatrix` as the calling matrix's LU factorization. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be factorized is close to singular. It is recommended to use `iarray` for pivot values. The function is *redefined* in the class `srbmatrix` and *inherited* in `srsmatrix`. See also `srmatrix`. Example:

```

using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix m(a, 3);
    srmatrix mLU(3), mLo(3), mUp(3);
    iarray naPivots(3);

    mLU.low_up (m, naPivots);

    mLo.identity ();

    mLo.diag(-2) = mLU.diag(-2);
    mLo.diag(-1) = mLU.diag(-1);
    mUp.diag(0) = mLU.diag(0);
    mUp.diag(1) = mLU.diag(1);
    mUp.diag(2) = mLU.diag(2);
}

```

```
std::cout << mLo << std::endl << mUp
          << std::endl << naPivots << std::endl;

mLU = mLo * mUp;
for (int i = 3; i >= 1; i--) {
    mLU.swap_rows (i, naPivots[i]);
}
std::cout << mLU;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
1.000e+00 0.000e+00 0.000e+00
3.333e-01 1.000e+00 0.000e+00
6.667e-01 5.000e-01 1.000e+00

3.000e+00 6.000e+00 1.000e+01
0.000e+00 2.000e+00 3.667e+00
0.000e+00 0.000e+00 -5.000e-01
```

3 3 3

```
1.000e+00 4.000e+00 7.000e+00
2.000e+00 5.000e+00 8.000e+00
3.000e+00 6.000e+00 1.000e+01
```

2.8.43 cond

Function

TR srmatrix::cond () const throw (cvmexception);

returns a reciprocal of a condition number of a calling matrix A in the **infinity-norm**:

$$\kappa_{\infty} = \|A\|_{\infty} \|A^{-1}\|_{\infty}.$$

Less value returned means that matrix A is closer to singular. Zero value returned means estimation underflow or that matrix A is singular. The condition number is used for error analysis of systems of linear equations. The function throws exception of type **cvmexception** in case of LAPACK subroutines failure. The function is *inherited* in the classes **srbmatrix** and **srsmatrix**. See also **srmatrix::solve**, **srmatrix**. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m(a, 3);
    std::cout << m.cond() << std::endl
               << m.det() << std::endl << std::endl;
    m(3,3) = 10.;
    std::cout << m.cond() << std::endl << m.det() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

0.000e+00
0.000e+00

7.519e-03
-3.000e+00
```

2.8.44 inv

Functions

```
srmatrix& srmatrix::inv (const srmatrix& m) throw (cvmexception);
srmatrix srmatrix::inv () const throw (cvmexception);
```

implement matrix inversion. The first version sets a calling matrix to be equal to *m* inverted and the second one creates an object of type *srmatrix* as inverted calling matrix. The functions throw exception of type *cvmexception* in case of inappropriate sizes of the operands or when the matrix to be inverted is close to singular. The function is *redefined* in the class *srsrmatrix* and *inherited* in *srbmatrix*. See also *srmatrix*. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (10);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix m(a, 3);
    srmatrix mi(3);
    mi.inv (m);
    std::cout << mi << std::endl << mi * m - eye_real(3);
    std::cout << std::endl << mi.inv() * mi - eye_real(3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-6.6666666667e-01 -6.6666666667e-01 1.0000000000e+00
-1.3333333333e+00 3.6666666667e+00 -2.0000000000e+00
1.0000000000e+00 -2.0000000000e+00 1.0000000000e+00

0.0000000000e+00 0.0000000000e+00 1.7763568394e-15
1.7763568394e-15 3.5527136788e-15 0.0000000000e+00
0.0000000000e+00 0.0000000000e+00 1.7763568394e-15

0.0000000000e+00 1.7763568394e-15 -1.7763568394e-15
-8.8817841970e-16 3.5527136788e-15 -3.5527136788e-15
0.0000000000e+00 0.0000000000e+00 -1.7763568394e-15
```

2.8.45 exp

Functions

```
srmatrix& srmatrix::exp (const srmatrix& m, TR tol = cvmMachSp ())
throw (cvmexception);
```

```
srmatrix srmatrix::exp (TR tol = cvmMachSp ()) const
throw (cvmexception);
```

compute an exponent of a calling matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1}N_{pq}(z) = 1 + z + \dots + z^p/p!,$$

where

$$N_{pq}(z) = \sum_{k=0}^p \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} z^k,$$

$$D_{pq}(z) = \sum_{k=0}^q \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} (-z)^k$$

along with the matrix normalizing as described in [2], p. 572. The functions use DMEXP (or SMEXP for float version) FORTRAN subroutine implementing the algorithm. The first version sets the calling matrix to be equal to the exponent of *m* and returns a reference to the matrix changed. The second version creates an object of type *srmatrix* as the exponent of the calling matrix. The algorithm uses parameter *tol* as $\varepsilon(p, q)$ in order to choose constants *p* and *q* so that

$$\varepsilon(p, q) \geq 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the **largest relative spacing** by default. The functions throw an exception of type **cvmexception** in case of inappropriate sizes of the operands or when LAPACK subroutine fails. The functions are *inherited* in the classes **srbmatrix** and **srsrmatrix**. The second version is *redefined* in **srbmatrix**. See also **srmatrix**. Example (see [2], p. 567, example 11.2.2):

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    srmatrix m(2);
    m(1,1) = -49.;
    m(1,2) = 24.;
```

```
m(2,1) = -64.;
m(2,2) = 31.;

std::cout << m << std::endl << m.exp();
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
-4.9000000000000000e+01 2.4000000000000000e+01
-6.4000000000000000e+01 3.1000000000000000e+01

-7.357587581448284e-01 5.518190996581556e-01
-1.471517599088415e+00 1.103638240715692e+00
```

Matlab output:

```
-7.357587581446907e-001    5.518190996580505e-001
-1.471517599088136e+000    1.103638240715478e+000
```

2.8.46 polynomial

Functions

```
srmatrix& srmatrix::polynom (const srmatrix& m, const rvector& v)
throw (cvmexception);
```

```
srmatrix srmatrix::polynom (const rvector& v) const
throw (cvmexception);
```

compute a matrix polynomial defined as

$$p(A) = b_0I + b_1A + \dots + b_qA^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^r B_k(A^s)^k, \quad s = \text{floor}(\sqrt{q}), \quad r = \text{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum_{i=0}^{s-1} b_{sk+i} A^i, & k = 0, 1, \dots, r-1 \\ \sum_{i=0}^{q-sr} b_{sr+i} A^i, & k = r. \end{cases}$$

See also [2], p. 568. The coefficients b_0, b_1, \dots, b_q are passed in the parameter v , where q is equal to $v.size() - 1$, so the functions compute matrix polynomial equal to

$$v[1] * I + v[2] * m + \dots + v[v.size()] * m^{v.size()-1}$$

The first version sets a calling matrix to be equal to the polynomial of m and the second one creates an object of type `srmatrix` as the polynomial of a calling matrix. The functions use DPOLY (or SPOLY for float version) FORTRAN subroutine implementing the Horner's algorithm. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in the class `srbmatrix` and *redefined* in `srsrmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    double a[] = {2.2, 1.3, 1.1, -0.9, 0.2,
                 -0.45, 45, -30, 10, 3, 3.2};
    const rvector v(a, 11);
    srmatrix m(2), mp(2);
```

```
m(1,1) = 1.;
m(1,2) = 0.5;
m(2,1) = -1.;
m(2,2) = 0.3;

mp.polynom (m, v);
std::cout << mp;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

prints

-7.963641665999998e+00 -7.551532476200001e+00
1.510306495240000e+01 2.608503800680002e+00
```

Matlab output:

```
-7.963641665999999e+000    -7.551532476200002e+000
1.510306495240000e+001    2.608503800680002e+000
```

2.8.47 eig

Functions

```
cvector srmatrix::eig (scmatrix& mEigVect, bool bRightVect = true) const
throw (cvmexception);
```

```
cvector srmatrix::eig () const throw (cvmexception);
```

solve a **nonsymmetric eigenvalue problem** and return a complex vector with eigenvalues of a calling matrix. The first version sets the output parameter `mEigVect` to be equal to the square matrix containing right (if parameter `bRightVect` is true, which is default value) or left (if parameter `bRightVect` is false) eigenvectors as columns. All the functions throw an exception of type **cvmexception** in case of inappropriate sizes of the operands or in case of convergence error. The functions are *inherited* in the class **srbmatrix** and *redefined* in **srsmatrix**. See also **cvector**, **scmatrix** and **srmatrix**. Example:

```
using namespace cvm;
```

```
try {
    scmatrix m(3), me(3);
    cvector vl(3);

    m(1,1) = 0.1;  m(1,2) = 0.2;  m(1,3) = 0.1;
    m(2,1) = 0.11; m(2,2) = -2.9; m(2,3) = -8.4;
    m(3,1) = 0.;  m(3,2) = 2.91; m(3,3) = 8.2;

    vl = m.eig (me);
    std::cout << vl;

    m(2,2) = 2.9;
    vl = m.eig (me);
    std::cout << vl << std::endl;

    std::cout.setf (std::ios::scientific | std::ios::showpos);
    std::cout.precision (1);

    std::cout << m * me(1) - me(1) * v(1);
    std::cout << m * me(2) - me(2) * v(2);
    std::cout << m * me(3) - me(3) * v(3);
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
}
```

prints

```
(-0.0555784,0) (0.285327,0) (5.17025,0)
(0.0968985,3.19501e-018) (5.55155,4.1733) (5.55155,-4.1733)

(+2.1e-002,-7.6e-002) (-3.9e-004,+1.4e-003) (+1.4e-004,-5.2e-004)
(+1.5e-001,-6.2e-002) (+5.4e+000,+3.3e+000) (-6.7e-002,-3.7e+000)
(+7.7e-002,-1.2e-001) (-2.1e+000,-5.3e+000) (+3.3e+000,+6.1e-001)
```

2.8.48 Cholesky

Function

```
srmatrix& srmatrix::cholesky (const srmatrix& m)
throw (cvmexception);
```

forms the Cholesky factorization of a symmetric positive-definite matrix A defined as

$$A = U^T U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the [LAPACK library](#). The function sets a calling matrix to be equal to the factorization of a symmetric positive-definite matrix m. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `srmatrix` and `srsrmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 1., 2., 5., -1., 1., -1., 20.};
    const srmatrix m(a, 3);
    srmatrix h(3);

    h.cholesky(m);
    std::cout << h << std::endl;
    std::cout << ~h * h - m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 1
0 1 -3
0 0 3.16228

0 0 0
0 0 0
0 0 0
```

2.8.49 Bunch-Kaufman

Function

```
srmatrix& srmatrix::bunch_kaufman (const srmatrix& m, int* pivots)  
throw (cvmexception);
```

forms the Bunch-Kaufman factorization of a symmetric matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is the input matrix passed in parameter m , P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D . It utilizes one of ?SYTRF routines of the [LAPACK library](#). The function sets a calling matrix to be equal to the factorization of a symmetric positive-definite matrix m . The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `srmatrix` and `srmatrix`. The function is mostly designed to be used for subsequent calls of ?SYTRS, ?SYCON and ?SYTRI routines of the [LAPACK library](#). Currently it's used internally in `srmatrix::det` flow when argument is symmetric but not positive-definite.

2.8.50 qr

Function

```
void srmatrix::qr (srmatrix& mQ, srmatrix& mR) const throw (cvmexception);
```

computes QR factorization as

$$M = QR$$

where M is a calling square matrix, orthogonal matrix Q and upper triangular matrix R are mQ and mR respectively. The function throws an exception of type `cvmexception` in case if inappropriate sizes of the operands passed. See also `rmatrix`, `rmatrix::qr`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const cvm::srmatrix m(a, 3);
cvm::srmatrix q(3), r(3);

m.qr(q,r);
std::cout << (eye_real(3) - ~q * q).norm()
           << " " << (m - q * r).norm() << std::endl;
```

prints

```
+5.2889959e-16 +7.0854500e-15
```

2.8.51 lq

Function

```
void srmatrix::lq (srmatrix& mL, srmatrix& mQ) const throw (cvmexception);
```

computes LQ factorization as

$$M = LQ$$

where M is a calling square matrix, lower triangular matrix L and orthogonal matrix Q are mL and mQ respectively. The function throws an exception of type `cvmexception` in case if inappropriate sizes of the operands passed. See also `rmatrix`, `rmatrix::lq`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const cvm::srmatrix m(a, 3);
cvm::srmatrix l(3), q(3);

m.lq(l,q);
std::cout << (eye_real(3) - ~q * q).norm()
           << " " << (m - l * q).norm() << std::endl;
```

prints

```
+7.3329369e-016 +8.1523942e-015
```

2.8.52 ql

Function

```
void srmatrix::ql (srmatrix& mQ, srmatrix& mL) const throw (cvmexception);
```

computes QL factorization as

$$M = QL$$

where M is a calling square matrix, orthogonal matrix Q and lower triangular matrix L are mQ and mL respectively. The function throws an exception of type `cvmexception` in case if inappropriate sizes of the operands passed. See also `rmatrix`, `rmatrix::ql`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const cvm::srmatrix m(a, 3);
cvm::srmatrix q(3), l(3);

m.ql(q,l);
std::cout << (eye_real(3) - ~q * q).norm()
           << " " << (m - q * l).norm() << std::endl;

prints

+1.6146017e-015 +4.3341378e-015
```

2.8.53 rq

Function

```
void srmatrix::rq (srmatrix& mR, srmatrix& mQ) const throw (cvmexception);
```

computes RQ factorization as

$$M = RQ$$

where M is a calling square matrix, upper triangular matrix R and orthogonal matrix Q are `mR` and `mQ` respectively. The function throws an exception of type `cvmexception` in case if inappropriate sizes of the operands passed. See also `rmatrix`, `rmatrix::rq`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const cvm::srmatrix m(a, 3);
cvm::srmatrix r(3), q(3);

m.rq(r,q);
std::cout << (eye_real(3) - ~q * q).norm()
           << " " << (m - r * q).norm() << std::endl;
```

prints

```
+3.7030557e-016 +2.2752801e-015
```

2.8.54 identity

Function

```
srmatrix& srmatrix::identity();
```

sets a calling matrix to be equal to identity matrix and returns a reference to the matrix changed. The function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (3);  
srmatrix m(3);  
m.randomize(0.,1.);
```

```
std::cout << m << std::endl;  
std::cout << m.identity();
```

prints

```
9.423e-01 2.950e-01 8.429e-01  
2.013e-01 3.250e-01 2.904e-01  
7.920e-01 2.405e-02 7.801e-01  
  
1.000e+00 0.000e+00 0.000e+00  
0.000e+00 1.000e+00 0.000e+00  
0.000e+00 0.000e+00 1.000e+00
```

2.8.55 vanish

Function

```
srmatrix& srmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `srmatrix::set(TR)` with zero operand passed. The function is *redefined* in the classes `srsmatrix` and `srbmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (3);  
srmatrix m(3);  
m.randomize(0.,1.);
```

```
std::cout << m << std::endl;  
std::cout << m.vanish ();
```

prints

```
1.747e-01 7.563e-01 5.163e-01  
9.657e-01 6.619e-01 8.036e-01  
6.392e-01 6.658e-01 6.495e-01  
  
0.000e+00 0.000e+00 0.000e+00  
0.000e+00 0.000e+00 0.000e+00  
0.000e+00 0.000e+00 0.000e+00
```

2.8.56 **randomize**

Function

```
srmatrix& srmatrix::randomize (TR dFrom, TR dTo);
```

fills a calling matrix with pseudo-random numbers distributed between dFrom and dTo. The function returns a reference to the matrix changed. See also [srmatrix](#). Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);
```

```
srmatrix m(3);
m.randomize(-2.,3.);
std::cout << m;
```

prints

```
-1.6790979e+00 5.0233467e-02 -1.9559008e+00
-1.7987609e-01 -5.2092044e-01 -1.8211615e+00
6.8242439e-01 9.0688803e-01 -1.7891171e+00
```

2.9 scmatrix

This is end-user class encapsulating a square matrix in Euclidean space of complex numbers.

```

template <typename TR, typename TC>
class scmatrix : public cmatrix <TR,TC>, public SqMatrix <TR,TC> {
public:
    scmatrix ();
    explicit scmatrix (int nMN);
    scmatrix (TC* pD, int nMN);
    scmatrix (const TC* pD, int nMN);
    scmatrix (const scmatrix& m);
    explicit scmatrix (const cmatrix& m);
    explicit scmatrix (const cvector& v);
    explicit scmatrix (const srmatrix& m, bool bRealPart = true);
    scmatrix (const TR* pRe, const TR* pIm, int nMN);
    scmatrix (const srmatrix& mRe, const srmatrix& mIm);
    scmatrix (cmatrix& m, int nRow, int nCol, int nSize);
    TC& operator () (int im, int in) throw (cvmexception);
    TC operator () (int im, int in) const throw (cvmexception);
    cvector operator () (int i) throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    cvector operator [] (int i) throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    const srmatrix real () const;
    const srmatrix imag () const;
    scmatrix& operator = (const scmatrix& m) throw (cvmexception);
    scmatrix& assign (const cvector& v);
    scmatrix& assign (const TC* pD);
    scmatrix& assign (int nRow, int nCol, const cmatrix& m)
        throw (cvmexception);
    scmatrix& set (TC x);
    scmatrix& assign_real (const srmatrix& mRe) throw (cvmexception);
    scmatrix& assign_imag (const srmatrix& mIm) throw (cvmexception);
    scmatrix& set_real (TR d);
    scmatrix& set_imag (TR d);
    scmatrix& resize (int nNewMN) throw (cvmexception);
    scmatrix& operator << (const scmatrix& m) throw (cvmexception);
    scmatrix operator + (const scmatrix& m) const
        throw (cvmexception);
    scmatrix operator - (const scmatrix& m) const
        throw (cvmexception);

```

```

scmatrix& sum (const scmatrix& m1,
              const scmatrix& m2) throw (cvmexception);
scmatrix& diff (const scmatrix& m1,
              const scmatrix& m2) throw (cvmexception);
scmatrix& operator += (const scmatrix& m) throw (cvmexception);
scmatrix& operator -= (const scmatrix& m) throw (cvmexception);
scmatrix operator - () const;
scmatrix& operator ++ ();
scmatrix& operator ++ (int);
scmatrix& operator -- ();
scmatrix& operator -- (int);
scmatrix operator * (TR d) const;
scmatrix operator / (TR d) const throw (cvmexception);
scmatrix operator * (TC z) const;
scmatrix operator / (TC z) const throw (cvmexception);
scmatrix& operator *= (TR d);
scmatrix& operator /= (TR d) throw (cvmexception);
scmatrix& operator *= (TC z);
scmatrix& operator /= (TC z) throw (cvmexception);
scmatrix& normalize ();
scmatrix operator ~ () const;
scmatrix& conj (const scmatrix& m) throw (cvmexception);
scmatrix& conj ();
cvector operator * (const cvector& v) const
    throw (cvmexception);
cmatrix operator * (const cmatrix& m) const
    throw (cvmexception);
scmatrix operator * (const scmatrix& m) const
    throw (cvmexception);
scmatrix& operator *= (const scmatrix& m)
    throw (cvmexception);
scmatrix& swap_rows (int n1, int n2) throw (cvmexception);
scmatrix& swap_cols (int n1, int n2) throw (cvmexception);
cvector solve (const cvector& vB) const throw (cvmexception);
cmatrix solve (const cmatrix& mB) const throw (cvmexception);
cvector solve (const cvector& vB, TR& dErr) const
    throw (cvmexception);
cmatrix solve (const cmatrix& mB, TR& dErr) const
    throw (cvmexception);
cvector solve_lu (const scmatrix& mLU, const int* pPivots,
                const cvector& vB, TR& dErr) throw (cvmexception);
cvector solve_lu (const scmatrix& mLU, const int* pPivots,
                const cvector& vB) throw (cvmexception);

```

```

cmatrix solve_lu (const scmatrix& mLU, const int* pPivots,
                 const cmatrix& mB, TR& dErr) throw (cvmexception);
cmatrix solve_lu (const scmatrix& mLU, const int* pPivots,
                 const cmatrix& mB) throw (cvmexception);
TC det () const throw (cvmexception);
scmatrix& low_up (const scmatrix& m,
                 int* nPivots) throw (cvmexception);
scmatrix low_up (int* nPivots) const throw (cvmexception);
TR cond () const throw (cvmexception);
scmatrix& inv (const scmatrix& mArg) throw (cvmexception);
scmatrix inv () const throw (cvmexception);
scmatrix& exp (const scmatrix& mArg, TR tol = cvmMachSp ())
              throw (cvmexception);
scmatrix exp (TR tol = cvmMachSp ()) const throw (cvmexception);
scmatrix& polynom (const scmatrix& m, const cvector& v)
                throw (cvmexception);
scmatrix polynom (const cvector& v) const
                throw (cvmexception);
cvector eig (scmatrix& mEigVect, bool bRightVect = true) const
            throw (cvmexception);
cvector eig () const throw (cvmexception);
scmatrix& cholesky (const schmatrix& m) throw (cvmexception);
scmatrix& bunch_kaufman (const schmatrix& m,
                       int* pivots) throw (cvmexception);
void qr(scmatrix& mQ, scmatrix& mR) const throw (cvmexception);
void lq(scmatrix& mL, scmatrix& mQ) const throw (cvmexception);
void rq(scmatrix& mR, scmatrix& mQ) const throw (cvmexception);
void ql(scmatrix& mQ, scmatrix& mL) const throw (cvmexception);
scmatrix& identity ();
scmatrix& vanish ();
scmatrix& randomize_real (TR dFrom, TR dTo);
scmatrix& randomize_imag (TR dFrom, TR dTo);
};

```

2.9.1 `scmatrix ()`

Constructor

```
scmatrix::scmatrix ();
```

creates an empty `scmatrix` object. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
scmatrix m;  
std::cout << m.msize() << std::endl  
           << m.nsize() << std::endl  
           << m.size() << std::endl;  
m.resize(3);  
std::cout << m;
```

prints

```
0  
0  
0  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)
```

2.9.2 *scmatrix* (int)

Constructor

```
explicit scmatrix::scmatrix (int nMN);
```

creates an $n \times n$ *scmatrix* object where n is passed in *nMN* parameter. The constructor throws an exception of type *cvmexception* in case of non-positive size passed or memory allocation failure. See also *scmatrix*. Example:

```
using namespace cvm;
```

```
scmatrix m (4);  
std::cout << m.msize() << std::endl  
          << m.nsize() << std::endl  
          << m.size() << std::endl << m;
```

prints

```
4  
4  
16  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)
```

2.9.3 *scmatrix* (TC*,int)

Constructor

```
scmatrix::scmatrix (TC* pD, int nMN);
```

creates an $n \times n$ *scmatrix* object where n is passed in *nMN* parameter. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by *pD*. The constructor throws an exception of type *cvmexception* in case of non-positive size passed. See also *scmatrix*, *scmatrix* (const TC*,int). Example:

```
using namespace cvm;
double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.,
              1., 1., 1., 1., 1., 1., 1., 1., 1.};
scmatrix m ((std::complex<double>*) a, 3);

m(1,1) = std::complex<double>(2.,3.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(2,3) (1,1) (1,1)
(1,1) (1,1) (1,1)
(1,1) (1,1) (1,1)
```

```
2 3 1 1 1 1
```

2.9.4 *scmatrix* (const TC*,int)

Constructor

```
scmatrix::scmatrix (const TC* pD, int nMN);
```

creates an $n \times n$ *scmatrix* object where n is passed in *nMN* parameter and copies $n * n$ elements of an array *pD* to it by **columns**. The constructor throws an exception of type ***cvexception*** in case of non-positive size passed. See also ***scmatrix***, ***scmatrix* (TC*,int)**. Example:

```
using namespace cvm;
const double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
                   1., 1., 1., 1., 1., 1., 1., 1., 1.};
scmatrix m ((const std::complex<double>*) a, 3);

m(1,1) = std::complex<double>(2.,3.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(2,3) (1,1) (1,1)
(1,1) (1,1) (1,1)
(1,1) (1,1) (1,1)

1 1 1 1 1 1
```

2.9.5 `scmatrix (const scmatrix&)`

Copy constructor

```
scmatrix::scmatrix (const scmatrix& m)
```

creates a `scmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
scmatrix m ((std::complex<double>*) a, 2);  
scmatrix mc(m);
```

```
m(1,1) = std::complex<double>(7.77,7.77);  
std::cout << m << std::endl << mc;
```

prints

```
(7.77,7.77) (5,6)  
(3,4) (7,8)
```

```
(1,2) (5,6)  
(3,4) (7,8)
```

2.9.6 `scmatrix (const cmatrix&)`

Constructor

```
explicit scmatrix::scmatrix (const cmatrix& m)
```

creates a `scmatrix` object as a copy of matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied. The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. Please note that this constructor is *not explicit* anymore. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.,  
              7., 8., 9., 10., 11., 12.};  
cmatrix m((std::complex<double>*) a, 2, 3);  
std::cout << m << std::endl;
```

```
m.resize(3, 3);  
scmatrix ms (m);  
std::cout << ms;
```

prints

```
(1,2) (5,6) (9,10)  
(3,4) (7,8) (11,12)
```

```
(1,2) (5,6) (9,10)  
(3,4) (7,8) (11,12)  
(0,0) (0,0) (0,0)
```

2.9.7 *scmatrix* (const *cvector*&)

Constructor

```
explicit scmatrix::scmatrix (const cvector& v);
```

creates a *scmatrix* object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scmatrix`, `cvector`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
cvector v((std::complex<double>*) a, 4);  
scmatrix m(v);  
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0) (0,0)  
(0,0) (3,4) (0,0) (0,0)  
(0,0) (0,0) (5,6) (0,0)  
(0,0) (0,0) (0,0) (7,8)
```

2.9.8 *scmatrix* (const *srmatrix*&,bool)

Constructor

```
explicit scmatrix::scmatrix (const srmatrix& m, bool bRealPart = true);
```

creates a *scmatrix* object having the same dimension as real matrix *m* and copies the matrix *m* to its real part if *bRealPart* is true or to its imaginary part otherwise. See also [scmatrix](#), [srmatrix](#). Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
const srmatrix m(a, 3);  
scmatrix mr(m), mi(m, false);  
std::cout << mr << std::endl << mi;
```

prints

```
(1,0) (4,0) (7,0)  
(2,0) (5,0) (8,0)  
(3,0) (6,0) (9,0)  
  
(0,1) (0,4) (0,7)  
(0,2) (0,5) (0,8)  
(0,3) (0,6) (0,9)
```

2.9.9 *scmatrix* (const TR*,const TR*,int)

Constructor

```
scmatrix::scmatrix (const TR* pRe, const TR1* pIm, int nMN);
```

creates a *scmatrix* object of size nMN by nMN and copies every element of arrays pointed to by pRe and pIm to a real and imaginary part of the matrix created respectively. Use NULL pointer to fill up appropriate part with zero values. The constructor throws an exception of type *cvexception* in case of memory allocation failure. See also *scmatrix*. Example:

```
using namespace cvm;
```

```
double re[] = {1., 2., 3., 4.};  
double im[] = {4., 3., 2., 1.};  
scmatrix m(re, im, 2);  
std::cout << m << std::endl;  
re[0] = 7.777;  
std::cout << m << std::endl;
```

```
const double rec[] = {1., 2., 3., 4.};  
const scmatrix mc (rec, NULL, 2);  
std::cout << mc;
```

prints

```
(1,4) (3,2)  
(2,3) (4,1)
```

```
(1,4) (3,2)  
(2,3) (4,1)
```

```
(1,0) (3,0)  
(2,0) (4,0)
```

2.9.10 *s*cmatrix (const *s*rmatrix&, const *s*rmatrix&)

Constructor

```
scmatrix::scmatrix (const srmatrix& mRe, const srmatrix& mIm);
```

creates a *s*cmatrix object of the same size as mRe and mIm has (the constructor throws an exception of type *cv*mexception if mRe and mIm have different sizes) and copies matrices mRe and mIm to a real and imaginary part of the matrix created respectively. The constructor throws an exception of type *cv*mexception in case of memory allocation failure. See also *s*cmatrix, *s*rmatrix. Example:

```
using namespace cvm;
```

```
srmatrix mr(3), mi(3);  
mr.set(1.);  
mi.set(2.);  
const scmatrix mc(mr, mi);  
std::cout << mc;
```

prints

```
(1,2) (1,2) (1,2)  
(1,2) (1,2) (1,2)  
(1,2) (1,2) (1,2)
```

2.9.11 **submatrix**

Submatrix constructor

```
scmatrix::scmatrix (cmatrix& m, int nRow, int nCol, int nSize);
```

creates a `scmatrix` object as a *submatrix* of `m`. It means that the matrix object created shares a memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are *l-based*) and its size (parameter `nSize`). See also `scmatrix`. Example:

```
using namespace cvm;
```

```
cmatrix m(4,5);  
scmatrix subm(m,2,2,2);  
subm.set(std::complex<double>(1.,2.));  
std::cout << m;
```

prints

```
(0,0) (0,0) (0,0) (0,0) (0,0)  
(0,0) (1,2) (1,2) (0,0) (0,0)  
(0,0) (1,2) (1,2) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0) (0,0)
```

2.9.12 operator (,)

Indexing operators

```
TC& scmatrix::operator () (int im, int in) throw (cvmexception);
TC scmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. The operators are *inherited* in the the class `scbmatrix` and *redefined* in the the class `schmatrix`. See also `scmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;

scmatrix m (3);
m.set(std::complex<double>(1.,2.));
std::cout << m(1,1) << std::endl;

m(2,2) = std::complex<double>(7.77,7.77);
std::cout << m;
```

prints

```
(1,2)
(1,2) (1,2) (1,2)
(1,2) (7.77,7.77) (1,2)
(1,2) (1,2) (1,2)
```

2.9.13 operator ()

Indexing operators

```
cvector scmatrix::operator () (int i) throw (cvmexception);
const rvector scmatrix::operator () (int i) const throw (cvmexception);
```

provide access to an *i*-th column of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the *i*-th column of the matrix in order to make possible write access to it. The second version creates a *copy* of a column and therefore is *not an l-value*. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if the parameter *i* is outside of `[1, nsize()]` range. The operators are *redefined* in the the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                 10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m ((std::complex<double>*)a, 3);
    scmatrix ms(3);
    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
(7,8) (9,10) (11,12)

(0,0) (13,14) (0,0)
(0,0) (15,16) (0,0)
(0,0) (17,18) (0,0)
```

2.9.14 operator []

Indexing operators

```
cvector scmatrix::operator [] (int i) throw (cvmexception);
const cvector scmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to an *i*-th row of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the *i*-th row of the matrix in order to make possible write access to it. The second version creates a *copy* of a row and therefore is *not an l-value*. Both operators are *l-based*. The operators throw an exception of type **cvmexception** if the parameter *i* is outside of [1,msize()] range. The operators are *redefined* in the the classes **scbmatrix** and **schmatrix**. See also **scmatrix**, **Matrix::msize()**. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m ((std::complex<double>*)a, 3);
    scmatrix ms(3);
    std::cout << m[2] << std::endl;

    ms[2] = m[3];
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
(3,4) (9,10) (15,16)

(0,0) (0,0) (0,0)
(5,6) (11,12) (17,18)
(0,0) (0,0) (0,0)
```

2.9.15 `real`

Function

```
const srmatrix scmatrix::real () const;
```

creates an object of type `const srmatrix` as a real part of a calling matrix. Please note that, unlike `cvector::real`, this function creates new object *not sharing* a memory with a real part of the calling matrix, i.e. the matrix returned is *not an l-value*. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `srmatrix`, `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
scmatrix m((std::complex<double>*) a, 2);  
std::cout << m << std::endl << m.real();
```

prints

```
(1,2) (5,6)  
(3,4) (7,8)
```

```
1 5  
3 7
```

2.9.16 *imag*

Function

```
const srmatrix scmatrix::imag () const;
```

creates an object of type `const srmatrix` as an imaginary part of a calling matrix. Please note that, unlike `cvector::imag`, this function creates new object *not sharing* a memory with an imaginary part of the calling matrix, i.e. the matrix returned is *not an l-value*. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `srmatrix`, `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
scmatrix m((std::complex<double>*) a, 2);  
std::cout << m << std::endl << m.imag();
```

prints

```
(1,2) (5,6)  
(3,4) (7,8)
```

```
2 6  
4 8
```

2.9.17 operator = (const *scmatrix*&)

Operator

```
scmatrix& scmatrix::operator = (const scmatrix& m)
throw (cvmexception);
```

sets an every element of a calling matrix to a value of appropriate element of a matrix *m* and returns a reference to the matrix changed. The operator throws an exception of type *cvmexception* in case of different matrix sizes. The operator is *redefined* in the the classes *scbmatrix* and *schmatrix*. See also *scmatrix*. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    const scmatrix m1((std::complex<double>*) a, 2);
    scmatrix m2(2);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6)
(3,4) (7,8)
```

2.9.18 assign (const TC*)

Function

```
scmatrix& scmatrix::assign (const cvector& v);  
scmatrix& scmatrix::assign (const TC* pD);
```

sets every element of a calling matrix to a value of appropriate element of a vector *v* or an array pointed to by *pD* and returns a reference to the matrix changed. The function is *redefined* in the classes *scbmatrix* and *schmatrix*. See also *scmatrix*. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12., 13., 14., 15., 16., 17., 18.};  
scmatrix m(3);  
m.assign((std::complex<double>*) a);  
std::cout << m;
```

prints

```
(1,2) (7,8) (13,14)  
(3,4) (9,10) (15,16)  
(5,6) (11,12) (17,18)
```

2.9.19 assign (int, int, const cmatrix&)

Function

```
scmatrix& scmatrix::assign (int nRow, int nCol, const cmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with *l*-based row nRow and column nCol to a matrix m and returns a reference to the matrix changed. The function throws an exception of type *cvmexception* if nRow or nCol are not positive or matrix m doesn't fit. The function is *redefined* in the class *scmatrix*. See also *cmatrix*, *scmatrix*. Example:

```
using namespace cvm;
```

```
scmatrix m1(5);
cmatrix m2(2,3);
m1.set(std::complex<double>(1.,1.));
m2.set(std::complex<double>(2.,2.));
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
(1,1) (1,1) (1,1) (1,1) (1,1)
(1,1) (1,1) (2,2) (2,2) (2,2)
(1,1) (1,1) (2,2) (2,2) (2,2)
(1,1) (1,1) (1,1) (1,1) (1,1)
(1,1) (1,1) (1,1) (1,1) (1,1)
```

2.9.20 set (TC)

Function

```
scmatrix& scmatrix::set (TC x);
```

sets every element of a calling matrix to a value of parameter *x* and returns a reference to the matrix changed. Use **vanish** to set every element of a calling matrix to be equal to zero. The function is *redefined* in the classes **scbmatrix** and *not applicable* to objects of the class **srsmatrix** (i.e. an exception of type **cvmexception** would be thrown in case of using it for objects of this class). See also **scmatrix**. Example:

```
using namespace cvm;
```

```
scmatrix m(3);  
m.set(std::complex<double>(1.,2.));  
std::cout << m;
```

prints

```
(1,2) (1,2) (1,2)  
(1,2) (1,2) (1,2)  
(1,2) (1,2) (1,2)
```

2.9.21 `assign_real`

Function

```
scmatrix& scmatrix::assign_real (const srmatrix& mRe)  
throw (cvmexception);
```

sets real part of every element of a calling matrix to a value of appropriate element of a matrix `mRe` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix` and `srmatrix`. Example:

```
using namespace cvm;  
  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
  
srmatrix m (3);  
scmatrix mc(3);  
m.randomize (0., 1.);  
  
mc.assign_real(m);  
std::cout << mc;  
  
prints  
  
(0.126835,0) (0.57271,0) (0.28312,0)  
(0.784417,0) (0.541673,0) (0.663869,0)
```

2.9.22 `assign_imag`

Function

```
scmatrix& scmatrix::assign_imag (const srmatrix& mIm)  
throw (cvmexception);
```

sets imaginary part of every element of a calling matrix to a value of appropriate element of a matrix `mIm` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix` and `srmatrix`. Example:

```
using namespace cvm;  
  
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
  
srmatrix m (3);  
scmatrix mc(3);  
m.randomize (0., 1.);  
  
mc.assign_imag(m);  
std::cout << mc;  
  
prints  
  
(0.00e+00,6.27e-01) (0.00e+00,1.57e-01) (0.00e+00,9.31e-01)  
(0.00e+00,8.50e-01) (0.00e+00,6.11e-01) (0.00e+00,1.00e+00)  
(0.00e+00,9.75e-01) (0.00e+00,7.38e-01) (0.00e+00,2.29e-01)
```

2.9.23 `set_real`

Function

```
scmatrix& scmatrix::set_real (TR d);
```

sets real part of every element of a calling matrix to a value of parameter `d` and returns a reference to the matrix changed. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
scmatrix m(3);  
m.set_real(1.);  
std::cout << m;
```

prints

```
(1,0) (1,0) (1,0)  
(1,0) (1,0) (1,0)  
(1,0) (1,0) (1,0)
```

2.9.24 `set_imag`

Function

```
scmatrix& scmatrix::set_imag (TR d);
```

sets imaginary part of every element of a calling matrix to a value of parameter `d` and returns a reference to the matrix changed. The function is *redefined* in the class `scbmatrix` and *not allowed* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
scmatrix m(3);  
m.set_imag(1.);  
std::cout << m;
```

prints

```
(0,1) (0,1) (0,1)  
(0,1) (0,1) (0,1)  
(0,1) (0,1) (0,1)
```

2.9.25 *resize*

Function

```
scmatrix& scmatrix::resize (int nNewMN);
throw (cvmexception);
```

changes a size of a calling matrix to *nNewMN* by *nNewMN* and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type *cvmexception* in case of negative size passed or memory allocation failure. The function is *redefined* in the classes *scbmatrix* and *schmatrix*. See also *scmatrix*. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    scmatrix m((std::complex<double>*) a, 2);

    std::cout << m << std::endl;
    m.resize (3);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6)
(3,4) (7,8)

(1,2) (5,6) (0,0)
(3,4) (7,8) (0,0)
(0,0) (0,0) (0,0)
```

2.9.26 operator <<

Operator

```
scmatrix& scmatrix::operator << (const scmatrix& m)
throw (cvmexception);
```

destroys a calling matrix, creates a new one as a copy of *m* and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
try {
    scmatrix m(3);
    scmatrix mc(1);
    m(1,2) = 1.;
    m(2,3) = 2.;
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,0) (1,0) (0,0)
(0,0) (0,0) (2,0)
(0,0) (0,0) (0,0)
```

```
(0,0)
```

```
(0,0) (1,0) (0,0)
(0,0) (0,0) (2,0)
(0,0) (0,0) (0,0)
```

2.9.27 operator +

Operator

```
scmatrix scmatrix::operator + (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a sum of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::sum`, `scmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    scmatrix m1((std::complex<double>*) a, 2);
    scmatrix m2((std::complex<double>*) b, 2);

    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(11,22) (55,66)
(33,44) (77,88)
```

```
(2,4) (10,12)
(6,8) (14,16)
```

2.9.28 operator -

Operator

```
scmatrix scmatrix::operator - (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a difference of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::diff`, `scmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    scmatrix m1((std::complex<double>*) a, 2);
    scmatrix m2((std::complex<double>*) b, 2);

    std::cout << m2 - m1 << std::endl << m1 - m1;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(9,18) (45,54)
(27,36) (63,72)
```

```
(0,0) (0,0)
(0,0) (0,0)
```

2.9.29 sum

Function

```
scmatrix& scmatrix::sum (const scmatrix& m1, const scmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of matrices *m1* and *m2* to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator +`, `scmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                 10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m1((std::complex<double>*)a, 3);
    scmatrix m2(3);
    scmatrix m(3);
    m2.set(std::complex<double>(1.,1.));

    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,3) (8,9) (14,15)
(4,5) (10,11) (16,17)
(6,7) (12,13) (18,19)

(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
(7,8) (13,14) (19,20)
```

2.9.30 diff

Function

```
scmatrix& scmatrix::diff (const scmatrix& m1, const scmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of matrices *m1* and *m2* to a calling matrix and returns a reference to the matrix changed. It throws an exception of type **cvmexception** in case of different sizes of the operands. The function is *redefined* in the classes **scbmatrix** and **schmatrix**. See also **scmatrix::operator -**, **scmatrix**. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m1((std::complex<double>*)a, 3);
    scmatrix m2(3);
    scmatrix m(3);
    m2.set(std::complex<double>(1.,1.));

    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
(0,1) (6,7) (12,13)
(2,3) (8,9) (14,15)
(4,5) (10,11) (16,17)

(-1,0) (5,6) (11,12)
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
```

2.9.31 operator +=

Operator

```
scmatrix& scmatrix::operator += (const scmatrix& m) throw (cvmexception);
```

adds a matrix *m* to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator +`, `scmatrix::sum`, `scmatrix`. Example:

```
using namespace cvm;
```

```
try {
    scmatrix m1(3);
    scmatrix m2(3);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(4,6) (4,6) (4,6)
(4,6) (4,6) (4,6)
(4,6) (4,6) (4,6)

(6,8) (6,8) (6,8)
(6,8) (6,8) (6,8)
(6,8) (6,8) (6,8)
```

2.9.32 operator -=

Operator

```
scmatrix& scmatrix::operator -= (const scmatrix& m) throw (cvmexception);
```

subtracts a matrix *m* from a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator -`, `scmatrix::diff`, `scmatrix`. Example:

```
using namespace cvm;
```

```
try {
    scmatrix m1(3);
    scmatrix m2(3);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-2,-2) (-2,-2) (-2,-2)
(-2,-2) (-2,-2) (-2,-2)
(-2,-2) (-2,-2) (-2,-2)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.33 operator - ()

Operator

```
scmatrix scmatrix::operator - () const throw (cvmexception);
```

creates an object of type *scmatrix* as a calling matrix multiplied by -1 . The operator is *redefined* in the classes *scbmatrix* and *schmatrix*. See also *scmatrix*. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
scmatrix m((std::complex<double>*) a, 2);  
std::cout << -m;
```

prints

```
(-1,-2) (-5,-6)  
(-3,-4) (-7,-8)
```

2.9.34 operator ++

Operator

```
scmatrix& scmatrix::operator ++ ();  
scmatrix& scmatrix::operator ++ (int);
```

adds identity matrix to a calling matrix and returns a reference to the matrix changed. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
scmatrix m(3);  
m.set(std::complex<double>(1.,1.));  
m++;  
std::cout << m << std::endl;  
std::cout << ++m;
```

prints

```
(2,1) (1,1) (1,1)  
(1,1) (2,1) (1,1)  
(1,1) (1,1) (2,1)
```

```
(3,1) (1,1) (1,1)  
(1,1) (3,1) (1,1)  
(1,1) (1,1) (3,1)
```

2.9.35 operator --

Operator

```
scmatrix& scmatrix::operator -- ();  
scmatrix& scmatrix::operator -- (int);
```

subtracts identity matrix from a calling matrix and returns a reference to the matrix changed. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
scmatrix m(3);  
m.set(std::complex<double>(1.,1.));  
m--;  
std::cout << m << std::endl;  
std::cout << --m;
```

prints

```
(0,1) (1,1) (1,1)  
(1,1) (0,1) (1,1)  
(1,1) (1,1) (0,1)  
  
(-1,1) (1,1) (1,1)  
(1,1) (-1,1) (1,1)  
(1,1) (1,1) (-1,1)
```

2.9.36 operator * (TR)

Operator

```
scmatrix scmatrix::operator * (TR d) const;
```

creates an object of type `scmatrix` as a product of a calling matrix and a real number `d`. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator *=`, `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12., 13., 14., 15., 16., 17., 18.};  
scmatrix m((std::complex<double>*) a, 3);  
std::cout << m * 5.;
```

prints

```
(5,10) (35,40) (65,70)  
(15,20) (45,50) (75,80)  
(25,30) (55,60) (85,90)
```

2.9.37 operator / (TR)

Operator

```
scmatrix scmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `scmatrix` as a quotient of a calling matrix and a real number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the **smallest normalized positive number**. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator /=` , `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
```

```
scmatrix m((std::complex<double>*) a, 3);
```

```
std::cout << m / 2.;
```

prints

```
(0.5,1) (3.5,4) (6.5,7)
```

```
(1.5,2) (4.5,5) (7.5,8)
```

```
(2.5,3) (5.5,6) (8.5,9)
```

2.9.38 operator * (TC)

Operator

```
scmatrix scmatrix::operator * (TC z) const;
```

creates an object of type `scmatrix` as a product of a calling matrix and a complex number `z`. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator *=`, `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
              10., 11., 12., 13., 14., 15., 16., 17., 18.};  
scmatrix m((std::complex<double>*) a, 3);  
std::cout << m * std::complex<double>(1.,1.);
```

prints

```
(-1,3) (-1,15) (-1,27)  
(-1,7) (-1,19) (-1,31)  
(-1,11) (-1,23) (-1,35)
```

2.9.39 operator / (TC)

Operator

```
scmatrix scmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `scmatrix` as a quotient of a calling matrix and a complex number `z`. It throws an exception of type `cvmexception` if `z` has an absolute value equal or less than the **smallest normalized positive number**. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator /=` , `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12., 13., 14., 15., 16., 17., 18.};  
scmatrix m((std::complex<double>*) a, 3);  
std::cout << m / std::complex<double>(1.,1.);
```

prints

```
(1.5,0.5) (7.5,0.5) (13.5,0.5)  
(3.5,0.5) (9.5,0.5) (15.5,0.5)  
(5.5,0.5) (11.5,0.5) (17.5,0.5)
```

2.9.40 operator *= (TR)

Operator

```
scmatrix& scmatrix::operator *= (TR d);
```

multiplies a calling matrix by a real number *d* and returns a reference to the matrix changed. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator *`, `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12., 13., 14., 15., 16., 17., 18.};  
scmatrix m((std::complex<double>*) a, 3);  
m *= 5.;  
std::cout << m;
```

prints

```
(5,10) (35,40) (65,70)  
(15,20) (45,50) (75,80)  
(25,30) (55,60) (85,90)
```

2.9.41 operator /= (TR)

Operator

```
scmatrix& scmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling matrix by a real number *d* and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if *d* has an absolute value equal or less than the `smallest normalized positive number`. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator / , scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12., 13., 14., 15., 16., 17., 18.};
```

```
scmatrix m((std::complex<double>*) a, 3);
```

```
m /= 2.;
```

```
std::cout << m;
```

prints

```
(0.5,1) (3.5,4) (6.5,7)
```

```
(1.5,2) (4.5,5) (7.5,8)
```

```
(2.5,3) (5.5,6) (8.5,9)
```

2.9.42 operator *= (TC)

Operator

```
scmatrix& scmatrix::operator *= (TC z);
```

multiplies a calling matrix by a complex number *z* and returns a reference to the matrix changed. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator *`, `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12., 13., 14., 15., 16., 17., 18.};  
scmatrix m((std::complex<double>*) a, 3);  
m *= std::complex<double>(2.,1.);  
std::cout << m;
```

prints

```
(0,5) (6,23) (12,41)  
(2,11) (8,29) (14,47)  
(4,17) (10,35) (16,53)
```

2.9.43 operator /= (TC)

Operator

```
scmatrix& scmatrix::operator /= (TC z) throw (cvmexception);
```

divides a calling matrix by a complex number *z* and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if *z* has an absolute value equal or less than the `smallest normalized positive number`. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator / , scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12., 13., 14., 15., 16., 17., 18.};  
scmatrix m((std::complex<double>*) a, 3);  
m /= std::complex<double>(2.,1.);  
std::cout << m;
```

prints

```
(0.8,0.6) (4.4,1.8) (8,3)  
(2,1) (5.6,2.2) (9.2,3.4)  
(3.2,1.4) (6.8,2.6) (10.4,3.8)
```

2.9.44 `normalize`

Function

```
scmatrix& scmatrix::normalize ();
```

normalizes a calling matrix so its **Euclidean norm** becomes equal to 1 if it was greater than the **smallest normalized positive number** before the call (otherwise the function does nothing). The function is *redefined* in the classes **scbmatrix** and **schmatrix**. See also **scmatrix**. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (5);
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
scmatrix m((std::complex<double>*) a, 2);  
m.normalize();  
std::cout << m << m.norm() << std::endl;
```

prints

```
(7.00140e-002,1.40028e-001) (3.50070e-001,4.20084e-001)  
(2.10042e-001,2.80056e-001) (4.90098e-001,5.60112e-001)  
1.00000e+000
```

2.9.45 conjugation

Operator and functions

```
scmatrix scmatrix::operator ~ () const throw (cvmexception);
scmatrix& scmatrix::conj (const scmatrix& m) throw (cvmexception);
scmatrix& scmatrix::conj () throw (cvmexception);
```

encapsulate complex matrix conjugation. First operator creates an object of type `scmatrix` as a conjugated calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `m` conjugated (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to conjugated itself (it also throws an exception of type `cvmexception` in case of memory allocation failure). The functions are *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
scmatrix mc(3);
std::cout << m << std::endl << ~m << std::endl ;
mc.conj(m);
std::cout << mc << std::endl;
mc.conj();
std::cout << mc;
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
```

```
(1,-2) (3,-4) (5,-6)
(7,-8) (9,-10) (11,-12)
(13,-14) (15,-16) (17,-18)
```

```
(1,-2) (3,-4) (5,-6)
(7,-8) (9,-10) (11,-12)
(13,-14) (15,-16) (17,-18)
```

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
```

2.9.46 operator * (const cvector&)

Operator

```
cvector scmatrix::operator * (const cvector& v) const  
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `cvector::mult` in order to get rid of a new object creation. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`, `cvector`. Example:

```
using namespace cvm;
```

```
try {  
    scmatrix m(3);  
    cvector v(3);  
    m.set(std::complex<double>(1.,1.));  
    v.set(std::complex<double>(1.,1.));  
    std::cout << m * v;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
(0,6) (0,6) (0,6)
```

2.9.47 operator * (const *cmatrix*&)

Operator

```
cmatrix scmatrix::operator * (const cmatrix& m) const  
throw (cvmexception);
```

creates an object of type *cmatrix* as a product of a calling matrix and a matrix *m*. It throws an exception of type *cvmexception* if the number of columns of the calling matrix differs from the number of rows of the matrix *m*. Use *cmatrix::mult* in order to get rid of a new object creation. The operator is *redefined* in the classes *scbmatrix* and *schmatrix*. See also *cmatrix*, *scmatrix*. Example:

```
using namespace cvm;
```

```
try {  
    scmatrix ms(3);  
    cmatrix m(3,2);  
    ms.set(std::complex<double>(1.,1.));  
    m.set(std::complex<double>(1.,1.));  
    std::cout << ms * m;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
(0,6) (0,6)  
(0,6) (0,6)  
(0,6) (0,6)
```

2.9.48 operator * (const *s*cmatrix&)

Operator

```
scmatrix scmatrix::operator * (const scmatrix& m) const  
throw (cvmexception);
```

creates an object of type *s*cmatrix as a product of a calling matrix and a matrix *m*. It throws an exception of type *cvmexception* if the operands have different sizes. Use *cmatrix::mult* in order to get rid of a new object creation. The operator is *inherited* in the class *scbmatrix* and *redefined* in *schmatrix*. See also *s*cmatrix. Example:

```
using namespace cvm;
```

```
try {  
    scmatrix m1(3), m2(3);  
    m1.set(std::complex<double>(1.,1.));  
    m2.set(std::complex<double>(1.,1.));  
    std::cout << m1 * m2;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
(0,6) (0,6) (0,6)  
(0,6) (0,6) (0,6)  
(0,6) (0,6) (0,6)
```

2.9.49 operator *= (const scmatrix&)

Operator

```
scmatrix& scmatrix::operator *= (const scmatrix& m)
throw (cvmexception);
```

sets a calling matrix to be equal to a product of itself by a matrix *m* and returns a reference to the object it changes. The operator throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *inherited* in the class `scbmatrix` and *redefined* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
try {
    scmatrix m1(3), m2(3);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(2.,1.));
    m1 *= m2;
    m2 *= m2;
    std::cout << m1 << std::endl << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,15) (0,15) (0,15)
(0,15) (0,15) (0,15)
(0,15) (0,15) (0,15)

(9,12) (9,12) (9,12)
(9,12) (9,12) (9,12)
(9,12) (9,12) (9,12)
```

2.9.50 swap_rows

Function

```
scmatrix& scmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. *n1* and *n2* are the numbers of rows to be swapped, both are *l*-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1,msize()]`. The function is *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `scmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                 10., 11., 12., 13., 14., 15., 16., 17., 18.};
    scmatrix m ((std::complex<double>*)a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_rows(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
```

```
(1,2) (7,8) (13,14)
(5,6) (11,12) (17,18)
(3,4) (9,10) (15,16)
```

2.9.51 swap_cols

Function

```
scmatrix& scmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. *n1* and *n2* are the numbers of columns to be swapped, both are *l*-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1, nsize())`. The function is *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `scmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                 10., 11., 12., 13., 14., 15., 16., 17., 18.};
    scmatrix m ((std::complex<double>*)a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_cols(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (13,14) (7,8)
(3,4) (15,16) (9,10)
(5,6) (17,18) (11,12)
```

2.9.52 solve

Functions

```
cvector scmatrix::solve (const cvector& vB) const throw (cvmexception);
cmatrix scmatrix::solve (const cmatrix& mB) const throw (cvmexception);
cvector scmatrix::solve (const cvector& vB, TR& dErr) const
throw (cvmexception);
cmatrix scmatrix::solve (const cmatrix& mB, TR& dErr) const
throw (cvmexception);
```

return a solution of a linear equation of kind $Ax = b$ or $AX = B$ where A is a calling matrix. The first and the third versions solve the equation $Ax = b$ where vector b is passed in the parameter vB and the second and fourth versions solve the equation $AX = B$ where matrix B is passed in the parameter mB . The last two versions also set output parameter $dErr$ to be equal to a norm of computation error. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the classes `scbmatrix` and `schmatrix`. See also `cvector::solve`, `cmatrix::solve`, `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (5);
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {-1., 2., -3., -4., 5., -6., 7., -8., 9.};
    scmatrix ma(re, im, 3);
    cmatrix mb(3,2);
    cmatrix mx(3,2);
    double dErr;

    mb(1).set(std::complex<double>(1.,1.));
    mb(1,2) = std::complex<double>(1.,1.);
    mb(2,2) = std::complex<double>(2.,2.);
    mb(3,2) = std::complex<double>(3.,3.);

    mx.solve (ma, mb, dErr);

    std::cout << mx << std::endl << ma * mx - mb;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
(-6.25000e-002,-2.70833e-001) (-1.25000e-001,+1.12500e+000)
(+1.25000e-001,+2.08333e-001) (+2.50000e-001,-2.50000e-001)
(+6.25000e-002,+6.25000e-002) (+1.25000e-001,+1.25000e-001)

(+0.00000e+000,+0.00000e+000) (+0.00000e+000,+0.00000e+000)
(-1.11022e-016,+0.00000e+000) (-2.22045e-016,+0.00000e+000)
(+0.00000e+000,+0.00000e+000) (+0.00000e+000,+0.00000e+000)
```

2.9.53 solve_lu

Functions

```

cvector
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cvector& vB, TR& dErr) throw (cvmexception);

cvector
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cvector& vB) throw (cvmexception);

cmatrix
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cmatrix& mB, TR& dErr) throw (cvmexception);

cmatrix
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cmatrix& mB) throw (cvmexception);

```

create an object of type `cvector` or `cmatrix` as a solution x or X of the matrix linear equation $Ax = b$ or $AX = B$ respectively. Here A is a calling matrix, parameter `mLU` is [LU factorization](#) of the matrix A , parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and parameters `vB` and `mB` are the vector b and matrix B respectively. The first and third version also set output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $Ax = b$ or $AX = B$ with the same matrix A and different vectors b or matrices B . In such case you save on matrix A factorization since it's needed to be performed just one time. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to cingular. The function is *inherited* in the classes `scbmatrix` and `schmatrix`. See also `cvector::solve`, `cmatrix`, `scmatrix`. Example:

```

using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    scmatrix ma(3);
    scmatrix mLU(3);
    cmatrix mb1(3,2); cvector vb1(3);
    cmatrix mb2(3,2); cvector vb2(3);
    cmatrix mx1(3,2); cvector vx1(3);
    cmatrix mx2(3,2); cvector vx2(3);
    iarray nPivots(3);
    double dErr = 0.;
    ma.randomize_real(-1.1,3.); ma.randomize_imag(-3.7,3.);
    mb1.randomize_real(-1.,3.); mb1.randomize_imag(-1.,3.);

```

```

vb1.randomize_real(-2.,3.); vb1.randomize_imag(-3.,1.);
mb2.randomize_real(-5.,1.); mb2.randomize_imag(-4.,1.);
vb2.randomize_real(-1.,6.); vb1.randomize_imag(-4.,4.);
mLU.low_up(ma, nPivots);
mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
std::cout << mx1 << dErr << std::endl;
mx2 = ma.solve_lu (mLU, nPivots, mb2);
std::cout << mx2 << std::endl;;
std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;
vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
std::cout << vx1 << dErr << std::endl;
vx2 = ma.solve_lu (mLU, nPivots, vb2);
std::cout << vx2 << std::endl;;
std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}

prints

(-3.91e-001,-1.62e-001) (-3.17e-001,2.39e-001)
(-3.00e-001,-7.91e-001) (4.71e-001,-9.13e-001)
(-2.34e-001,1.09e+000) (1.10e-001,2.72e-001)
2.78e-015
(-2.60e-001,-5.48e-001) (-3.09e-002,-9.62e-001)
(8.77e-001,8.41e-001) (-6.02e-001,1.87e+000)
(4.20e-003,-9.72e-001) (6.18e-001,-5.64e-001)

(0.00e+000,-2.64e-016) (-1.11e-016,1.11e-016)
(0.00e+000,-2.22e-016) (2.22e-016,-4.44e-016)
(0.00e+000,0.00e+000) (-5.55e-017,0.00e+000)

(6.66e-016,-2.22e-016) (6.18e-016,0.00e+000)
(0.00e+000,1.11e-016) (0.00e+000,0.00e+000)
(0.00e+000,0.00e+000) (-4.44e-016,0.00e+000)
(2.61e-001,2.97e-001) (1.95e+000,-1.07e-001) (-5.51e-001,-1.03e-001)
1.96e-015
(1.26e-001,4.07e-001) (-4.82e-001,-1.14e-002) (2.59e-001,1.60e-001)

(1.11e-016,0.00e+000) (5.55e-017,0.00e+000) (1.11e-016,-2.22e-016)

(-1.11e-016,-7.61e-017) (-2.22e-016,-8.94e-017) (0.00e+000,4.07e-017)

```

2.9.54 `det`

Function

```
TC scmatrix::det () const throw (cvmexception);
```

returns a determinant of a calling matrix. It uses the [LU factorization](#) inside and may throw the same exceptions as the factorizer. The function is *inherited* in the classes [scbmatrix](#) and [schmatrix](#). See also [scmatrix](#). Example:

```
using namespace cvm;
```

```
try {  
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};  
    double im[] = {-1., 2., -3., -4., 5., -6., 7., -8., 9.};  
    const scmatrix m(re, im, 3);  
  
    std::cout << m << std::endl << m.det() << std::endl;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
(1,-1) (4,-4) (7,7)
```

```
(2,2) (5,5) (8,-8)
```

```
(3,-3) (6,-6) (9,9)
```

```
(-192,-192)
```

2.9.55 low_up

Functions

```
scmatrix& scmatrix::low_up (const scmatrix& m, int* nPivots)
throw (cvmexception);
```

```
scmatrix scmatrix::low_up (int* nPivots) const
throw (cvmexception);
```

compute the LU factorization of a calling matrix as

$$A = PLU$$

where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and U is an upper triangular matrix. All the functions store the result as the matrix L without main diagonal combined with U. All the functions return pivot indices as an array of integers (it should support at least `m.size()` elements) pointed to by `nPivots` so *i*-th row was interchanged with `nPivots[i]`-th row. The first version sets a calling matrix to be equal to the *m*'s LU factorization and the second one creates an object of type `scmatrix` as the calling matrix's LU factorization. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be factorized is close to singular. It is recommended to use `iarray` for pivot values. The function is *redefined* in the class `scbmatrix` and *inherited* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.1};
    scmatrix m(re, im, 3);
    scmatrix mLU(3), mLo(3), mUp(3);
    iarray naPivots(3);

    mLU.low_up (m, naPivots);

    mLo.identity ();
    mLo.diag(-2) = mLU.diag(-2);
    mLo.diag(-1) = mLU.diag(-1);
    mUp.diag(0) = mLU.diag(0);
    mUp.diag(1) = mLU.diag(1);
    mUp.diag(2) = mLU.diag(2);

    std::cout << mLo << std::endl << mUp
```

```

        << std::endl << naPivots << std::endl;

    mLU = mLo * mUp;
    for (int i = 3; i >= 1; i--) {
        mLU.swap_rows (i, naPivots[i]);
    }
    std::cout << mLU << std::endl << m - mLU;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

```

prints

```

(1,0) (0,0) (0,0)
(0.333333,0) (1,0) (0,0)
(0.666667,0) (0.5,0) (1,0)

(3,3) (6,6) (9,9.1)
(0,0) (2,2) (4,3.96667)
(0,0) (0,0) (-1.11022e-016,-0.05)

3 3 3

(1,1) (4,4) (7,7)
(2,2) (5,5) (8,8)
(3,3) (6,6) (9,9.1)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)

```

2.9.56 cond

Function

TR scmatrix::cond () const throw (cvmexception);

returns a reciprocal of a condition number of a calling matrix A in the **infinity-norm**:

$$\kappa_{\infty} = \|A\|_{\infty} \|A^{-1}\|_{\infty}.$$

Less value returned means that matrix A is closer to singular. Zero value returned means estimation underflow or that matrix A is singular. The condition number is used for error analysis of systems of linear equations. The function throws exception of type **cvmexception** in case of LAPACK subroutines failure. The function is *inherited* in the classes **scbmatrix** and **schmatrix**. See also **scmatrix::solve**, **scmatrix**. Example:

```
using namespace cvm;

try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};

    scmatrix m(re, im, 3);
    std::cout << m.cond() << std::endl
              << m.det() << std::endl << std::endl;

    m(3,3) = std::complex<double>(9.,10.);
    std::cout << m.cond() << std::endl << m.det() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
1.54198e-018
(1.33227e-015, -1.33227e-015)
```

```
0.0050679
(6, -1.33227e-015)
```

2.9.57 inv

Functions

```
scmatrix& scmatrix::inv (const scmatrix& m) throw (cvmexception);
scmatrix scmatrix::inv () const throw (cvmexception);
```

implement matrix inversion. The first version sets a calling matrix to be equal to *m* inverted and the second one creates an object of type *scmatrix* as inverted calling matrix. The functions throw exception of type *cvmexception* in case of inappropriate sizes of the operands or when the matrix to be inverted is close to cingular. The function is *redefined* in the class *schmatrix* and *inherited* in *scbmatrix*. See also *scmatrix*. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    scmatrix m(re, im, 3);
    scmatrix mi(3);

    mi.inv (m);
    std::cout << mi << std::endl << mi * m - eye_complex(3);
    std::cout << std::endl << mi.inv() * mi - eye_complex(3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-8.33e-001,-1.67e-001) (6.67e-001,1.33e+000) (4.81e-016,-1.00e+000)
(3.33e-001,1.67e+000) (-1.67e-001,-3.83e+000) (-5.37e-016,2.00e+000)
(2.22e-016,-1.00e+000) (-4.44e-016,2.00e+000) (2.22e-016,-1.00e+000)

(-1.11e-016,1.11e-016) (0.00e+000,2.22e-016) (8.33e-017,1.26e-015)
(4.44e-016,-5.00e-016) (0.00e+000,7.77e-016) (7.22e-016,-1.15e-015)
(-1.11e-016,1.11e-016) (2.22e-016,-6.66e-016) (0.00e+000,1.11e-016)

(4.44e-016,4.44e-016) (-1.18e-015,1.33e-015) (6.66e-016,-7.77e-016)
(-6.85e-016,8.88e-016) (1.33e-015,3.16e-030) (-8.33e-016,-1.77e-030)
(-5.09e-016,6.66e-016) (7.96e-016,-1.78e-015) (-4.44e-016,1.11e-016)
```

2.9.58 exp

Functions

```

scmatrix& scmatrix::exp (const scmatrix& m, TR tol = cvmMachSp ())
throw (cvmexception);
scmatrix scmatrix::exp (TR tol = cvmMachSp ()) const
throw (cvmexception);

```

compute an exponent of a calling matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \dots + z^p/p!,$$

where

$$N_{pq}(z) = \sum_{k=0}^p \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} z^k,$$

$$D_{pq}(z) = \sum_{k=0}^q \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} (-z)^k$$

along with the matrix normalizing as described in [2], p. 572. The functions use ZMEXP (or CMEXP for float version) FORTRAN subroutine implementing the algorithm. The first version sets the calling matrix to be equal to the exponent of *m* and returns the reference to the matrix changed. The second version creates an object of type *scmatrix* as the exponent of the calling matrix. The algorithm uses parameter *tol* as $\varepsilon(p, q)$ in order to choose constants *p* and *q* so that

$$\varepsilon(p, q) \geq 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the **largest relative spacing** by default. The functions throw an exception of type **cvmexception** in case of inappropriate sizes of the operands or when LAPACK subroutine fails. The functions are *inherited* in the classes **scbmatrix** and **schmatrix**. The second version is *redefined* in **scbmatrix**. See also **scmatrix**. Example:

```
using namespace cvm;
```

```

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    scmatrix m(2);
    m(1,1) = std::complex<double>(-49., 1.);
    m(1,2) = std::complex<double>(24., 1.);
    m(2,1) = std::complex<double>(-64., 1.);
    m(2,2) = std::complex<double>(31., 1.);

```

```

    m = m.exp();
    std::cout << m(1,1) << std::endl << "  "
              << m(1,2) << std::endl;
    std::cout << m(2,1) << std::endl << "  "
              << m(2,2) << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

```

prints

```

(-4.508497580070061e-001,7.900659666739228e-001)
 (3.199576050798058e-001,-6.081804753524478e-001)
(-7.584316151932173e-001,1.666747485117903e+000)
 (5.295040786048336e-001,-1.278050361026397e+000)

```

MATLAB output:

Column 1

```

-4.508497580070262e-001 +7.900659666739607e-001i
-7.584316151932523e-001 +1.666747485117982e+000i

```

Column 2

```

3.199576050798204e-001 -6.081804753524764e-001i
5.295040786048589e-001 -1.278050361026457e+000i

```

2.9.59 polynomial

Functions

```
scmatrix& scmatrix::polynom (const scmatrix& m, const cvector& v)
throw (cvmexception);
```

```
scmatrix scmatrix::polynom (const cvector& v) const
throw (cvmexception);
```

compute a matrix polynomial defined as

$$p(A) = b_0I + b_1A + \dots + b_qA^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^r B_k(A^s)^k, \quad s = \text{floor}(\sqrt{q}), \quad r = \text{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum_{i=0}^{s-1} b_{sk+i}A^i, & k = 0, 1, \dots, r-1 \\ \sum_{i=0}^{q-sr} b_{sr+i}A^i, & k = r. \end{cases}$$

See also [2], p. 568. The coefficients b_0, b_1, \dots, b_q are passed in the parameter v , where q is equal to $v.size() - 1$, so the functions compute matrix polynomial equal to

$$v[1] * I + v[2] * m + \dots + v[v.size()] * m^{v.size()-1}$$

The first version sets a calling matrix to be equal to the polynomial of m and the second one creates an object of type `scmatrix` as the polynomial of a calling matrix. The functions use ZPOLY (or CPOLY for float version) FORTRAN subroutine implementing the Horner's algorithm. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in the class `scbmatrix` and *redefined* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    const double re[] = {2.2, 1.3, 1.1, -0.9, 0.2,
                        -0.45, 45., -30., 10., 3., 0.};
    const double im[] = {0.5, -2, 0, 1, 3,
                        -3., 30., 0., -9., 0., 1.};
```

```

const cvector v(re, im, 11);
scmatrix m(2), mp(2);
m(1,1) = std::complex<double>(0.1, -0.2);
m(1,2) = std::complex<double>(0.1, -0.2);
m(2,1) = std::complex<double>(0.5, -0.6);
m(2,2) = std::complex<double>(0.3, -0.4);

mp.polynom (m, v);
std::cout << mp(1,1) << std::endl << "  "
          << mp(1,2) << std::endl;
std::cout << mp(2,1) << std::endl << "  "
          << mp(2,2) << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}

```

prints

```

(2.4856526656000000e+000,3.7912633088000001e+000)
(2.8177861760000004e-001,2.3019428608000001e+000)
(-8.8350698880000001e-001,8.0520286208000002e+000)
(1.9030098624000001e+000,6.6663061888000002e+000)

```

MATLAB output:

Column 1

```

2.4856526656000000e+000 +3.7912633088000001e+000i
-8.8350698879999991e-001 +8.0520286208000002e+000i

```

Column 2

```

2.8177861760000000e-001 +2.3019428608000001e+000i
1.9030098623999999e+000 +6.6663061888000003e+000i

```

2.9.60 eig

Functions

```
cvector scmatrix::eig (scmatrix& mEigVect, bool bRightVect = true) const
throw (cvmexception);
```

```
cvector scmatrix::eig () const throw (cvmexception);
```

solve a **nonsymmetric eigenvalue problem** and return a complex vector with eigenvalues of a calling matrix. The first version sets the output parameter `mEigVect` to be equal to the square matrix containing right (if parameter `bRightVect` is true, which is default value) or left (if parameter `bRightVect` is false) eigenvectors as columns. All the functions throw an exception of type **cvmexception** in case of inappropriate sizes of the operands or in case of convergence error. The functions are *inherited* in the class **scbmatrix** and *redefined* in **schmatrix**. See also **cvector**, **scmatrix**. Example:

```
using namespace cvm;
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    scmatrix m(re, im, 3);
    scmatrix me(3);
    cvector vl(3);

    vl = m.eig (me);
    std::cout << vl << std::endl;
    std::cout.setf (std::ios::scientific | std::ios::left);
    std::cout.precision (2);
    std::cout << m * me(1) - me(1) * vl(1);
    std::cout << m * me(2) - me(2) * vl(2);
    std::cout << m * me(3) - me(3) * vl(3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(16.1096,16.7004) (-1.09351,-0.88358) (-0.0161248,0.183218)
(-9.44e-016,-3.55e-015) (-1.11e-016,-1.07e-014) (2.66e-015,-1.42e-014)
(-5.55e-016,-4.44e-016) (-1.80e-015,9.44e-016) (-2.00e-015,7.22e-016)
(9.92e-016,1.24e-015) (1.05e-015,2.78e-017) (1.64e-015,9.30e-016)
```

2.9.61 Cholesky

Function

```
schmatrix& schmatrix::cholesky (const schmatrix& m)
throw (cvmexception);
```

forms the Cholesky factorization of a hermitian positive-definite matrix A defined as

$$A = U^T U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the [LAPACK library](#). The function sets a calling matrix to be equal to the factorization of a hermitian positive-definite matrix m. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `schmatrix` and `schmatrix`. Example:

```
using namespace cvm;

try {
    double r[] = {1., 2., 1., 2., 15., -1., 1., -1., 20.};
    double i[] = {0., -1., 2., 1., 0., 3., -2., -3., 0.};
    const schmatrix m(r, i, 3);
    schmatrix c(3);

    c.cholesky(m);

    std::cout << c << std::endl;
    std::cout << ~c * c - m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1,0) (2,1) (1,-2)
(0,-0) (3.16228,0) (-0.316228,0.632456)
(0,0) (-0,0) (3.80789,0)

(0,0) (0,0) (0,0)
(0,0) (1.77636e-015,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.62 Bunch-Kaufman

Function

```
scmatrix& scmatrix::bunch_kaufman (const schmatrix& m, int* pivots)  
throw (cvmexception);
```

forms the Bunch-Kaufman factorization of a hermitian matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is the input matrix passed in parameter m , P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D . It utilizes one of ?SYTRF routines of the [LAPACK library](#). The function sets a calling matrix to be equal to the factorization of a hermitian positive-definite matrix m . The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `scmatrix` and `schmatrix`. The function is mostly designed to be used for subsequent calls of ?SYTRS, ?SYCON and ?SYTRI routines of the [LAPACK library](#). Currently it's used internally in `scmatrix::det` flow when argument is hermitian but not positive-definite matrix.

2.9.63 qr

Function

```
void scmatrix::qr (scmatrix& mQ, scmatrix& mR) const throw (cvmexception);
```

computes QR factorization as

$$M = QR$$

where M is a calling square matrix, unitary matrix Q and upper triangular matrix R are mQ and mR respectively. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `cmatrix`, `cmatrix::qr`, `scmatrix`. Example:

```
using namespace cvm;
```

```
treal ar[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
treal ai[] = {1., -1., 2., -2., 3., -3., 4., -4., 5.};
const cvm::scmatrix m(ar, ai, 3);
cvm::scmatrix q(3), r(3);
```

```
m.qr(q,r);
std::cout << (eye_complex(3) - ~q * q).norm()
           << " " << (m - q * r).norm() << std::endl;
```

prints

```
+3.7957075e-16 +1.9641850e-15
```

2.9.64 lq

Function

```
void scmatrix::lq (scmatrix& mL, scmatrix& mQ) const throw (cvmexception);
```

computes LQ factorization as

$$M = LQ$$

where M is a calling square matrix, lower triangular matrix L and unitary matrix Q are mL and mQ respectively. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `cmatrix`, `cmatrix::lq`, `scmatrix`. Example:

```
using namespace cvm;
```

```
treal ar[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
treal ai[] = {1., -1., 2., -2., 3., -3., 4., -4., 5.};
const cvm::scmatrix m(ar, ai, 3);
cvm::scmatrix l(3), q(3);
```

```
m.lq(l,q);
std::cout << (eye_complex(3) - ~q * q).norm()
           << " " << (m - l * q).norm() << std::endl;
```

prints

```
+3.5331153e-016 +3.0526086e-015
```

2.9.65 `rq`

Function

```
void scmatrix::rq (scmatrix& mR, scmatrix& mQ) const throw (cvmexception);
```

computes RQ factorization as

$$M = RQ$$

where M is a calling square matrix, upper triangular matrix R and unitary matrix Q are `mR` and `mQ` respectively. The function throws an exception of type `cvmexception` in case if inappropriate sizes of the operands passed. See also `cmatrix`, `cmatrix::rq`, `srmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
treal ai[] = {1., -1., 2., -2., 3., -3., 4., -4., 5.};
const cvm::scmatrix m(ar, ai, 3);
cvm::scmatrix r(3), q(3);

m.rq(r,q);
std::cout << (eye_complex(3) - ~q * q).norm()
           << " " << (m - r * q).norm() << std::endl;

prints

+3.3357248e-016 +3.4488467e-015
```

2.9.66 ql

Function

```
void scmatrix::ql (scmatrix& mQ, scmatrix& mL) const throw (cvmexception);
```

computes QL factorization as

$$M = QL$$

where M is a calling square matrix, unitary matrix Q and lower triangular matrix L are mQ and mL respectively. The function throws an exception of type `cvmexception` in case if inappropriate sizes of the operands passed. See also `cmatrix`, `cmatrix::ql`, `scmatrix`.
Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
treal ai[] = {1., -1., 2., -2., 3., -3., 4., -4., 5.};
const cvm::scmatrix m(ar, ai, 3);
cvm::scmatrix q(3), l(3);

m.ql(q,l);
std::cout << (eye_complex(3) - ~q * q).norm()
           << " " << (m - q * l).norm() << std::endl;

prints

+8.8904536e-016 +4.5789129e-015
```

2.9.67 **identity**

Function

```
scmatrix& scmatrix::identity();
```

sets a calling matrix to be equal to identity matrix and returns a reference to the matrix changed. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
scmatrix m(3);  
m.randomize_real(0.,3.);  
m.randomize_imag(-1.,2.);
```

```
std::cout << m << std::endl;  
std::cout << m.identity();
```

prints

```
(1.31162,-0.52501) (2.8612,-0.531144) (1.31849,0.547838)  
(1.19929,1.48253) (0.535417,0.41316) (0.459883,1.7019)  
(0.415937,-0.491134) (2.0969,-0.218024) (0.545305,1.17866)
```

```
(1,0) (0,0) (0,0)  
(0,0) (1,0) (0,0)  
(0,0) (0,0) (1,0)
```

2.9.68 **vanish**

Function

```
scmatrix& scmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `scmatrix::set(TC)` with zero operand passed. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
scmatrix m(3);  
m.randomize_real(0.,3.);  
m.randomize_imag(-1.,2.);
```

```
std::cout << m << std::endl;  
std::cout << m.vanish();
```

prints

```
(1.34834,-0.758385) (0.837825,-0.225532) (0.367687,0.791833)  
(2.23698,-0.183142) (2.6878,0.741111) (0.495865,0.698904)  
(0.584124,0.00491348) (1.31574,0.687643) (0.482131,1.66482)  
  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)
```

2.9.69 `randomize_real`

Function

```
scmatrix& scmatrix::randomize_real (TR dFrom, TR dTo);
```

fills a real part of a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
scmatrix m(3);  
m.randomize_real(0.,3.);  
std::cout << m;
```

prints

```
(1.56e+000,0.00e+000) (2.39e+000,0.00e+000) (2.41e+000,0.00e+000)  
(3.73e-002,0.00e+000) (2.61e+000,0.00e+000) (1.36e+000,0.00e+000)  
(2.71e+000,0.00e+000) (1.69e+000,0.00e+000) (2.68e+000,0.00e+000)
```

2.9.70 `randomize_imag`

Function

```
scmatrix& scmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills an imaginary part of a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
```

```
std::cout.precision (2);
```

```
scmatrix m(3);
```

```
m.randomize_imag(0.,3.);
```

```
std::cout << m;
```

prints

```
(0.00e+000,1.58e+000) (0.00e+000,2.38e+000) (0.00e+000,2.64e+000)
```

```
(0.00e+000,1.62e-002) (0.00e+000,4.26e-002) (0.00e+000,2.21e+000)
```

```
(0.00e+000,2.39e+000) (0.00e+000,6.95e-001) (0.00e+000,4.30e-001)
```

2.10 BandMatrix

This base class contains member functions common for band matrices. This class is not designed to be instantiated.

```
template <typename TR, typename TC>
class BandMatrix {
public:
    int lsize () const;
    int usize () const;
};
```

2.10.1 lsize

Function

```
int BandMatrix<TR,TC>::lsize () const;
```

returns a number lower sub-diagonals a calling matrix. The function is *inherited* in the classes `srbmatrix` and `scbmatrix`. See also `BandMatrix`. Example:

```
using namespace cvm;
```

```
srbmatrix m(4,2,1);  
m.set(1.);  
std::cout << m;  
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;  
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
1 1 0 0  
1 1 1 0  
1 1 1 1  
0 1 1 1  
4 4 16 2 1
```

2.10.2 `usize`

Function

```
int BandMatrix<TR,TC>::usize () const;
```

returns a number upper sub-diagonals a calling matrix. The function is *inherited* in the classes `srbmatrix` and `scbmatrix`. See also `BandMatrix`. Example:

```
using namespace cvm;
```

```
srbmatrix m(4,2,1);  
m.set(1.);  
std::cout << m;  
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;  
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
1 1 0 0  
1 1 1 0  
1 1 1 1  
0 1 1 1  
4 4 16 2 1
```

2.11 srbmatrix

This is end-user class encapsulating a square band matrix in Euclidean space of real numbers. This class utilizes [band storage](#) for its elements.

```

template <typename TR>
class srbmatrix : public srmatrix <TR>, public BandMatrix <TR,TR> {
public:
    srbmatrix ();
    explicit srbmatrix (int nMN);
    srbmatrix (int nMN);
    srbmatrix (TR* pD, int nMN, int nKL, int nKU);
    srbmatrix (const TR* pD, int nMN, int nKL, int nKU);
    srbmatrix (const srbmatrix& m);
    srbmatrix (const rmatrix& m, int nKL, int nKU);
    explicit srbmatrix (const rvector& v);
    TR& operator () (int im, int in) throw (cvmexception);
    TR operator () (int im, int in) const throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    srbmatrix& operator = (const srbmatrix& m) throw (cvmexception);
    srbmatrix& assign (const rvector& v);
    srbmatrix& assign (const TR* pD);
    srbmatrix& set (TR x);
    srbmatrix& resize (int nNewMN) throw (cvmexception);
    srbmatrix& resize_lu (int nNewKL, int nNewKU) throw (cvmexception);
    bool operator == (const srbmatrix& v) const;
    bool operator != (const srbmatrix& v) const;
    srbmatrix& operator << (const srbmatrix& m) throw (cvmexception);
    srbmatrix operator + (const srbmatrix& m) const
        throw (cvmexception);
    srbmatrix operator - (const srbmatrix& m) const
        throw (cvmexception);
    srbmatrix& sum (const srbmatrix& m1,
        const srbmatrix& m2) throw (cvmexception);
    srbmatrix& diff (const srbmatrix& m1,
        const srbmatrix& m2) throw (cvmexception);
    srbmatrix& operator += (const srbmatrix& m) throw (cvmexception);
    srbmatrix& operator -= (const srbmatrix& m) throw (cvmexception);
    srbmatrix operator - () const;
    srbmatrix& operator ++ ();
    srbmatrix& operator ++ (int);
    srbmatrix& operator -- ();

```

```
srbmatrix& operator -- (int);
srbmatrix operator * (TR d) const;
srbmatrix operator / (TR d) const throw (cvmexception);
srbmatrix& operator *= (TR d);
srbmatrix& operator /= (TR d) throw (cvmexception);
srbmatrix& normalize ();
srbmatrix operator ~ () const throw (cvmexception);
srbmatrix& transpose (const srbmatrix& m) throw (cvmexception);
srbmatrix& transpose () throw (cvmexception);
rvector operator * (const rvector& v) const throw (cvmexception);
rmatrix operator * (const rmatrix& m) const throw (cvmexception);
srmatrix operator * (const srmatrix& m) const throw (cvmexception);
srbmatrix operator * (const srbmatrix& m) const throw (cvmexception);
srbmatrix& low_up (const srbmatrix& m,
                 int* nPivots) throw (cvmexception);
srbmatrix low_up (int* nPivots) const throw (cvmexception);
srbmatrix& identity ();
srbmatrix& vanish ();
srbmatrix& randomize (TR dFrom, TR dTo);
};
```

2.11.1 srbmatrix ()

Constructor

```
srbmatrix::srbmatrix ();
```

creates an empty srbmatrix object. See also [srbmatrix](#). Example:

```
using namespace cvm;
```

```
srbmatrix m;  
std::cout << m.msize() << " " << m.nsize() << " " << m.size();  
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
```

```
m.resize (3);  
m.resize_lu(1,0);  
m.set(1.);  
std::cout << m;
```

prints

```
0 0 0 0 0  
1 0 0  
1 1 0  
0 1 1
```

2.11.2 srbmatrix (int)

Constructor

```
explicit srbmatrix::srbmatrix (int nMN);
```

creates an $n \times n$ srbmatrix object where n is passed in `nMN` parameter. The matrix created is diagonal, i.e. $k_l = k_u = 0$. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `srbmatrix`.
Example:

```
using namespace cvm;
```

```
srbmatrix m(4);  
std::cout << m.msize() << " " << m.nsize() << " " << m.size();  
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
```

```
m.set(1.);  
std::cout << m;
```

prints

```
4 4 4 0 0  
1 0 0 0  
0 1 0 0  
0 0 1 0  
0 0 0 1
```

2.11.3 srbmatrix (int,int,int)

Constructor

```
srbmatrix::srbmatrix (int nMN, int nKL, int nKU);
```

creates an $n \times n$ srbmatrix object where n is passed in nMN parameter. The matrix created has nKL sub-diagonals and nKU super-diagonals. The constructor throws an exception of type `cvmexception` in case of non-positive size or negative number of sub-diagonals or super-diagonals passed or in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
srbmatrix m(5,1,1);  
m.set(1.);  
std::cout << m << std::endl  
          << m.msize() << " " << m.nsize() << " " << m.size()  
          << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
1 1 0 0 0  
1 1 1 0 0  
0 1 1 1 0  
0 0 1 1 1  
0 0 0 1 1  
  
5 5 15 1 1
```

2.11.4 srbmatrix (TR*,int,int,int)

Constructor

```
srbmatrix::srbmatrix (TR* pD, int nMN, int nKL, int nKU);
```

creates an $n \times n$ srbmatrix object where n is passed in nMN parameter. The matrix created has $k_l=nKL$ sub-diagonals and $k_u=nKU$ super-diagonals. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by pD. Please note that this array must contain at least $(k_l + k_u + 1) * n$ elements. The constructor throws an exception of type `cvmexception` in case of non-positive size or negative number of sub- or super-diagonals passed. See also `srbmatrix`, `srbmatrix (const TR*,int,int,int)`. Example:

```
using namespace cvm;
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix ml(a,4,1,0);
srbmatrix mu(a,4,0,1);
ml(2,1) = 9.;
std::cout << ml << std::endl << mu << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
1 0 0 0
9 3 0 0
0 4 5 0
0 0 6 7

9 3 0 0
0 4 5 0
0 0 6 7
0 0 0 8

1 9 3 4 5 6
```

2.11.5 srbmatrix (const TR*,int,int,int)

Constructor

```
srbmatrix::srbmatrix (const TR* pD, int nMN, int nKL, int nKU);
```

creates an $n \times n$ srbmatrix object where n is passed in nMN parameter. The matrix created has $k_l=nKL$ sub-diagonals and $k_u=nKU$ super-diagonals. Then constructor copies $(k_l + k_u + 1) * n$ elements of an array pD to the matrix according to **band storage**. The constructor throws an exception of type **cvmexception** in case of non-positive size or negative number of sub- or super-diagonals passed. See also **srbmatrix**, **srbmatrix (TR*,int,int,int)**. Example:

```
using namespace cvm;
const double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix ml(a,4,1,0);
srbmatrix mu(a,4,0,1);
ml(2,1) = 9.;
std::cout << ml << std::endl << mu << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
1 0 0 0
9 3 0 0
0 4 5 0
0 0 6 7
```

```
2 3 0 0
0 4 5 0
0 0 6 7
0 0 0 8
```

```
1 2 3 4 5 6
```

2.11.6 srbmatrix (const srbmatrix&)

Copy constructor

```
srbmatrix::srbmatrix (const srbmatrix& m);
```

creates a srbmatrix object as a copy of m. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m(a,4,1,0);  
srbmatrix mc(m);  
m(1,1) = 7.77;  
std::cout << m << std::endl << mc;
```

prints

```
7.77 0 0 0  
2 3 0 0  
0 4 5 0  
0 0 6 7
```

```
1 0 0 0  
2 3 0 0  
0 4 5 0  
0 0 6 7
```

2.11.7 srbmatrix (const rmatrix&,int,int)

Constructor

```
srbmatrix::srbmatrix (const rmatrix& m, int nKL, int nKU);
```

creates a srbmatrix object as a copy of “sliced” matrix *m*, i.e. it copies main diagonal, *nKL* sub-diagonals and *nKU* super-diagonals of the matrix *m*. It’s assumed that $m \times n$ matrix *m* must have equal sizes, i.e. $m = n$ is satisfied. The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
srbmatrix m(a,4);
srbmatrix mb(m,1,2);
std::cout << m << std::endl << mb;
```

prints

```
1 5 9 13
2 6 10 14
3 7 11 15
4 8 12 16
```

```
1 5 9 0
2 6 10 14
0 7 11 15
0 0 12 16
```

2.11.8 srbmatrix (const rvector&)

Constructor

```
explicit srbmatrix::srbmatrix (const rvector& v);
```

creates a srbmatrix object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srbmatrix`, `rvector`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5.};  
rvector v(a, 5);  
srbmatrix m(v);  
std::cout << m;
```

prints

```
1 0 0 0 0  
0 2 0 0 0  
0 0 3 0 0  
0 0 0 4 0  
0 0 0 0 5
```

2.11.9 operator (,)

Indexing operators

```
TR& srbmatrix::operator () (int im, int in) throw (cvmexception);
TR srbmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a band matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Only elements located on main diagonal or on non-zero sub- or super-diagonals are l-values. All other values located outside this area are not writable. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range or in case of attempt to write to a non-writable element⁹. See also `srbmatrix`, `BandMatrix::lsize()` and `BandMatrix::usize()`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix m (a,4,1,0);

    m(2,2) = 7.77;
    std::cout << m << std::endl;
    std::cout << m(3,2) << " " << m(1,4) << std::endl;

    m(1,3) = 7.77;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1.00e+000 0.00e+000 0.00e+000 0.00e+000
2.00e+000 7.77e+000 0.00e+000 0.00e+000
0.00e+000 4.00e+000 5.00e+000 0.00e+000
0.00e+000 0.00e+000 6.00e+000 7.00e+000
```

```
4.00e+000 0.00e+000
```

```
Exception: Attempt to change a read-only element
```

⁹Here I use `type_proxy<T>` class originally described in [4], p. 217.

2.11.10 operator ()

Indexing operator

```
const rvector srbmatrix::operator () (int i) const throw (cvmexception);
```

provides access to an *i*-th column of a band matrix. Unlike `srmatrix::operator ()`, this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is *l-based*. The operator throws an exception of type `cvmexception` if the parameter *i* is outside of `[1, nsize()]` range. See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m (a,4,1,0);
```

```
std::cout << m << std::endl;  
std::cout << m(2);
```

prints

```
1 0 0 0  
2 3 0 0  
0 4 5 0  
0 0 6 7  
  
0 3 4 0
```

2.11.11 operator []

Indexing operator

```
const rvector srbmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to an *i*-th row of a band matrix. Unlike `srmatrix::operator []`, this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is *l-based*. The operator throws an exception of type `cvmexception` if the parameter *i* is outside of `[1, nsize()]` range. See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m (a,4,1,0);
```

```
std::cout << m << std::endl;
```

```
std::cout << m[2];
```

prints

```
1 0 0 0
```

```
2 3 0 0
```

```
0 4 5 0
```

```
0 0 6 7
```

```
2 3 0 0
```

2.11.12 operator = (const srbmatrix&)

Operator

```
srbmatrix& srbmatrix::operator = (const srbmatrix& m)
throw (cvmexception);
```

sets an every element of a calling band matrix to a value of appropriate element of a band matrix `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different matrix sizes or in case of different numbers of sub- or super-diagonals. See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    const srbmatrix m1(a,4,1,0);
    srbmatrix m2(4,1,0);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7
```

2.11.13 assign (const TR*)

Function

```
srbmatrix& srbmatrix::assign (const rvector& v);  
srbmatrix& srbmatrix::assign (const TR* pD);
```

sets every element of a calling band matrix to a value of appropriate element of a vector v or an array pointed to by pD and returns a reference to the matrix changed. Please note that this array must contain at least $(k_l + k_u + 1)n$ elements. See also [srbmatrix](#). Example:

```
using namespace cvm;
```

```
const double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m(4,0,1);
```

```
m.assign(a);  
std::cout << m;
```

prints

```
2 3 0 0  
0 4 5 0  
0 0 6 7  
0 0 0 8
```

2.11.14 set (TR)

Function

```
srbmatrix& srbmatrix::set (TR x);
```

sets every element of a calling band matrix to a value of parameter `x` and returns a reference to the matrix changed. Use `vanish` to set every element of a calling matrix to be equal to zero. See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
srbmatrix m(8,2,1);  
m.set(3.);  
std::cout << m;
```

prints

```
3 3 0 0 0 0 0 0  
3 3 3 0 0 0 0 0  
3 3 3 3 0 0 0 0  
0 3 3 3 3 0 0 0  
0 0 3 3 3 3 0 0  
0 0 0 3 3 3 3 0  
0 0 0 0 3 3 3 3  
0 0 0 0 0 3 3 3
```

2.11.15 resize

Function

```
srbmatrix& srbmatrix::resize (int nNewMN) throw (cvmexception);
```

changes a size of a calling band matrix to nNewMN by nNewMN and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. This function doesn't change a number of sub- or super-diagonals. Like any band matrix class member function, this function doesn't change **non-referred elements**. See number 8 appearing after resize in example below. The function throws an exception of type **cvmexception** in case of non-positive size passed or memory allocation failure. See also **srbmatrix.resize_lu**, **srbmatrix**. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix m(a,4,1,0);
    std::cout << m << std::endl;
    m.resize (5);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

1 0 0 0 0
2 3 0 0 0
0 4 5 0 0
0 0 6 7 0
0 0 0 8 0
```

2.11.16 `resize_lu`

Function

```
srbmatrix& srbmatrix::resize_lu (int nNewKL, int nNewKU)
throw (cvmexception);
```

changes a number of sub- and super-diagonals of a calling band matrix to `nNewKL` by `nNewKU` respectively and returns a reference to the matrix changed. In case of increasing of the numbers, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative number passed or memory allocation failure. See also `srbmatrix::resize`, `srbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix m(a,4,1,0);
    std::cout << m << std::endl;
    m.resize_lu (0,1);
    m.diag(1).set(9.);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7
```

```
1 9 0 0
0 3 9 0
0 0 5 9
0 0 0 7
```

2.11.17 operator ==

Operator

```
bool srbmatrix::operator == (const srbmatrix& m) const;
```

compares a calling band matrix with a band matrix `m` and returns `true` if they have the same sizes, the same numbers of sub- and super-diagonals and their appropriate elements differ by not more than the **smallest normalized positive number**. Returns `false` otherwise. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
srbmatrix m1(a,2,1,0);
srbmatrix m2(2,1,0);

std::cout << m1 << std::endl;

m2(1,1) = 1.;
m2(2,1) = 2.; m2(2,2) = 3.;

std::cout << (m1 == m2) << std::endl;

prints

1 0
2 3

1
```

2.11.18 operator !=

Operator

```
bool srbmatrix::operator != (const srbmatrix& m) const;
```

compares a calling band matrix with a band matrix `m` and returns `true` if they have different sizes, different numbers of sub- or super-diagonals or at least one of their appropriate elements differs by more than the **smallest normalized positive number**. Returns `false` otherwise. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
srbmatrix m1(a,2,1,0);
srbmatrix m2(2,1,0);

std::cout << m1 << std::endl;

m2(1,1) = 1.;
m2(2,1) = 2.; m2(2,2) = 3.;

std::cout << (m1 != m2) << std::endl;

prints

1 0
2 3

0
```

2.11.19 operator <<

Operator

```
srbmatrix& srbmatrix::operator << (const srbmatrix& m)
throw (cvmexception);
```

destroys a calling band matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    srbmatrix m(3,1,0);
    srmatrix mc(1);
    m(2,1) = 1.;
    m(2,2) = 2.;
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
0 0 0
1 2 0
0 0 0
```

```
0
```

```
0 0 0
1 2 0
0 0 0
```

2.11.20 operator +

Operator

```
srbmatrix srbmatrix::operator + (const srbmatrix& m) const
throw (cvmexception);
```

creates an object of type `srbmatrix` as a sum of a calling band matrix and a band matrix `m`. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::sum`, `srbmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (1);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    srbmatrix m1(a,4,1,0);
    srbmatrix m2(b,4,1,0);

    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1.1e+01 0.0e+00 0.0e+00 0.0e+00
2.2e+01 3.3e+01 0.0e+00 0.0e+00
0.0e+00 4.4e+01 5.5e+01 0.0e+00
0.0e+00 0.0e+00 6.6e+01 7.7e+01

2.0e+00 0.0e+00 0.0e+00 0.0e+00
4.0e+00 6.0e+00 0.0e+00 0.0e+00
0.0e+00 8.0e+00 1.0e+01 0.0e+00
0.0e+00 0.0e+00 1.2e+01 1.4e+01
```

2.11.21 operator -

Operator

```
srbmatrix srbmatrix::operator - (const srbmatrix& m) const
throw (cvmexception);
```

creates an object of type `srbmatrix` as a difference of a calling band matrix and a band matrix `m`. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::diff`, `srbmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (1);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    srbmatrix m1(a,4,1,0);
    srbmatrix m2(b,4,1,0);

    std::cout << m2 - m1 << std::endl << m1 - m1;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
9.0e+00 0.0e+00 0.0e+00 0.0e+00
1.8e+01 2.7e+01 0.0e+00 0.0e+00
0.0e+00 3.6e+01 4.5e+01 0.0e+00
0.0e+00 0.0e+00 5.4e+01 6.3e+01

0.0e+00 0.0e+00 0.0e+00 0.0e+00
0.0e+00 0.0e+00 0.0e+00 0.0e+00
0.0e+00 0.0e+00 0.0e+00 0.0e+00
0.0e+00 0.0e+00 0.0e+00 0.0e+00
```

2.11.22 sum

Function

```
srbmatrix& srbmatrix::sum (const srbmatrix& m1, const srbmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of band matrices m1 and m2 to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator +`, `srbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const srbmatrix m1(a,3,1,0);
    srbmatrix m2(3,1,0);
    srbmatrix m(3,1,0);
    m2.set(1.);
    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0
2 3 0
0 4 5
```

```
1 0 0
1 1 0
0 1 1
```

```
2 0 0
3 4 0
0 5 6
```

```
3 0 0
4 5 0
0 6 7
```

2.11.23 diff

Function

```
srbmatrix& srbmatrix::diff (const srbmatrix& m1, const srbmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of band matrices m1 and m2 to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator -`, `srbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const srbmatrix m1(a,3,1,0);
    srbmatrix m2(3,1,0);
    srbmatrix m(3,1,0);
    m2.set(1.);
    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0
2 3 0
0 4 5
```

```
1 0 0
1 1 0
0 1 1
```

```
0 0 0
1 2 0
0 3 4
```

```
-1 0 0
0 1 0
0 2 3
```

2.11.24 operator +=

Operator

```
srbmatrix& srbmatrix::operator += (const srbmatrix& m)
throw (cvmexception);
```

adds a band matrix `m` to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator +`, `srbmatrix::sum`, `srbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    srbmatrix m1(4,0,1);
    srbmatrix m2(4,0,1);
    m1.set(1.);
    m2.set(2.);

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
3 3 0 0
0 3 3 0
0 0 3 3
0 0 0 3
```

```
4 4 0 0
0 4 4 0
0 0 4 4
0 0 0 4
```

2.11.25 operator -=

Operator

```
srbmatrix& srbmatrix::operator -= (const srbmatrix& m)
throw (cvmexception);
```

subtracts a band matrix `m` from a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator -`, `srbmatrix::diff`, `srbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    srbmatrix m1(4,0,1);
    srbmatrix m2(4,0,1);
    m1.set(1.);
    m2.set(4.);

    m2 -= m1;
    std::cout << m2 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
3 3 0 0
0 3 3 0
0 0 3 3
0 0 0 3

0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0
```

2.11.26 operator - ()

Operator

```
srbmatrix srbmatrix::operator - () const throw (cvmexception);
```

creates an object of type `srbmatrix` as a calling band matrix multiplied by -1 . See also [srbmatrix](#). Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific |  
                std::ios::left |  
                std::ios::showpos);
```

```
std::cout.precision (1);
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
```

```
srbmatrix m(a,4,1,0);
```

```
std::cout << -m;
```

prints

```
-1.0e+00 +0.0e+00 +0.0e+00 +0.0e+00  
-2.0e+00 -3.0e+00 +0.0e+00 +0.0e+00  
+0.0e+00 -4.0e+00 -5.0e+00 +0.0e+00  
+0.0e+00 +0.0e+00 -6.0e+00 -7.0e+00
```

2.11.27 operator ++

Operator

```
srbmatrix& srbmatrix::operator ++ ();  
srbmatrix& srbmatrix::operator ++ (int);
```

adds identity matrix to a calling band matrix and returns a reference to the matrix changed.
See also [srbmatrix](#). Example:

```
using namespace cvm;  
  
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m (a,4,1,0);
```

```
m++;  
std::cout << m << std::endl;  
std::cout << ++m;
```

prints

```
2 0 0 0  
2 4 0 0  
0 4 6 0  
0 0 6 8
```

```
3 0 0 0  
2 5 0 0  
0 4 7 0  
0 0 6 9
```

2.11.28 operator --

Operator

```
srbmatrix& srbmatrix::operator -- ();  
srbmatrix& srbmatrix::operator -- (int);
```

subtracts identity matrix from a calling band matrix and returns a reference to the matrix changed. See also [srbmatrix](#). Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m (a,4,1,0);
```

```
m--;  
std::cout << m << std::endl;  
std::cout << --m;
```

prints

```
0 0 0 0  
2 2 0 0  
0 4 4 0  
0 0 6 6
```

```
-1 0 0 0  
2 1 0 0  
0 4 3 0  
0 0 6 5
```

2.11.29 operator * (TR)

Operator

```
srbmatrix srbmatrix::operator * (TR d) const;
```

creates an object of type `srbmatrix` as a product of a calling band matrix and a number `d`.
See also `srbmatrix::operator *=` , `srbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m (a,4,1,0);  
std::cout << m * 5. << std::endl;
```

prints

```
5 0 0 0  
10 15 0 0  
0 20 25 0  
0 0 30 35
```

2.11.30 operator / (TR)

Operator

```
srbmatrix srbmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `srbmatrix` as a quotient of a calling band matrix and a number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the **smallest normalized positive number**. See also `srbmatrix::operator /=` , `srbmatrix`.
Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m (a,4,1,0);  
std::cout << m / 2. << std::endl;
```

prints

```
0.5 0 0 0  
1 1.5 0 0  
0 2 2.5 0  
0 0 3 3.5
```

2.11.31 operator *= (TR)

Operator

```
srbmatrix& srbmatrix::operator *= (TR d);
```

multiplies a calling band matrix by a number d and returns a reference to the matrix changed. See also `srbmatrix::operator *`, `srbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m (a,4,1,0);  
m *= 2.;  
std::cout << m;
```

prints

```
2 0 0 0  
4 6 0 0  
0 8 10 0  
0 0 12 14
```

2.11.32 operator /= (TR)

Operator

```
srbmatrix& srbmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling matrix by a number d and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if d has an absolute value equal or less than the `smallest normalized positive number`. See also `srbmatrix::operator /`, `srbmatrix`.
Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m (a,4,1,0);  
m /= 2.;  
std::cout << m;
```

prints

```
0.5 0 0 0  
1 1.5 0 0  
0 2 2.5 0  
0 0 3 3.5
```

2.11.33 normalize

Function

```
srbmatrix& srbmatrix::normalize ();
```

normalizes a calling band matrix so its **Euclidean norm** becomes equal to 1 if it was greater than the **smallest normalized positive number** before the call (otherwise the function does nothing). See also **srbmatrix**. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (3);  
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
srbmatrix m (a,4,1,0);
```

```
m.normalize();  
std::cout << m << m.norm() << std::endl;
```

prints

```
8.452e-02 0.000e+00 0.000e+00 0.000e+00  
1.690e-01 2.535e-01 0.000e+00 0.000e+00  
0.000e+00 3.381e-01 4.226e-01 0.000e+00  
0.000e+00 0.000e+00 5.071e-01 5.916e-01  
1.000e+00
```

2.11.34 transposition

Operator and functions

```
srbmatrix srbmatrix::operator ~ () const throw (cvmexception);
srbmatrix& srbmatrix::transpose (const srbmatrix& m) throw (cvmexception);
srbmatrix& srbmatrix::transpose () throw (cvmexception);
```

encapsulate band matrix transposition. First operator creates an object of type `srbmatrix` as a transposed calling band matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling band matrix to be equal to a band matrix `m` transposed (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands or in case of memory allocation failure), third one makes it to be equal to transposed itself. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
srbmatrix mt(4,0,1);
std::cout << ~m << std::endl ;
mt.transpose(m);
std::cout << mt << std::endl;
mt.transpose();
std::cout << mt;
```

prints

```
1 2 0 0
0 3 4 0
0 0 5 6
0 0 0 7
```

```
1 2 0 0
0 3 4 0
0 0 5 6
0 0 0 7
```

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7
```

2.11.35 operator * (const rvector&)

Operator

```
rvector srbmatrix::operator * (const rvector& v) const  
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling band matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `rvector::mult` in order to get rid of a new object creation. See also `srbmatrix` and `rvector`. Example:

```
using namespace cvm;
```

```
try {  
    srbmatrix m (4,1,0);  
    rvector v(4);  
    m.set(1.);  
    v.set(1.);  
  
    std::cout << m * v;  
}  
catch (exception& e) {  
    std::cout << "Exception: " << e.what () << std::endl;  
}
```

prints

```
1 2 2 2
```

2.11.36 operator * (const rmatrix&)

Operator

```
rmatrix srbmatrix::operator * (const rmatrix& m) const  
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling band matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `rmatrix::mult` in order to get rid of a new object creation. See also `rmatrix` and `srbmatrix`. Example:

```
using namespace cvm;
```

```
try {  
    srbmatrix mb (4,1,0);  
    rmatrix m(4,2);  
    mb.set(1.);  
    m.set(1.);  
  
    std::cout << mb * m;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
1 1  
2 2  
2 2  
2 2
```

2.11.37 operator * (const srmatrix&)

Operator

```
srmatrix srbmatrix::operator * (const srmatrix& m) const  
throw (cvmexception);
```

creates an object of type `srmatrix` as a product of a calling band matrix and a matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `srmatrix::mult` in order to get rid of a new object creation. See also `srmatrix` and `srbmatrix`. Example:

```
using namespace cvm;
```

```
try {  
    srbmatrix mb(4,1,0);  
    srmatrix ms(4);  
    mb.set(1.);  
    ms.set(1.);  
  
    std::cout << mb * ms;  
}  
catch (exception& e) {  
    std::cout << "Exception " << e.what () << std::endl;  
}
```

prints

```
1 1 1 1  
2 2 2 2  
2 2 2 2  
2 2 2 2
```

2.11.38 operator * (const srbmatrix&)

Operator

```
srbmatrix srbmatrix::operator * (const srbmatrix& m) const
throw (cvmexception);
```

creates an object of type `srbmatrix` as a product of a calling band matrix and band matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `rmatrix::mult` in order to get rid of a new object creation. See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    srbmatrix m1(7,1,0);
    srbmatrix m2(7,1,1);
    m1.set(1.);
    m2.set(1.);

    std::cout << m1 * m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 1 0 0 0 0 0
2 2 1 0 0 0 0
1 2 2 1 0 0 0
0 1 2 2 1 0 0
0 0 1 2 2 1 0
0 0 0 1 2 2 1
0 0 0 0 1 2 2
```

2.11.39 low_up

Functions

```
srbmatrix&
srbmatrix::low_up (const srbmatrix& m, int* nPivots) throw (cvmexception);
srbmatrix
srbmatrix::low_up (int* nPivots) const throw (cvmexception);
```

compute the LU factorization of a calling band matrix as

$$A = PLU$$

where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and U is an upper triangular matrix. All the functions store the result as the matrix L without main diagonal combined with U. All the functions return pivot indices as an array of integers (it should support at least `m.size()` elements) pointed to by `nPivots` so *i*-th row was interchanged with `nPivots[i]`-th row. The first version sets a calling matrix to be equal to the *m*'s LU factorization and the second one creates an object of type `srbmatrix` as the calling band matrix's LU factorization. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be factorized is close to singular. The first version also changes numbers of super-diagonals to be equal to $k_l + k_u$ in order to keep the result of factorization. It is recommended to use `iarray` for pivot values. This function is provided mostly for solving multiple systems of linear equations using `srbmatrix::solve_lu` function, See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix ma(a,4,1,0);
    srbmatrix mLU(4,1,0);
    rmatrix mb1(4,2); rvector vb1(4);
    rmatrix mb2(4,2); rvector vb2(4);
    rmatrix mx1(4,2); rvector vx1(4);
    rmatrix mx2(4,2); rvector vx2(4);
    iarray nPivots(4);
    double dErr = 0.;
    mb1.randomize(-1.,3.); vb1.randomize(-2.,4.);
    mb2.randomize(-2.,5.); vb2.randomize(-3.,1.);

    mLU.low_up(ma, nPivots);
    mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
```

```

std::cout << mx1 << dErr << std::endl << std::endl;
mx2 = ma.solve_lu (mLU, nPivots, mb2);
std::cout << mx2 << std::endl;;
std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;

vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
std::cout << vx1 << dErr << std::endl;
vx2 = ma.solve_lu (mLU, nPivots, vb2);
std::cout << vx2 << std::endl;;
std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
prints
7.6327e-02 -4.7386e-01
-2.9523e-01 9.7577e-01
1.7288e-01 -3.5093e-01
1.0595e-01 4.7363e-01
1.1832e-15

3.1963e+00 4.8622e+00
-4.9904e-01 -2.6575e+00
8.2183e-01 2.3294e+00
-6.1693e-01 -1.8015e+00

0.0000e+00 0.0000e+00
0.0000e+00 -2.2204e-16
0.0000e+00 -4.4409e-16
0.0000e+00 0.0000e+00

0.0000e+00 0.0000e+00
8.8818e-16 0.0000e+00
0.0000e+00 -4.4409e-16
-4.4409e-16 4.4409e-16
7.8933e-01 7.0543e-01 -1.6338e-02 -2.6206e-01
1.4832e-15
-1.5505e+00 5.8987e-01 -8.4977e-01 7.3059e-01

0.0000e+00 0.0000e+00 0.0000e+00 0.0000e+00

-2.2204e-16 0.0000e+00 0.0000e+00 4.4409e-16

```

2.11.40 identity

Function

```
srbmatrix& srbmatrix::identity();
```

sets a calling band matrix to be equal to identity matrix and returns a reference to the matrix changed. The function doesn't change numbers of sub- and super-diagonals. See also [srbmatrix](#). Example:

```
using namespace cvm;
```

```
srbmatrix m(4);  
m.randomize(0.,1.);  
std::cout << m << std::endl;  
std::cout << m.identity();
```

prints

```
0.327372 0 0 0  
0 0.955718 0 0  
0 0 0.0960723 0  
0 0 0 0.291818
```

```
1 0 0 0  
0 1 0 0  
0 0 1 0  
0 0 0 1
```

2.11.41 vanish

Function

```
srbmatrix& srbmatrix::vanish();
```

sets every element of a calling band matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `srbmatrix::set(TR)` with zero operand passed. See also `srbmatrix`. Example:

```
using namespace cvm;
```

```
srbmatrix m(4,1,0);  
m.randomize(0.,1.);  
std::cout << m << std::endl;  
std::cout << m.vanish();
```

prints

```
0.337138 0 0 0  
0.101199 0.522843 0 0  
0 0.258522 0.123447 0  
0 0 0.591723 0.661489
```

```
0 0 0 0  
0 0 0 0  
0 0 0 0  
0 0 0 0
```

2.11.42 `randomize`

Function

```
srbmatrix& srbmatrix::randomize (TR dFrom, TR dTo);
```

fills a calling band matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. See also [srbmatrix](#). Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::showpos);  
std::cout.precision (7);  
srbmatrix m(4,1,0);  
m.randomize(0.,1.);  
std::cout << m << std::endl;
```

prints

```
+3.4281442e-01 +0.0000000e+00 +0.0000000e+00 +0.0000000e+00  
+7.9808954e-01 +5.9761345e-01 +0.0000000e+00 +0.0000000e+00  
+0.0000000e+00 +1.1670278e-01 +6.5645314e-02 +0.0000000e+00  
+0.0000000e+00 +0.0000000e+00 +4.2225410e-01 +7.5563829e-02
```

2.12 scbmatrix

This is end-user class encapsulating a square band matrix in Euclidean space of complex numbers. This class utilizes [band storage](#) for its elements.

```
template <typename TR, typename TC>
class scbmatrix : public scmatrix <TR,TC>, public BandMatrix <TR,TC> {
public:
    scbmatrix ();
    explicit scbmatrix (int nMN);
    scbmatrix (int nMN);
    scbmatrix (TC* pD, int nMN, int nKL, int nKU);
    scbmatrix (const TC* pD, int nMN, int nKL, int nKU);
    scbmatrix (const scbmatrix& m);
    scbmatrix (const cmatrix& m, int nKL, int nKU);
    explicit scbmatrix (const cvector& v);
    explicit scbmatrix (const srbmatrix& m, bool bRealPart = true);
    scbmatrix (const srbmatrix& mRe, const srbmatrix& mIm);
    TC& operator () (int im, int in) throw (cvmexception);
    TC operator () (int im, int in) const throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    const srbmatrix real () const;
    const srbmatrix imag () const;
    scbmatrix& operator = (const scbmatrix& m) throw (cvmexception);
    scbmatrix& assign (const cvector& v);
    scbmatrix& assign (const TC* pD);
    scbmatrix& set (TC z);
    scbmatrix& assign_real (const srbmatrix& mRe) throw (cvmexception);
    scbmatrix& assign_imag (const srbmatrix& mIm) throw (cvmexception);
    scbmatrix& set_real (TR d);
    scbmatrix& set_imag (TR d);
    scbmatrix& resize (int nNewMN) throw (cvmexception);
    scbmatrix& resize_lu (int nNewKL, int nNewKU) throw (cvmexception);
    bool operator == (const scbmatrix& v) const;
    bool operator != (const scbmatrix& v) const;
    scbmatrix& operator << (const scbmatrix& m) throw (cvmexception);
    scbmatrix operator + (const scbmatrix& m) const
        throw (cvmexception);
    scbmatrix operator - (const scbmatrix& m) const
        throw (cvmexception);
    scbmatrix& sum (const scbmatrix& m1,
        const scbmatrix& m2) throw (cvmexception);
```

```

scbmatrix& diff (const scbmatrix& m1,
                const scbmatrix& m2) throw (cvmexception);
scbmatrix& operator += (const scbmatrix& m) throw (cvmexception);
scbmatrix& operator -= (const scbmatrix& m) throw (cvmexception);
scbmatrix operator - () const;
scbmatrix& operator ++ ();
scbmatrix& operator ++ (int);
scbmatrix& operator -- ();
scbmatrix& operator -- (int);
scbmatrix operator * (TR d) const;
scbmatrix operator / (TR d) const throw (cvmexception);
scbmatrix operator * (TC z) const;
scbmatrix operator / (TC z) const throw (cvmexception);
scbmatrix& operator *= (TR d);
scbmatrix& operator /= (TR d) throw (cvmexception);
scbmatrix& operator *= (TC z);
scbmatrix& operator /= (TC z) throw (cvmexception);
scbmatrix& normalize ();
scbmatrix operator ~ () const;
scbmatrix& conj (const scbmatrix& m) throw (cvmexception);
scbmatrix& conj ();
cvector operator * (const cvector& v) const throw (cvmexception);
cmatrix operator * (const cmatrix& m) const throw (cvmexception);
scmatrix operator * (const scmatrix& m) const throw (cvmexception);
scbmatrix operator * (const scbmatrix& m) const throw (cvmexception);
scbmatrix& low_up (const scbmatrix& m,
                 int* nPivots) throw (cvmexception);
scbmatrix low_up (int* nPivots) const throw (cvmexception);
scbmatrix& identity ();
scbmatrix& vanish ();
scbmatrix& randomize_real (TR dFrom, TR dTo);
scbmatrix& randomize_imag (TR dFrom, TR dTo);
};

```

2.12.1 scbmatrix ()

Constructor

```
scbmatrix::scbmatrix ();
```

creates an empty scbmatrix object. See also [scbmatrix](#). Example:

```
using namespace cvm;
```

```
scbmatrix m;  
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;  
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;  
m.resize(3);  
m.resize_lu(1,0);  
m.set(std::complex<double>(1.,2.));  
std::cout << m;
```

prints

```
0 0 0 0 0  
(1,2) (0,0) (0,0)  
(1,2) (1,2) (0,0)  
(0,0) (1,2) (1,2)
```

2.12.2 scbmatrix (int)

Constructor

```
explicit scbmatrix::scbmatrix (int nMN);
```

creates an $n \times n$ `scbmatrix` object where n is passed in `nMN` parameter. The matrix created is diagonal, i.e. $k_l = k_u = 0$. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `scbmatrix`.
Example:

```
using namespace cvm;
```

```
scbmatrix m(4);
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
m.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
4 4 4 0 0
(1,2) (0,0) (0,0) (0,0)
(0,0) (1,2) (0,0) (0,0)
(0,0) (0,0) (1,2) (0,0)
(0,0) (0,0) (0,0) (1,2)
```

2.12.3 scbmatrix (int,int,int)

Constructor

```
scbmatrix::scbmatrix (int nMN, int nKL, int nKU);
```

creates an $n \times n$ `scbmatrix` object where n is passed in `nMN` parameter. The matrix created has `nKL` sub-diagonals and `nKU` super-diagonals. The constructor throws an exception of type `cvmexception` in case of non-positive size or negative number of sub-diagonals or super-diagonals passed or in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;
```

```
scbmatrix m(4,1,1);
m.set(std::complex<double>(1.,2.));
std::cout << m << std::endl
          << m.msize() << " " << m.nsize() << " " << m.size()
          << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
(1,2) (1,2) (0,0) (0,0)
(1,2) (1,2) (1,2) (0,0)
(0,0) (1,2) (1,2) (1,2)
(0,0) (0,0) (1,2) (1,2)
```

```
4 4 12 1 1
```

2.12.4 scbmatrix (TC*,int,int,int)

Constructor

```
scbmatrix::scbmatrix (TC* pD, int nMN, int nKL, int nKU);
```

creates an $n \times n$ `scbmatrix` object where n is passed in `nMN` parameter. The matrix created has $k_l=nKL$ sub-diagonals and $k_u=nKU$ super-diagonals. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD`. Please note that this array must contain at least $(k_l + k_u + 1) * n$ elements. The constructor throws an exception of type `cvmexception` in case of non-positive size or negative number of sub- or super-diagonals passed. See also `scbmatrix`, `scbmatrix (const TC*,int,int,int)`. Example:

```
using namespace cvm;
double a[] = {1., 1., 1., 1., 1., 1., 1., 1.,
              1., 1., 1., 1., 1., 1., 1., 1.};
scbmatrix m1 ((std::complex<double>*)a,4,1,0);
scbmatrix mu ((std::complex<double>*)a,4,0,1);
m1(2,1) = std::complex<double>(5.,5.);
std::cout << m1 << std::endl << mu << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
           << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(1,1) (0,0) (0,0) (0,0)
(5,5) (1,1) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0)
(0,0) (0,0) (1,1) (1,1)
```

```
(5,5) (1,1) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0)
(0,0) (0,0) (1,1) (1,1)
(0,0) (0,0) (0,0) (1,1)
```

```
1 1 5 5 1 1
```

2.12.5 scbmatrix (const TC*,int,int,int)

Constructor

```
scbmatrix::scbmatrix (const TC* pD, int nMN, int nKL, int nKU);
```

creates an $n \times n$ scbmatrix object where n is passed in nMN parameter. The matrix created has $k_l=nKL$ sub-diagonals and $k_u=nKU$ super-diagonals. Then constructor copies $(k_l + k_u + 1) * n$ elements of an array pD to the matrix according to **band storage**. The constructor throws an exception of type **cvmexception** in case of non-positive size or negative number of sub- or super-diagonals passed. See also **scbmatrix**, **scbmatrix (TC*,int,int,int)**. Example:

```
using namespace cvm;
const double a[] = {1., 1., 1., 1., 1., 1., 1., 1.,
                   1., 1., 1., 1., 1., 1., 1., 1.};
scbmatrix ml ((const std::complex<double>*)a,4,1,0);
scbmatrix mu ((const std::complex<double>*)a,4,0,1);
ml(2,1) = std::complex<double>(5.,5.);
std::cout << ml << std::endl << mu << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(1,1) (0,0) (0,0) (0,0)
(5,5) (1,1) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0)
(0,0) (0,0) (1,1) (1,1)
```

```
(1,1) (1,1) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0)
(0,0) (0,0) (1,1) (1,1)
(0,0) (0,0) (0,0) (1,1)
```

```
1 1 1 1 1 1
```

2.12.6 scbmatrix (const scbmatrix&)

Copy constructor

```
scbmatrix::scbmatrix (const scbmatrix& m);
```

creates a `scbmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
scbmatrix m ((std::complex<double>*)a,4,1,0);
scbmatrix mc(m);
m(1,1) = 7.77;
std::cout << m << std::endl << mc;
```

prints

```
(7.77,0) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)

(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)
```

2.12.7 scbmatrix (const cmatrix&,int,int)

Constructor

```
scbmatrix::scbmatrix (const cmatrix& m, int nKL, int nKU);
```

creates a scbmatrix object as a copy of “sliced” matrix *m*, i.e. it copies main diagonal, *nKL* sub-diagonals and *nKU* super-diagonals of the matrix *m*. It’s assumed that $m \times n$ matrix *m* must have equal sizes, i.e. $m = n$ is satisfied. The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*)a,3);
scbmatrix mb(m,1,0);
std::cout << m << std::endl << mb;
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (0,0) (0,0)
(3,4) (9,10) (0,0)
(0,0) (11,12) (17,18)
```

2.12.8 `scbmatrix (const cvector&)`

Constructor

```
explicit scbmatrix::scbmatrix (const cvector& v);
```

creates a `scbmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scbmatrix`, `cvector`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6.};  
cvector v((std::complex<double>*)a,3);  
scbmatrix m(v);  
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0)  
(0,0) (3,4) (0,0)  
(0,0) (0,0) (5,6)
```

2.12.9 scbmatrix (const srbmatrix&,bool)

Constructor

```
explicit scbmatrix::scbmatrix (const srbmatrix& m, bool bRealPart = true);
```

creates a scbmatrix object having the same dimension and the same numbers of sub- and super-diagonals as real matrix m and copies the matrix m to its real part if bRealPart is true or to its imaginary part otherwise. See also [scbmatrix](#), [srbmatrix](#). Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};  
const srbmatrix m(a,4,1,0);  
scbmatrix mr(m), mi(m, false);  
std::cout << mr << std::endl << mi;
```

prints

```
(1,0) (0,0) (0,0) (0,0)  
(2,0) (3,0) (0,0) (0,0)  
(0,0) (4,0) (5,0) (0,0)  
(0,0) (0,0) (6,0) (7,0)
```

```
(0,1) (0,0) (0,0) (0,0)  
(0,2) (0,3) (0,0) (0,0)  
(0,0) (0,4) (0,5) (0,0)  
(0,0) (0,0) (0,6) (0,7)
```

2.12.10 scbmatrix (const srbmatrix&, const srbmatrix&)

Constructor

```
scbmatrix::scbmatrix (const srbmatrix& mRe, const srbmatrix& mIm);
```

creates a scbmatrix object of the same size as mRe and mIm has (the constructor throws an exception of type `cvmexception` if mRe and mIm have different sizes or different numbers of sub- or super-diagonals) and copies matrices mRe and mIm to a real and imaginary part of the matrix created respectively. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scbmatrix`, `srbmatrix`. Example:

```
using namespace cvm;
```

```
srbmatrix mr(4,1,0), mi(4,1,0);  
mr.set(1.);  
mi.set(2.);  
const scbmatrix m(mr,mi);  
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0) (0,0)  
(1,2) (1,2) (0,0) (0,0)  
(0,0) (1,2) (1,2) (0,0)  
(0,0) (0,0) (1,2) (1,2)
```

2.12.11 operator (,)

Indexing operators

```
TC& scbmatrix::operator () (int im, int in) throw (cvmexception);
TC scbmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a band matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Only elements located on main diagonal or on non-zero sub- or super-diagonals are l-values. All other values located outside this area are not writable. Both operators are *l-based*. The operators throw an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range or in case of attempt to write to a non-writable element¹⁰. See also `scbmatrix`, `BandMatrix::lsize()` and `BandMatrix::usize()`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    srbmatrix m (a,3,1,0);

    m(2,1) = 7.77;
    std::cout << m << std::endl;
    std::cout << m(3,2) << " " << m(1,3) << std::endl;

    m(1,3) = 7.77;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}

prints

1.00e+00 0.00e+00 0.00e+00
7.77e+00 3.00e+00 0.00e+00
0.00e+00 4.00e+00 5.00e+00

4.00e+00 0.00e+00
Exception: Attempt to change a read-only element
```

¹⁰Here I use `type_proxy<T>` class originally described in [4], p. 217.

2.12.12 operator ()

Indexing operator

```
const cvector scbmatrix::operator () (int i) const throw (cvmexception);
```

provides access to an *i*-th column of a band matrix. Unlike `scmatrix::operator ()`, this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is *l-based*. The operator throws an exception of type `cvmexception` if the parameter *i* is outside of `[1, nsize()]` range. See also `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};  
scbmatrix m ((std::complex<double>*)a,3,1,0);  
std::cout << m << std::endl;  
std::cout << m(2);
```

prints

```
(1,2) (0,0) (0,0)  
(3,4) (5,6) (0,0)  
(0,0) (7,8) (9,10)  
  
(0,0) (5,6) (7,8)
```

2.12.13 operator []

Indexing operator

```
const cvector scbmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to an *i*-th row of a band matrix. Unlike `scmatrix::operator []`, this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is *l-based*. The operator throws an exception of type `cvmexception` if the parameter *i* is outside of `[1, nsize()]` range. See also `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};  
scbmatrix m ((std::complex<double>*)a,3,1,0);  
std::cout << m << std::endl;  
std::cout << m[3];
```

prints

```
(1,2) (0,0) (0,0)  
(3,4) (5,6) (0,0)  
(0,0) (7,8) (9,10)  
  
(0,0) (7,8) (9,10)
```

2.12.14 `real`

Function

```
const srbmatrix scbmatrix::real () const;
```

creates an object of type `const srbmatrix` as a real part of a calling band matrix. Please note that, unlike `cvector::real`, this function creates new object *not sharing* a memory with a real part of the calling matrix, i.e. the matrix returned is *not an l-value*. See also `srbmatrix`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};  
scbmatrix m ((std::complex<double>*)a,3,1,0);  
std::cout << m << std::endl;  
std::cout << m.real();
```

prints

```
(1,2) (0,0) (0,0)  
(3,4) (5,6) (0,0)  
(0,0) (7,8) (9,10)
```

```
1 0 0  
3 5 0  
0 7 9
```

2.12.15 `imag`

Function

```
const srbmatrix scbmatrix::imag () const;
```

creates an object of type `const srbmatrix` as an imaginary part of a calling band matrix. Please note that, unlike `cvector::imag`, this function creates new object *not sharing* a memory with an imaginary part of the calling matrix, i.e. the matrix returned is *not an l-value*. See also `srbmatrix`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};  
scbmatrix m ((std::complex<double>*)a,3,1,0);  
std::cout << m << std::endl;  
std::cout << m.imag();
```

prints

```
(1,2) (0,0) (0,0)  
(3,4) (5,6) (0,0)  
(0,0) (7,8) (9,10)
```

```
2 0 0  
4 6 0  
0 8 10
```

2.12.16 operator = (const scbmatrix&)

Operator

```
scbmatrix& scbmatrix::operator = (const scbmatrix& m)
throw (cvmexception);
```

sets an every element of a calling band matrix to a value of appropriate element of a band matrix *m* and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different matrix sizes or in case of different numbers of sub- or super-diagonals. See also `scbmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    scbmatrix m1((std::complex<double>*)a,3,1,0);
    scbmatrix m2(3,1,0);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1.00e+00,2.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
(3.00e+00,4.00e+00) (5.00e+00,6.00e+00) (0.00e+00,0.00e+00)
(0.00e+00,0.00e+00) (7.00e+00,8.00e+00) (9.00e+00,1.00e+01)
```

2.12.17 assign (const TC*)

Function

```
scbmatrix& scbmatrix::assign (const cvector& v);  
scbmatrix& scbmatrix::assign (const TC* pD);
```

sets every element of a calling band matrix to a value of appropriate element of a vector *v* or an array pointed to by *pD* and returns a reference to the matrix changed. Please note that this array must contain at least $(k_l + k_u + 1)n$ elements. See also [scbmatrix](#). Example:

```
using namespace cvm;
```

```
const double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};  
scbmatrix m(3,0,1);  
m.assign((const std::complex<double>*)a);  
std::cout << m;
```

prints

```
(3,4) (5,6) (0,0)  
(0,0) (7,8) (9,10)  
(0,0) (0,0) (11,12)
```

2.12.18 set (TC)

Function

```
scbmatrix& scbmatrix::set (TC z);
```

sets every element of a calling band matrix to a value of parameter *z* and returns a reference to the matrix changed. Use **vanish** to set every element of a calling matrix to be equal to zero. See also **scbmatrix**. Example:

```
using namespace cvm;
```

```
scbmatrix m(4,1,0);  
m.set(std::complex<double>(1.,2.));  
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0) (0,0)  
(1,2) (1,2) (0,0) (0,0)  
(0,0) (1,2) (1,2) (0,0)  
(0,0) (0,0) (1,2) (1,2)
```

2.12.19 assign_real

Function

```
scbmatrix& scbmatrix::assign_real (const srbmatrix& mRe)  
throw (cvmexception);
```

sets real part of every element of a calling band matrix to a value of appropriate element of a band matrix `mRe` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. See also `scbmatrix` and `srbmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
srbmatrix m (3,0,1);  
scbmatrix mc(3,0,1);  
m.randomize (0., 1.);
```

```
mc.assign_real(m);  
std::cout << mc;
```

prints

```
(5.44e-01,0.00e+00) (5.48e-02,0.00e+00) (0.00e+00,0.00e+00)  
(0.00e+00,0.00e+00) (3.66e-01,0.00e+00) (3.49e-01,0.00e+00)  
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (8.00e-01,0.00e+00)
```

2.12.20 assign_imag

Function

```
scbmatrix& scbmatrix::assign_imag (const srbmatrix& mIm)  
throw (cvmexception);
```

sets imaginary part of every element of a calling band matrix to a value of appropriate element of a bandmatrix `mIm` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. See also `scbmatrix` and `srbmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
srbmatrix m (3,0,1);  
scbmatrix mc(3,0,1);  
m.randomize (0., 1.);
```

```
mc.assign_imag(m);  
std::cout << mc;
```

prints

```
(0.00e+00,5.53e-01) (0.00e+00,2.16e-01) (0.00e+00,0.00e+00)  
(0.00e+00,0.00e+00) (0.00e+00,1.57e-01) (0.00e+00,1.12e-01)  
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (0.00e+00,7.03e-01)
```

2.12.21 set_real

Function

```
scbmatrix& scbmatrix::set_real (TR d);
```

sets real part of every element of a calling band matrix to a value of parameter *d* and returns a reference to the matrix changed. See also [scbmatrix](#). Example:

```
using namespace cvm;
```

```
scbmatrix m(4,0,1);  
m.set_real(1.);  
std::cout << m;
```

prints

```
(1,0) (1,0) (0,0) (0,0)  
(0,0) (1,0) (1,0) (0,0)  
(0,0) (0,0) (1,0) (1,0)  
(0,0) (0,0) (0,0) (1,0)
```

2.12.22 set_imag

Function

```
scbmatrix& scbmatrix::set_imag (TR d);
```

sets imaginary part of every element of a calling band matrix to a value of parameter *d* and returns a reference to the matrix changed. See also [scbmatrix](#). Example:

```
using namespace cvm;
```

```
scbmatrix m(4,0,1);  
m.set_imag(1.);  
std::cout << m;
```

prints

```
(0,1) (0,1) (0,0) (0,0)  
(0,0) (0,1) (0,1) (0,0)  
(0,0) (0,0) (0,1) (0,1)  
(0,0) (0,0) (0,0) (0,1)
```

2.12.23 resize

Function

```
scbmatrix& scbmatrix::resize (int nNewMN) throw (cvmexception);
```

changes a size of a calling band matrix to nNewMN by nNewMN and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. This function doesn't change a number of sub- or super-diagonals. Like any band matrix class member function, this function doesn't change **non-referred elements**. See number (11,12) appearing after resize in example below. The function throws an exception of type **cvmexception** in case of non-positive size passed or memory allocation failure. See also **scbmatrix.resize_lu**, **scbmatrix**. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    scbmatrix m((std::complex<double>*)a,3,1,0);
    std::cout << m << std::endl;
    m.resize (4);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}

prints

(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (0,0)
```

2.12.24 resize_lu

Function

```
scbmatrix& scbmatrix::resize_lu (int nNewKL, int nNewKU)
throw (cvmexception);
```

changes a number of sub- and super-diagonals of a calling band matrix to nNewKL by nNewKU respectively and returns a reference to the matrix changed. In case of increasing of the numbers, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative number passed or memory allocation failure. See also `scbmatrix::resize`, `scbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    scbmatrix m((std::complex<double>*)a,3,1,0);
    std::cout << m << std::endl;
    m.resize_lu (0,1);
    m.diag(1).set(std::complex<double>(9.,9.));
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}

prints

(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,2) (9,9) (0,0)
(0,0) (5,6) (9,9)
(0,0) (0,0) (9,10)
```

2.12.25 operator ==

Operator

```
bool scbmatrix::operator == (const scbmatrix& m) const;
```

compares a calling band matrix with a band matrix `m` and returns `true` if they have the same sizes, the same numbers of sub- and super-diagonals and their appropriate elements differ by not more than the **smallest normalized positive number**. Returns `false` otherwise. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scbmatrix m1((std::complex<double>*)a,2,1,0);
scbmatrix m2(2,1,0);
std::cout << m1 << std::endl;

m2(1,1) = std::complex<double>(1.,2.);
m2(2,1) = std::complex<double>(3.,4.);
m2(2,2) = std::complex<double>(5.,6.);

std::cout << (m1 == m2) << std::endl;

prints

(1,2) (0,0)
(3,4) (5,6)
```

1

2.12.26 operator !=

Operator

```
bool scbmatrix::operator != (const scbmatrix& m) const;
```

compares a calling band matrix with a band matrix *m* and returns true if they have different sizes, different numbers of sub- or super-diagonals or at least one of their appropriate elements differs by more than the **smallest normalized positive number**. Returns false otherwise. See also [scbmatrix](#). Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scbmatrix m1((std::complex<double>*)a,2,1,0);
scbmatrix m2(2,1,0);
std::cout << m1 << std::endl;

m2(1,1) = std::complex<double>(1.,2.);
m2(2,1) = std::complex<double>(3.,4.);
m2(2,2) = std::complex<double>(5.,6.00001);

std::cout << (m1 != m2) << std::endl;

prints

(1,2) (0,0)
(3,4) (5,6)
```

1

2.12.27 operator <<

Operator

```
scbmatrix& scbmatrix::operator << (const scbmatrix& m)
throw (cvmexception);
```

destroys a calling band matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

try {
    scbmatrix m(3,1,0);
    scbmatrix mc(1);
    m(2,1) = std::complex<double>(1.,2.);
    m(2,2) = std::complex<double>(3.,4.);
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}

prints

(0,0) (0,0) (0,0)
(1,2) (3,4) (0,0)
(0,0) (0,0) (0,0)

(0,0)

(0,0) (0,0) (0,0)
(1,2) (3,4) (0,0)
(0,0) (0,0) (0,0)
```

2.12.28 operator +

Operator

```
scbmatrix scbmatrix::operator + (const scbmatrix& m) const
throw (cvmexception);
```

creates an object of type `scbmatrix` as a sum of a calling band matrix and a band matrix `m`. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::sum`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
                  9., 10., 11., 12.};
    double b[] = {10., 20., 30., 40., 50., 60.,
                  70., 80., 90., 100., 110., 120.};
    scbmatrix m1((std::complex<double>*)a,3,0,1);
    scbmatrix m2((std::complex<double>*)b,3,0,1);

    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)
(0,0) (0,0) (11,12)

(30,40) (50,60) (0,0)
(0,0) (70,80) (90,100)
(0,0) (0,0) (110,120)

(33,44) (55,66) (0,0)
(0,0) (77,88) (99,110)
(0,0) (0,0) (121,132)

(6,8) (10,12) (0,0)
(0,0) (14,16) (18,20)
(0,0) (0,0) (22,24)
```

2.12.29 operator -

Operator

```
scbmatrix scbmatrix::operator - (const scbmatrix& m) const
throw (cvmexception);
```

creates an object of type `scbmatrix` as a difference of a calling band matrix and a band matrix `m`. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::diff`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
                  9., 10., 11., 12.};
    double b[] = {10., 20., 30., 40., 50., 60.,
                  70., 80., 90., 100., 110., 120.};
    scbmatrix m1((std::complex<double>*)a,3,0,1);
    scbmatrix m2((std::complex<double>*)b,3,0,1);

    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m1 - m2 << std::endl << m1 - m1;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)
(0,0) (0,0) (11,12)

(30,40) (50,60) (0,0)
(0,0) (70,80) (90,100)
(0,0) (0,0) (110,120)

(-27,-36) (-45,-54) (0,0)
(0,0) (-63,-72) (-81,-90)
(0,0) (0,0) (-99,-108)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.12.30 sum

Function

```
scbmatrix& scbmatrix::sum (const scbmatrix& m1, const scbmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of band matrices m1 and m2 to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::operator +`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
              9., 10., 11., 12.};
const scbmatrix m1((std::complex<double>*)a,3,1,0);
scbmatrix m2(3,1,0);
scbmatrix m(3,1,0);
m2.set(std::complex<double>(1.,1.));
std::cout << m1 << std::endl << m2 << std::endl;
std::cout << m.sum(m1, m2) << std::endl;
std::cout << m.sum(m, m2);
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,1) (0,0) (0,0)
(1,1) (1,1) (0,0)
(0,0) (1,1) (1,1)

(2,3) (0,0) (0,0)
(4,5) (6,7) (0,0)
(0,0) (8,9) (10,11)

(3,4) (0,0) (0,0)
(5,6) (7,8) (0,0)
(0,0) (9,10) (11,12)
```

2.12.31 diff

Function

```
scbmatrix& scbmatrix::diff (const scbmatrix& m1, const scbmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of band matrices m1 and m2 to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::operator -`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
              9., 10., 11., 12.};
const scbmatrix m1((std::complex<double>*)a,3,1,0);
scbmatrix m2(3,1,0);
scbmatrix m(3,1,0);
m2.set(std::complex<double>(1.,1.));
std::cout << m1 << std::endl << m2 << std::endl;
std::cout << m.diff(m1, m2) << std::endl;
std::cout << m.diff(m, m2);
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)
```

```
(1,1) (0,0) (0,0)
(1,1) (1,1) (0,0)
(0,0) (1,1) (1,1)
```

```
(0,1) (0,0) (0,0)
(2,3) (4,5) (0,0)
(0,0) (6,7) (8,9)
```

```
(-1,0) (0,0) (0,0)
(1,2) (3,4) (0,0)
(0,0) (5,6) (7,8)
```

2.12.32 operator +=

Operator

```
scbmatrix& scbmatrix::operator += (const scbmatrix& m)
throw (cvmexception);
```

adds a band matrix *m* to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::operator +`, `scbmatrix::sum`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    scbmatrix m1(4,0,1);
    scbmatrix m2(4,0,1);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(4,6) (4,6) (0,0) (0,0)
(0,0) (4,6) (4,6) (0,0)
(0,0) (0,0) (4,6) (4,6)
(0,0) (0,0) (0,0) (4,6)
```

```
(6,8) (6,8) (0,0) (0,0)
(0,0) (6,8) (6,8) (0,0)
(0,0) (0,0) (6,8) (6,8)
(0,0) (0,0) (0,0) (6,8)
```

2.12.33 operator -=

Operator

```
scbmatrix& scbmatrix::operator -= (const scbmatrix& m)
throw (cvmexception);
```

subtracts a band matrix `m` from a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::operator -`, `scbmatrix::diff`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    scbmatrix m1(4,0,1);
    scbmatrix m2(4,0,1);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(-2,-2) (-2,-2) (0,0) (0,0)
(0,0) (-2,-2) (-2,-2) (0,0)
(0,0) (0,0) (-2,-2) (-2,-2)
(0,0) (0,0) (0,0) (-2,-2)

(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.12.34 operator - ()

Operator

```
scbmatrix scbmatrix::operator - () const throw (cvmexception);
```

creates an object of type `scbmatrix` as a calling band matrix multiplied by -1 . See also [scbmatrix](#). Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific |  
                std::ios::left |  
                std::ios::showpos);  
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,  
              9., 10., 11., 12.};  
scbmatrix m((std::complex<double>*)a,3,1,0);
```

```
std::cout << -m;
```

prints

```
(-1.00e+000,-2.00e+000) (+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000)  
(-3.00e+000,-4.00e+000) (-5.00e+000,-6.00e+000) (+0.00e+000,+0.00e+000)  
(+0.00e+000,+0.00e+000) (-7.00e+000,-8.00e+000) (-9.00e+000,-1.00e+001)
```

2.12.35 operator ++

Operator

```
scbmatrix& scbmatrix::operator ++ ();  
scbmatrix& scbmatrix::operator ++ (int);
```

adds identity matrix to a calling band matrix and returns a reference to the matrix changed.
See also [scbmatrix](#). Example:

```
using namespace cvm;  
  
scbmatrix m(4,1,0);  
m.set(std::complex<double>(1.,1.));  
  
m++;  
std::cout << m << std::endl;  
std::cout << ++m;
```

prints

```
(2,1) (0,0) (0,0) (0,0)  
(1,1) (2,1) (0,0) (0,0)  
(0,0) (1,1) (2,1) (0,0)  
(0,0) (0,0) (1,1) (2,1)  
  
(3,1) (0,0) (0,0) (0,0)  
(1,1) (3,1) (0,0) (0,0)  
(0,0) (1,1) (3,1) (0,0)  
(0,0) (0,0) (1,1) (3,1)
```

2.12.36 operator --

Operator

```
scbmatrix& scbmatrix::operator -- ();
scbmatrix& scbmatrix::operator -- (int);
```

subtracts identity matrix from a calling band matrix and returns a reference to the matrix changed. See also [scbmatrix](#). Example:

```
using namespace cvm;
```

```
scbmatrix m(4,1,0);
m.set(std::complex<double>(1.,1.));
```

```
m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
(0,1) (0,0) (0,0) (0,0)
(1,1) (0,1) (0,0) (0,0)
(0,0) (1,1) (0,1) (0,0)
(0,0) (0,0) (1,1) (0,1)
```

```
(-1,1) (0,0) (0,0) (0,0)
(1,1) (-1,1) (0,0) (0,0)
(0,0) (1,1) (-1,1) (0,0)
(0,0) (0,0) (1,1) (-1,1)
```

2.12.37 operator * (TR)

Operator

```
scbmatrix scbmatrix::operator * (TR d) const;
```

creates an object of type `scbmatrix` as a product of a calling band matrix and a real number `d`. See also `scbmatrix::operator *=`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12.};  
scbmatrix m((std::complex<double>*)a,3,0,1);  
std::cout << m * 5.;
```

prints

```
(15,20) (25,30) (0,0)  
(0,0) (35,40) (45,50)  
(0,0) (0,0) (55,60)
```

2.12.38 operator / (TR)

Operator

```
scbmatrix scbmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `scbmatrix` as a quotient of a calling band matrix and a real number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the **smallest normalized positive number**. See also `scbmatrix::operator /=`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
              10., 11., 12.};  
scbmatrix m((std::complex<double>*)a,3,0,1);  
std::cout << m / 2.;
```

prints

```
(1.5,2) (2.5,3) (0,0)  
(0,0) (3.5,4) (4.5,5)  
(0,0) (0,0) (5.5,6)
```

2.12.39 operator * (TC)

Operator

```
scbmatrix scbmatrix::operator * (TC z) const;
```

creates an object of type `scbmatrix` as a product of a calling band matrix and a complex number `z`. See also `scbmatrix::operator *=` , `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12.};  
scbmatrix m((std::complex<double>*)a,3,0,1);  
std::cout << m * std::complex<double>(1.,1.);
```

prints

```
(-1,7) (-1,11) (0,0)  
(0,0) (-1,15) (-1,19)  
(0,0) (0,0) (-1,23)
```

2.12.40 operator / (TC)

Operator

```
scbmatrix scbmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `scbmatrix` as a quotient of a calling band matrix and a complex number `z`. It throws an exception of type `cvmexception` if `z` has an absolute value equal or less than the `smallest normalized positive number`. See also `scbmatrix::operator /=`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
              10., 11., 12.};
```

```
scbmatrix m((std::complex<double>*)a,3,0,1);  
std::cout << m / std::complex<double>(1.,1.);
```

prints

```
(3.5,0.5) (5.5,0.5) (0,0)  
(0,0) (7.5,0.5) (9.5,0.5)  
(0,0) (0,0) (11.5,0.5)
```

2.12.41 operator *= (TR)

Operator

```
scbmatrix& scbmatrix::operator *= (TR d);
```

multiplies a calling band matrix by a real number d and returns a reference to the matrix changed. See also `scbmatrix::operator *`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12.};  
scbmatrix m((std::complex<double>*)a,3,0,1);  
m *= 5.;  
std::cout << m;
```

prints

```
(15,20) (25,30) (0,0)  
(0,0) (35,40) (45,50)  
(0,0) (0,0) (55,60)
```

2.12.42 operator /= (TR)

Operator

```
scbmatrix& scbmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling band matrix by a real number *d* and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if *d* has an absolute value equal or less than the `smallest normalized positive number`. See also `scbmatrix::operator /`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12.};
```

```
scbmatrix m((std::complex<double>*)a,3,0,1);
```

```
m /= 2.;
```

```
std::cout << m;
```

prints

```
(1.5,2) (2.5,3) (0,0)
```

```
(0,0) (3.5,4) (4.5,5)
```

```
(0,0) (0,0) (5.5,6)
```

2.12.43 operator *= (TC)

Operator

```
scbmatrix& scbmatrix::operator *= (TC z);
```

multiplies a calling band matrix by a complex number z and returns a reference to the matrix changed. See also `scbmatrix::operator *`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
m *= std::complex<double>(1.,1.);
std::cout << m;
```

prints

```
(-1,7) (-1,11) (0,0)
(0,0) (-1,15) (-1,19)
(0,0) (0,0) (-1,23)
```

2.12.44 operator /= (TC)

Operator

```
scbmatrix& scbmatrix::operator /= (TC z) throw (cvmexception);
```

divides a calling band matrix by a complex number z and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if z has an absolute value equal or less than the `smallest normalized positive number`. See also `scbmatrix::operator /`, `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12.};
```

```
scbmatrix m((std::complex<double>*)a,3,0,1);
```

```
m /= std::complex<double>(1.,1.);
```

```
std::cout << m;
```

prints

```
(3.5,0.5) (5.5,0.5) (0,0)  
(0,0) (7.5,0.5) (9.5,0.5)  
(0,0) (0,0) (11.5,0.5)
```

2.12.45 normalize

Function

```
scbmatrix& scbmatrix::normalize ();
```

normalizes a calling band matrix so its **Euclidean norm** becomes equal to 1 if it was greater than the **smallest normalized positive number** before the call (otherwise the function does nothing). See also **scbmatrix**. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
```

```
std::cout.precision (2);
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,  
             10., 11., 12.};
```

```
scbmatrix m((std::complex<double>*)a,3,0,1);
```

```
m.normalize();
```

```
std::cout << m << m.norm() << std::endl;
```

prints

```
(1.18e-001,1.57e-001) (1.97e-001,2.36e-001) (0.00e+000,0.00e+000)  
(0.00e+000,0.00e+000) (2.76e-001,3.15e-001) (3.54e-001,3.94e-001)  
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (4.33e-001,4.72e-001)  
1.00e+000
```

2.12.46 conjugation

Operator and functions

```
scbmatrix scbmatrix::operator ~ () const throw (cvmexception);
scbmatrix& scbmatrix::conj (const scbmatrix& m) throw (cvmexception);
scbmatrix& scbmatrix::conj () throw (cvmexception);
```

encapsulate complex band matrix conjugation. First operator creates an object of type `scbmatrix` as a conjugated calling band matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `m` conjugated (it throws an exception of type `cvmexception` in case of not appropriate sizes or numbers of sub- or super-diagonals of the operands), third one makes it to be equal to conjugated itself (it also throws an exception of type `cvmexception` in case of memory allocation failure). See also `scbmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
scbmatrix m((std::complex<double>*)a,4,1,0);
scbmatrix mc(4,0,1);
std::cout << m << std::endl << ~m << std::endl ;
mc.conj(m);
std::cout << mc << std::endl;
mc.conj();
std::cout << mc;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)

(1,-2) (3,-4) (0,0) (0,0)
(0,0) (5,-6) (7,-8) (0,0)
(0,0) (0,0) (9,-10) (11,-12)
(0,0) (0,0) (0,0) (13,-14)

(1,-2) (3,-4) (0,0) (0,0)
(0,0) (5,-6) (7,-8) (0,0)
(0,0) (0,0) (9,-10) (11,-12)
(0,0) (0,0) (0,0) (13,-14)
```

(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)

2.12.47 operator * (const cvector&)

Operator

```
cvector scbmatrix::operator * (const cvector& v) const  
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling band matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `cvector::mult` in order to get rid of a new object creation. See also `scbmatrix` and `cvector`. Example:

```
using namespace cvm;
```

```
try {  
    scbmatrix m (4,1,0);  
    cvector v(4);  
    m.set(std::complex<double>(1.,1.));  
    v.set(std::complex<double>(1.,1.));  
  
    std::cout << m * v;  
}  
catch (exception& e) {  
    std::cout << "Exception: " << e.what () << std::endl;  
}
```

prints

```
(0,2) (0,4) (0,4) (0,4)
```

2.12.48 operator * (const cmatrix&)

Operator

```
cmatrix scbmatrix::operator * (const cmatrix& m) const  
throw (cvmexception);
```

creates an object of type `cmatrix` as a product of a calling band matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `cmatrix::mult` in order to get rid of a new object creation. See also `cmatrix` and `scbmatrix`. Example:

```
using namespace cvm;
```

```
try {  
    scbmatrix mb(4,1,0);  
    cmatrix m(4,2);  
    mb.set(std::complex<double>(1.,1.));  
    m.set(std::complex<double>(1.,1.));  
  
    std::cout << mb * m;  
}  
catch (exception& e) {  
    std::cout << "Exception: " << e.what () << std::endl;  
}
```

prints

```
(0,2) (0,2)  
(0,4) (0,4)  
(0,4) (0,4)  
(0,4) (0,4)
```

2.12.49 operator * (const scmatrix&)

Operator

```
scmatrix scbmatrix::operator * (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a product of a calling band matrix and a matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `cmatrix::mult` in order to get rid of a new object creation. See also `scmatrix` and `scbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    scbmatrix mb(4,1,0);
    scmatrix m(4);
    mb.set(std::complex<double>(1.,1.));
    m.set(std::complex<double>(1.,1.));

    std::cout << mb * m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,2) (0,2) (0,2) (0,2)
(0,4) (0,4) (0,4) (0,4)
(0,4) (0,4) (0,4) (0,4)
(0,4) (0,4) (0,4) (0,4)
```

2.12.50 operator * (const scbmatrix&)

Operator

```
scbmatrix scbmatrix::operator * (const scbmatrix& m) const
throw (cvmexception);
```

creates an object of type `scbmatrix` as a product of a calling band matrix and a band matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `cmatrix::mult` in order to get rid of a new object creation. See also `scbmatrix`. Example:

```
using namespace cvm;
```

```
try {
    scbmatrix m1(5,1,0);
    scbmatrix m2(5,1,1);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(1.,1.));

    std::cout << m1 * m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,2) (0,2) (0,0) (0,0) (0,0)
(0,4) (0,4) (0,2) (0,0) (0,0)
(0,2) (0,4) (0,4) (0,2) (0,0)
(0,0) (0,2) (0,4) (0,4) (0,2)
(0,0) (0,0) (0,2) (0,4) (0,4)
```

2.12.51 low_up

Functions

```
scbmatrix&
scbmatrix::low_up (const scbmatrix& m, int* nPivots) throw (cvmexception);
scbmatrix
scbmatrix::low_up (int* nPivots) const throw (cvmexception);
```

compute the LU factorization of a calling band matrix as

$$A = PLU$$

where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and U is an upper triangular matrix. All the functions store the result as the matrix L without main diagonal combined with U. All the functions return pivot indices as an array of integers (it should support at least `m.size()` elements) pointed to by `nPivots` so *i*-th row was interchanged with `nPivots[i]`-th row. The first version sets a calling matrix to be equal to the *m*'s LU factorization and the second one creates an object of type `scbmatrix` as the calling band matrix's LU factorization. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be factorized is close to singular. The first version also changes numbers of super-diagonals to be equal to $k_l + k_u$ in order to keep the result of factorization. It is recommended to use `iarray` for pivot values. This function is provided mostly for solving multiple systems of linear equations using `scmatrix::solve_lu` function, See also `scbmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12.};
    scbmatrix ma((std::complex<double>*)a,3,1,0);
    scbmatrix mLU(3,1,0);
    cmatrix mb1(3,2); cvector vb1(3);
    cmatrix mb2(3,2); cvector vb2(3);
    cmatrix mx1(3,2); cvector vx1(3);
    cmatrix mx2(3,2); cvector vx2(3);
    iarray nPivots(3);
    double dErr = 0.;
    mb1.randomize_real(-1.,3.); mb1.randomize_imag(1.,5.);
    mb2.randomize_real(-2.,5.); mb2.randomize_imag(-3.,0.);
    vb1.randomize_real(-2.,4.); vb1.randomize_imag(-4.,1.);
    vb2.randomize_real(-3.,1.); vb2.randomize_imag(4.,5.);
```

```

    mLU.low_up(ma, nPivots);
    mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
    std::cout << mx1 << dErr << std::endl << std::endl;
    mx2 = ma.solve_lu (mLU, nPivots, mb2);
    std::cout << mx2 << std::endl;;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;

    vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
    std::cout << vx1 << dErr << std::endl;
    vx2 = ma.solve_lu (mLU, nPivots, vb2);
    std::cout << vx2 << std::endl;;
    std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
}

prints

(1.20e+000,4.02e-002) (1.82e+000,1.23e+000)
(-6.55e-001,1.37e-001) (-6.41e-001,-8.72e-001)
(7.75e-001,4.70e-002) (5.35e-001,8.11e-001)
1.45e-015

(-4.52e-001,-2.68e-002) (-1.09e+000,2.01e-001)
(6.08e-001,-4.76e-001) (5.48e-001,-1.95e-001)
(-3.46e-001,1.57e-001) (-3.38e-001,-7.54e-002)

(0.00e+000,4.44e-016) (-2.22e-016,8.88e-016)
(-2.22e-016,2.22e-016) (0.00e+000,0.00e+000)
(-1.11e-016,0.00e+000) (-3.33e-016,-6.66e-016)

(0.00e+000,0.00e+000) (2.22e-016,2.22e-016)
(0.00e+000,0.00e+000) (4.44e-016,-2.22e-016)
(8.88e-016,5.55e-016) (0.00e+000,0.00e+000)
(-1.28e+000,-5.12e-001) (8.22e-001,1.59e-001) (-6.45e-001,-3.74e-001)
1.31e-015
(1.26e+000,1.50e+000) (-5.13e-001,-4.66e-001) (5.97e-001,7.01e-001)

(0.00e+000,8.88e-016) (-4.44e-016,4.44e-016) (-8.88e-016,0.00e+000)

(2.22e-016,-8.88e-016) (4.44e-016,-8.88e-016) (-2.22e-016,8.88e-016)

```

2.12.52 identity

Function

```
scbmatrix& scbmatrix::identity();
```

sets a calling band matrix to be equal to identity matrix and returns a reference to the matrix changed. The function doesn't change numbers of sub- and super-diagonals. See also [scbmatrix](#). Example:

```
using namespace cvm;
```

```
srbmatrix m(4);  
m.randomize(0.,1.);  
std::cout << m << std::endl;  
std::cout << m.identity();
```

prints

```
(0.576128,1.42595) (0,0) (0,0) (0,0)  
(0.956359,-0.919523) (0.869716,-0.704093) (0,0) (0,0)  
(0,0) (0.0959807,0.0616779) (0.632618,1.1793) (0,0)  
(0,0) (0,0) (0.532182,-0.870724) (0.338023,1.22892)  
  
(1,0) (0,0) (0,0) (0,0)  
(0,0) (1,0) (0,0) (0,0)  
(0,0) (0,0) (1,0) (0,0)  
(0,0) (0,0) (0,0) (1,0)
```

2.12.53 vanish

Function

```
scbmatrix& scbmatrix::vanish();
```

sets every element of a calling band matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `scbmatrix::set(TR)` with zero operand passed. See also `scbmatrix`. Example:

```
using namespace cvm;
```

```
scbmatrix m(4,1,0);  
m.randomize_real(0.,1.);  
m.randomize_imag(-1.,2.);  
std::cout << m << std::endl;  
std::cout << m.vanish();
```

prints

```
(0.584094,0.985931) (0,0) (0,0) (0,0)  
(0.197546,0.0150761) (0.483413,-0.733848) (0,0) (0,0)  
(0,0) (0.844356,1.97848) (0.814692,1.50194) (0,0)  
(0,0) (0,0) (0.118931,-0.720756) (0.936796,-0.582232)  
  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)
```

2.12.54 `randomize_real`

Function

```
scbmatrix& scbmatrix::randomize_real (TR dFrom, TR dTo);
```

fills a real part of a calling band matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. See also [scbmatrix](#).

Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
```

```
std::cout.precision (2);
```

```
scbmatrix m(3,0,1);
```

```
m.randomize_real(0.,3.);
```

```
std::cout << m;
```

prints

```
(1.78e+000,0.00e+000) (1.17e+000,0.00e+000) (0.00e+000,0.00e+000)
```

```
(0.00e+000,0.00e+000) (1.09e-002,0.00e+000) (6.05e-001,0.00e+000)
```

```
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (2.49e+000,0.00e+000)
```

2.12.55 `randomize_imag`

Function

```
scbmatrix& scbmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills an imaginary part of a calling band matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. See also [scbmatrix](#). Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
scbmatrix m(3,0,1);  
m.randomize_imag(0.,3.);  
std::cout << m;
```

prints

```
(0.00e+000,1.80e+000) (0.00e+000,1.68e-001) (0.00e+000,0.00e+000)  
(0.00e+000,0.00e+000) (0.00e+000,1.05e+000) (0.00e+000,1.40e+000)  
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,1.98e+000)
```

2.13 srsmatrix

This is end-user class encapsulating a symmetric matrix in Euclidean space of real numbers.

```

template <typename TR>
class srsmatrix : public srmatrix <TR> {
public:
    srsmatrix ();
    explicit srsmatrix (int nMN);
    srsmatrix (TR* pD, int nMN, TR tol = cvmMachSp());
    srsmatrix (const TR* pD, int nMN, TR tol = cvmMachSp());
    srsmatrix (const srsmatrix& m);
    explicit srsmatrix (const rmatrix& m, TR tol = cvmMachSp());
    explicit srsmatrix (const rvector& v);
    srsmatrix (srsmatrix& m, int nRowCol, int nSize);
    TR operator () (int im, int in) const throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    const rvector diag (int i) const throw (cvmexception);
    srsmatrix& operator = (const srsmatrix& m) throw (cvmexception);
    srsmatrix& assign (const rvector& v, TR tol = cvmMachSp())
        throw (cvmexception);
    srsmatrix& assign (const TR* pD, TR tol = cvmMachSp())
        throw (cvmexception);
    srsmatrix& assign (int nRowCol, const srsmatrix& m)
        throw (cvmexception);
    srsmatrix& set (TR x);
    srsmatrix& set (int nRow, int nCol, TR x);
    srsmatrix& set_diag (int i, const rvector& v) throw (cvmexception);
    srsmatrix& resize (int nNewMN) throw (cvmexception);
    bool operator == (const srsmatrix& m) const;
    bool operator != (const srsmatrix& m) const;
    srsmatrix& operator << (const srsmatrix& m) throw (cvmexception);
    srsmatrix operator + (const srsmatrix& m) const
        throw (cvmexception);
    srsmatrix operator - (const srsmatrix& m) const
        throw (cvmexception);
    srsmatrix& sum (const srsmatrix& m1,
        const srsmatrix& m2) throw (cvmexception);
    srsmatrix& diff (const srsmatrix& m1,
        const srsmatrix& m2) throw (cvmexception);
    srsmatrix& operator += (const srsmatrix& m) throw (cvmexception);

```

```

srsmatrix& operator -= (const srsmatrix& m) throw (cvmexception);
srsmatrix operator - () const;
srsmatrix& operator ++ ();
srsmatrix& operator ++ (int);
srsmatrix& operator -- ();
srsmatrix& operator -- (int);
srsmatrix operator * (TR d) const;
srsmatrix operator / (TR d) const throw (cvmexception);
srsmatrix& operator *= (TR d);
srsmatrix& operator /= (TR d) throw (cvmexception);
srsmatrix& normalize ();
srsmatrix operator ~ () const throw (cvmexception);
srsmatrix& transpose (const srsmatrix& m) throw (cvmexception);
srsmatrix& transpose ();
rvector operator * (const rvector& v) const throw (cvmexception);
rmatrix operator * (const rmatrix& m) const throw (cvmexception);
srmatrix operator * (const srmatrix& m) const throw (cvmexception);
srsmatrix& syrk (TR alpha,
                const rvector& v, TR beta) throw (cvmexception);
srsmatrix& syrk (bool bTransp, TR alpha,
                const rmatrix& m, TR beta) throw (cvmexception);
srsmatrix& syr2k (TR alpha,
                const rvector& v1, const rvector& v2, TR beta)
                throw (cvmexception);
srsmatrix& syr2k (bool bTransp, TR alpha,
                const rmatrix& m1, const rmatrix& m2, TR beta)
                throw (cvmexception);
srsmatrix& inv (const srsmatrix& mArg) throw (cvmexception);
srsmatrix inv () const throw (cvmexception);
srsmatrix& exp (const srsmatrix& m,
                TR tol = cvmMachSp()) throw (cvmexception);
srsmatrix exp (TR tol = cvmMachSp()) const throw (cvmexception);
srsmatrix& polynom (const srsmatrix& m, const rvector& v)
                throw (cvmexception);
srsmatrix polynom (const rvector& v) const throw (cvmexception);
rvector eig (srmatrix& mEigVect) const throw (cvmexception);
rvector eig () const throw (cvmexception);
srmatrix cholesky () const throw (cvmexception);
srmatrix bunch_kaufman () const throw (cvmexception);
srsmatrix& identity ();
srsmatrix& vanish ();
srsmatrix& randomize (TR dFrom, TR dTo);
};

```

2.13.1 srsmatrix ()

Constructor

```
srsmatrix::srsmatrix ();
```

creates an empty srsmatrix object. See also [srsmatrix](#). Example:

```
using namespace cvm;
```

```
srsmatrix m;  
std::cout << m.msize() << " " << m.nsize() << " " << m.size()  
          << std::endl << std::endl;  
m.resize (3);  
m.set(1.);  
std::cout << m;
```

prints

```
0 0 0
```

```
1 1 1
```

```
1 1 1
```

```
1 1 1
```

2.13.2 srsmatrix (int)

Constructor

```
explicit srsmatrix::srsmatrix (int nMN);
```

creates an $n \times n$ srsmatrix object where n is passed in nMN parameter. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
srsmatrix m(4);  
std::cout << m.msize() << " " << m.nsize() << " " << m.size()  
          << std::endl << std::endl;  
m.set(1.);  
std::cout << m;
```

prints

```
4 4 16
```

```
1 1 1 1  
1 1 1 1  
1 1 1 1  
1 1 1 1
```

2.13.3 srsmatrix (TR*,int)

Constructor

```
srsmatrix::srsmatrix (TR* pD, int nMN, TR tol = cvmMachSp());
```

creates an $n \times n$ srsmatrix object where n is passed in nMN parameter. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by pD. The constructor throws an exception of type `cvmexception` if the matrix created doesn't appear to be symmetric or in case of non-positive size passed. The symmetry tolerance is set by parameter tol. If subsequent application flow would change the array passed so it becomes not symmetric matrix anymore then results are not predictable. See also `srsmatrix`, `srsmatrix (const TR*,int)`. Example:

```
using namespace cvm;
double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.};
srsmatrix m (a, 3);
m.set(2,1,5.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
1 5 1
5 1 1
1 1 1

1 5 1 5 1 1
```

2.13.4 srsmatrix (const TR*,int)

Constructor

```
srsmatrix::srsmatrix (const TR* pD, int nMN, TR tol = cvmMachSp());
```

creates an $n \times n$ srsmatrix object where n is passed in nMN parameter and copies $n * n$ elements of an array pD to it by columns. The constructor throws an exception of type `cvmexception` if the matrix created doesn't appear to be symmetric or in case of non-positive size passed. The symmetry tolerance is set by parameter tol. See also `srsmatrix`, `srsmatrix (TR*,int)`. Example:

```
using namespace cvm;
const double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.};
srsmatrix m (a, 3);
m.set(2,1,5.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
1 5 1
5 1 1
1 1 1

1 1 1 1 1 1
```

2.13.5 srsmatrix (const srsmatrix&)

Copy constructor

```
srsmatrix::srsmatrix (const srsmatrix& m);
```

creates a srsmatrix object as a copy of m. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m (a, 3);  
srmatrix mc(m);  
m.set(2,3,7.77);  
std::cout << m << std::endl << mc;
```

prints

```
1 2 3  
2 5 7.77  
3 7.77 9
```

```
1 2 3  
2 5 6  
3 6 9
```

2.13.6 srsmatrix (const rmatrix&)

Constructor

```
explicit srsmatrix::srsmatrix (const rmatrix& m, TR tol = cvmMachSp());
```

creates a srsmatrix object as a copy of matrix m. It's assumed that $m \times n$ matrix m must have equal sizes, i.e. $m = n$ is satisfied, and must be symmetric. The symmetry tolerance is set by parameter tol. The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
rmatrix m(a, 3, 3);  
srsmatrix ms(m);  
std::cout << ms;
```

prints

```
1 2 3  
2 5 6  
3 6 9
```

2.13.7 srsmatrix (const rvector&)

Constructor

```
explicit srsmatrix::srsmatrix (const rvector& v);
```

creates a srsmatrix object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srsmatrix`, `rvector`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5.};  
const rvector v(a, 5);  
srsmatrix m(v);  
std::cout << m;
```

prints

```
1 0 0 0 0  
0 2 0 0 0  
0 0 3 0 0  
0 0 0 4 0  
0 0 0 0 5
```

2.13.8 submatrix

Submatrix constructor

```
srsmatrix::srsmatrix (srsmatrix& m, int nRowCol, int nSize);
```

creates a srsmatrix object as a *submatrix* of symmetric matrix m. It means that the matrix object created shares a memory with some part of m. This part is defined by its upper left corner (parameter nRowCol, *l-based*) and its size (parameter nSize). See also [srsmatrix](#). Example:

```
using namespace cvm;
```

```
srsmatrix m(5);  
srsmatrix subm(m, 2, 2);  
subm.set(1.);  
std::cout << m;
```

prints

```
0 0 0 0 0  
0 1 1 0 0  
0 1 1 0 0  
0 0 0 0 0
```

2.13.9 operator (,)

Indexing operator

```
TR srsmatrix::operator () (int im, int in) const throw (cvmexception);
```

provides access to an element of a matrix. Unlike indexing operators in other classes, this operator doesn't return an *l-value* because this would make the matrix non-symmetric. The operator is *l-based*. The operator throws an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. See also `srsmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a, 3);  
std::cout << m(1,1) << " " << m(2,3) << std::endl;
```

prints

```
1 6
```

2.13.10 operator ()

Indexing operator

```
const rvector srsmatrix::operator () (int i) const throw (cvmexception);
```

provides access to an i -th column of a matrix. Unlike indexing operators in other classes, this operator doesn't return an *l-value* because this would make the matrix non-symmetric. The operator creates an object of class `rvector` as a *copy* of a column and therefore it's *not an l-value*. The operator is *ℓ-based*. The operator throws an exception of type `cvmexception` if the parameter i is outside of $[1, \text{nsize}()]$ range. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a, 3);  
std::cout << m(1) << m(2) << m(3);
```

prints

```
1 2 3  
2 5 6  
3 6 9
```

2.13.11 operator []

Indexing operator

```
const rvector srsmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to an *i*-th row of a matrix. Unlike indexing operators in other classes, this operator doesn't return an *l-value* because this would make the matrix non-symmetric. The operator creates an object of class `rvector` as a *copy* of a row and therefore it's *not an l-value*. The operator is *ℓ-based*. The operator throws an exception of type `cvmexception` if the parameter *i* is outside of `[1,msize()]` range. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a, 3);  
std::cout << m[1] << m[2] << m[3];
```

prints

```
1 2 3  
2 5 6  
3 6 9
```

2.13.12 diag

Function

```
const rvector srsmatrix::diag (int i) const throw (cvmexception);
```

provides access to an i -th diagonal of a matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. Unlike `diag` function in other classes, this one doesn't return an *l-value* because this would make the matrix non-symmetric. The function creates an object of class `rvector` as a *copy* of a diagonal and therefore it's *not an l-value*. The function is *l-based*. The function throws an exception of type `cvmexception` if the parameter `i` is outside of `[-msize()+1, nsize()-1]` range. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a, 3);  
std::cout << m << std::endl  
          << m.diag(-2) << m.diag(-1) << m.diag(0)  
          << m.diag(1) << m.diag(2);
```

prints

```
1 2 3  
2 5 6  
3 6 9  
  
3  
2 6  
1 5 9  
2 6  
3
```

2.13.13 operator = (const srsmatrix&)

Operator

```
srsmatrix& srsmatrix::operator = (const srsmatrix& m)
throw (cvmexception);
```

sets an every element of a calling symmetric matrix to a value of appropriate element of symmetric matrix `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
const srsmatrix m1(a, 3);
srsmatrix m2(3);
```

```
m2 = m1;
std::cout << m2;
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.14 assign (const TR*)

Function

```
srsmatrix& srsmatrix::assign (const rvector& v, TR tol = cvmMachSp())  
throw (cvmexception);
```

```
srsmatrix& srsmatrix::assign (const TR* pD, TR tol = cvmMachSp())  
throw (cvmexception);
```

sets every element of a calling matrix to a value of appropriate element of a vector *v* or an array pointed to by *pD* and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if the matrix changed doesn't appear to be symmetric. The symmetry tolerance is set by parameter *tol*. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(3);  
m.assign(a);  
std::cout << m;
```

prints

```
1 2 3  
2 5 6  
3 6 9
```

2.13.15 assign (int, int, const srsmatrix&)

Function

```
srsmatrix& srsmatrix::assign (int nRowCol, const srsmatrix& m)
throw (cvmexception);
```

sets main sub-matrix of a calling symmetric matrix beginning with ℓ -based row nRowCol to a symmetric matrix m and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if nRowCol is not positive or matrix m doesn't fit. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
srsmatrix m1(5);
srsmatrix m2(2);
m1.set(1.);
m2.set(2.);
m1.assign(2,m2);
std::cout << m1;
```

prints

```
1 1 1 1 1
1 2 2 1 1
1 2 2 1 1
1 1 1 1 1
1 1 1 1 1
```

2.13.16 set (TR)

Function

```
srsmatrix& srsmatrix::set (TR x);
```

sets every element of a calling matrix to a value of parameter `x` and returns a reference to the matrix changed. See also [srsmatrix](#). Example:

```
using namespace cvm;
```

```
srsmatrix m(3);  
m.set(3.);  
std::cout << m;
```

prints

```
3 3 3  
3 3 3  
3 3 3
```

2.13.17 set (int,int,TR)

Function

```
srsmatrix& srsmatrix::set (int nRow, int nCol, TR x);
```

sets both elements located on nRow's row and nCol's column and on nCol's row and nRow's column to a value of parameter x and returns a reference to the matrix changed (thus the matrix remains symmetric). The parameters passed are *ℓ-based*. The function throws an exception of type *cvmexception* if anyone of the parameters passed is outside of [1,msize()] range. See also *srsmatrix*. Example:

```
using namespace cvm;
```

```
srsmatrix m(3);  
m.set(3.);  
m.set(1,3,7.);  
std::cout << m;
```

prints

```
3 3 7  
3 3 3  
7 3 3
```

2.13.18 set_diag (int,rvector)

Function

```
srsmatrix& srsmatrix::set_diag (int i, const rvector& v)
throw (cvmexception);
```

assigns vector v to an i -th diagonal of a matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. If $i \neq 0$, then the function assigns the vector to both i -th and $-i$ -th diagonals (thus the matrix remains symmetric). The function returns a reference to the matrix changed. The function is *l*-based. The function throws an exception of type `cvmexception` if the parameter i is outside of $[-m\text{size}()+1, n\text{size}()-1]$ range or if the vector v passed has a size not equal to $m\text{size}()-\text{abs}(i)$. See also `srsmatrix`.
Example:

```
using namespace cvm;
```

```
srsmatrix m(3);
rvector v(2);
m.set(3.);
v.set(1.);
m.set_diag(1,v);
std::cout << m;
```

prints

```
3 1 3
1 3 1
3 1 3
```

2.13.19 `resize`

Function

```
srsmatrix& srsmatrix::resize (int nNewMN) throw (cvmexception);
```

changes a size of a calling matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a,3);  
std::cout << m << std::endl;  
m.resize(4);  
std::cout << m;
```

prints

```
1 2 3  
2 5 6  
3 6 9
```

```
1 2 3 0  
2 5 6 0  
3 6 9 0  
0 0 0 0
```

2.13.20 operator ==

Operator

```
bool srsmatrix::operator == (const srsmatrix& m) const;
```

compares a calling symmetric matrix with symmetric matrix `m` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the **smallest normalized positive number**. Returns `false` otherwise. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 2., 3.};  
srsmatrix m1(a, 2);  
srsmatrix m2(2);  
m2.set(1,1,1.);  
m2.set(1,2,2.);  
m2.set(2,2,3.);
```

```
std::cout << (m1 == m2) << std::endl;
```

prints

1

2.13.21 operator !=

Operator

```
bool srsmatrix::operator != (const srsmatrix& m) const;
```

compares a calling matrix with a matrix `m` and returns `true` if they have different sizes or at least one of their appropriate elements differs by more than the **smallest normalized positive number**. Returns `false` otherwise. See also [srsmatrix](#). Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 2., 3.};  
srsmatrix m1(a, 2);  
srsmatrix m2(2);  
m2.set(1,1,1.0001);  
m2.set(1,2,2.);  
m2.set(2,2,3.);
```

```
std::cout << (m1 != m2) << std::endl;
```

prints

```
1
```

2.13.22 operator <<

Operator

```
srsmatrix& srsmatrix::operator << (const srsmatrix& m)
throw (cvmexception);
```

destroys a calling matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
srsmatrix mc(1);
std::cout << m << std::endl << mc << std::endl;
mc << m;
std::cout << mc;
```

prints

```
1 2 3
2 5 6
3 6 9
```

```
0
```

```
1 2 3
2 5 6
3 6 9
```

2.13.23 operator +

Operator

```
srsmatrix srsmatrix::operator + (const srsmatrix& m) const  
throw (cvmexception);
```

creates an object of type `srsmatrix` as a sum of a calling symmetric matrix and symmetric matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::sum`, `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m1(a, 3);  
srsmatrix m2(3);  
m2.set(1.);  
std::cout << m1 + m2 << std::endl << m1 + m1;
```

prints

```
2 3 4  
3 6 7  
4 7 10
```

```
2 4 6  
4 10 12  
6 12 18
```

2.13.24 operator -

Operator

```
srsmatrix srsmatrix::operator - (const srsmatrix& m) const  
throw (cvmexception);
```

creates an object of type `srsmatrix` as a difference of a calling symmetric matrix and symmetric matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::diff`, `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m1(a, 3);  
srsmatrix m2(3);  
m2.set(1.);  
std::cout << m1 - m2 << std::endl << m1 - m1;
```

prints

```
0 1 2  
1 4 5  
2 5 8
```

```
0 0 0  
0 0 0  
0 0 0
```

2.13.25 sum

Function

```
srsmatrix& srsmatrix::sum (const srsmatrix& m1, const srsmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of symmetric matrices m1 and m2 to a calling symmetric matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator +`, `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
const srsmatrix m1(a, 3);
srsmatrix m2(3);
srsmatrix m(3);
m2.set(1.);
```

```
std::cout << m.sum(m1, m2) << std::endl;
std::cout << m.sum(m, m2);
```

prints

```
2 3 4
3 6 7
4 7 10
```

```
3 4 5
4 7 8
5 8 11
```

2.13.26 diff

Function

```
srsmatrix& srsmatrix::diff (const srsmatrix& m1, const srsmatrix& m2)  
throw (cvmexception);
```

assigns a result of subtraction of symmetric matrices m1 and m2 to a calling symmetric matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator -`, `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
const srsmatrix m1(a, 3);  
srsmatrix m2(3);  
srsmatrix m(3);  
m2.set(1.);
```

```
std::cout << m.diff(m1, m2) << std::endl;  
std::cout << m.diff(m, m2);
```

prints

```
0 1 2  
1 4 5  
2 5 8
```

```
-1 0 1  
0 3 4  
1 4 7
```

2.13.27 operator +=

Operator

```
srsmatrix& srsmatrix::operator += (const srsmatrix& m)
throw (cvmexception);
```

adds symmetric matrix `m` to a calling symmetric matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator +`, `srsmatrix::sum`, `srsmatrix`. Example:

```
using namespace cvm;
```

```
srsmatrix m1(3);
srsmatrix m2(3);
m1.set(1.);
m2.set(2.);
```

```
m1 += m2;
std::cout << m1 << std::endl;
```

```
// well, you can do this too, but temporary object would be created
m2 += m2;
std::cout << m2;
```

prints

```
3 3 3
3 3 3
3 3 3
```

```
4 4 4
4 4 4
4 4 4
```

2.13.28 operator -=

Operator

```
srsmatrix& srsmatrix::operator -= (const srsmatrix& m)
throw (cvmexception);
```

subtracts symmetric matrix `m` from a calling symmetric matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator -`, `srsmatrix::diff`, `srsmatrix`. Example:

```
using namespace cvm;
```

```
srsmatrix m1(3);
srsmatrix m2(3);
m1.set(1.);
m2.set(2.);
```

```
m1 -= m2;
std::cout << m1 << std::endl;
```

```
// well, you can do this too, but temporary object would be created
m2 -= m2;
std::cout << m2;
```

prints

```
-1 -1 -1
-1 -1 -1
-1 -1 -1
```

```
0 0 0
0 0 0
0 0 0
```

2.13.29 operator - ()

Operator

```
srsmatrix srsmatrix::operator - () const throw (cvmexception);
```

creates an object of type `srsmatrix` as a calling symmetric matrix multiplied by -1 . See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a, 3);  
std::cout << -m;
```

prints

```
-1 -2 -3  
-2 -5 -6  
-3 -6 -9
```

2.13.30 operator ++

Operator

```
srsmatrix& srsmatrix::operator ++ ();  
srsmatrix& srsmatrix::operator ++ (int);
```

adds identity matrix to a calling symmetric matrix and returns a reference to the matrix changed. See also [srsmatrix](#). Example:

```
using namespace cvm;
```

```
srsmatrix m(4);  
m.set(4.);  
m++;  
std::cout << m << std::endl;  
std::cout << ++m;
```

prints

```
5 4 4 4  
4 5 4 4  
4 4 5 4  
4 4 4 5
```

```
6 4 4 4  
4 6 4 4  
4 4 6 4  
4 4 4 6
```

2.13.31 operator --

Operator

```
srsmatrix& srsmatrix::operator -- ();  
srsmatrix& srsmatrix::operator -- (int);
```

subtracts identity matrix from a calling symmetric matrix and returns a reference to the matrix changed. See also [srsmatrix](#). Example:

```
using namespace cvm;  
  
srsmatrix m(4);  
m.set(4.);  
m--;  
std::cout << m << std::endl;  
std::cout << --m;
```

prints

```
3 4 4 4  
4 3 4 4  
4 4 3 4  
4 4 4 3  
  
2 4 4 4  
4 2 4 4  
4 4 2 4  
4 4 4 2
```

2.13.32 operator * (TR)

Operator

```
srsmatrix srsmatrix::operator * (TR d) const;
```

creates an object of type `srsmatrix` as a product of a calling symmetric matrix and a number `d`. See also `srsmatrix::operator *=`, `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a,3);  
std::cout << m * 5.;
```

prints

```
5 10 15  
10 25 30  
15 30 45
```

2.13.33 operator / (TR)

Operator

```
srsmatrix srsmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `srsmatrix` as a quotient of a calling symmetric matrix and a number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the **smallest normalized positive number**. See also `srsmatrix::operator /=`, `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a,3);  
std::cout << m / 4.;
```

prints

```
0.25 0.5 0.75  
0.5 1.25 1.5  
0.75 1.5 2.25
```

2.13.34 operator *= (TR)

Operator

```
srsmatrix& srsmatrix::operator *= (TR d);
```

multiplies a calling symmetric matrix by a number d and returns a reference to the matrix changed. See also [srsmatrix::operator *](#) , [srsmatrix](#). Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a,3);  
m *= 2.;  
std::cout << m;
```

prints

```
2 4 6  
4 10 12  
6 12 18
```

2.13.35 operator /= (TR)

Operator

```
srsmatrix& srsmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling symmetric matrix by a number d and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if d has an absolute value equal or less than the `smallest normalized positive number`. See also `srsmatrix::operator /`, `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a,3);  
m /= 2.;  
std::cout << m;
```

prints

```
0.5 1 1.5  
1 2.5 3  
1.5 3 4.5
```

2.13.36 normalize

Function

```
srsmatrix& srsmatrix::normalize ();
```

normalizes a calling symmetric matrix so its **Euclidean norm** becomes equal to 1 if it was greater than the **smallest normalized positive number** before the call (otherwise the function does nothing). See also **srsmatrix**. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (3);  
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a,3);  
m.normalize();  
std::cout << m << m.norm() << std::endl;
```

prints

```
6.984e-002 1.397e-001 2.095e-001  
1.397e-001 3.492e-001 4.191e-001  
2.095e-001 4.191e-001 6.286e-001  
1.000e+000
```

2.13.37 transposition

Operator and functions

```
srsmatrix srsmatrix::operator ~ () const throw (cvmexception);  
srsmatrix& srsmatrix::transpose (const srsmatrix& m) throw (cvmexception);  
srsmatrix& srsmatrix::transpose ();
```

do nothing since a matrix calling is symmetric. They are provided to override the same member functions and operator of the class `srmatrix`. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a,3);  
std::cout << m - ~m;
```

prints

```
0 0 0  
0 0 0  
0 0 0
```

2.13.38 operator * (const rvector&)

Operator

```
rvector srsmatrix::operator * (const rvector& v) const  
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling symmetric matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `rvector::mult` in order to get rid of a new object creation. See also `srsmatrix` and `rvector`. Example:

```
using namespace cvm;
```

```
try {  
    srsmatrix m (4);  
    rvector v(4);  
    m.set(1.);  
    v.set(1.);  
  
    std::cout << m * v;  
}  
catch (exception& e) {  
    std::cout << "Exception: " << e.what () << std::endl;  
}
```

prints

```
4 4 4 4
```

2.13.39 operator * (const rmatrix&)

Operator

```
rmatrix srsmatrix::operator * (const rmatrix& m) const  
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling symmetric matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `rmatrix::mult` in order to get rid of a new object creation. See also `rmatrix` and `srsmatrix`. Example:

```
using namespace cvm;
```

```
try {  
    srsmatrix ms(4);  
    rmatrix m(4,2);  
    ms.set(1.);  
    m.set(2.);  
    std::cout << ms * m;  
}  
catch (exception& e) {  
    std::cout << "Exception: " << e.what () << std::endl;  
}
```

```
prints
```

```
8 8  
8 8  
8 8  
8 8
```

2.13.40 operator * (const srmatrix&)

Operator

```
srmatrix srsmatrix::operator * (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a product of a calling symmetric matrix and a matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `srmatrix::mult` in order to get rid of a new object creation. See also `srmatrix` and `srsmatrix`. Example:

```
using namespace cvm;
```

```
try {
    srsmatrix ms(3);
    srmatrix m(3);
    ms.set(1.);
    m.set(2.);
    std::cout << ms * m << std::endl;

    double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
    const srsmatrix ms2(a, 3);
    std::cout << ms2 * ms;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
6 6 6
6 6 6
6 6 6
```

```
6 6 6
13 13 13
18 18 18
```

2.13.41 syrk

Functions

```
srsmatrix&
srsmatrix::syrk (TR alpha, const rvector& v, TR beta)
throw (cvmexception);
```

```
srsmatrix&
srsmatrix::syrk (bool bTransp, TR alpha, const rmatrix& m, TR beta)
throw (cvmexception);
```

call one of ?SYRK routines of the [BLAS library](#) performing a matrix-vector operation defined for the first version as rank-1 update operation

$$C = \alpha v \cdot v' + \beta C,$$

and for the second version as

$$C = \alpha M \cdot M^T + \beta C \quad \text{or} \quad c = \alpha M^T \cdot M + \beta C.$$

Here α and β are real numbers (parameters alpha and beta), M is a matrix (parameter m), C is a calling symmetric matrix and v is a vector (parameter v). First operation for the second version of the function is performed if `bTransp` passed is `false` and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also [rvector](#), [rmatrix](#) and [srsmatrix](#) Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
rvector v(a,4);
srsmatrix ms(4);
ms.set(1.);
ms.syrk (2., v, 1.);
std::cout << ms << std::endl;

rmatrix m(4,2);
m(1) = v;
m(2).set(1.);
ms.syrk (false, 2., m, 0.);
std::cout << ms << std::endl;

srsmatrix ms2(2);
ms2.syrk (true, 1., m, 0.);
std::cout << ms2;
```

prints

```
3 5 7 9
5 9 13 17
7 13 19 25
9 17 25 33
```

```
4 6 8 10
6 10 14 18
8 14 20 26
10 18 26 34
```

```
30 10
10 4
```

2.13.42 syr2k

Functions

```
srsmatrix&
srsmatrix::syr2k (TR alpha, const rvector& v1,
                 const rvector& v2, TR beta) throw (cvmexception);
```

```
srsmatrix&
srsmatrix::syr2k (bool bTransp, TR alpha, const rmatrix& m1,
                 const rmatrix& m2, TR beta) throw (cvmexception);
```

call one of ?SYR2K routines of the [BLAS library](#) performing a matrix-vector operation defined for the first version as rank-1 update operation

$$C = \alpha v_1 \cdot v_2' + \alpha v_2 \cdot v_1' + \beta C,$$

and for the second version as

$$C = \alpha M_1 \cdot M_2' + \alpha M_2 \cdot M_1' + \beta C \quad \text{or} \quad C = \alpha M_1' \cdot M_2 + \alpha M_2' \cdot M_1 + \beta C.$$

Here α and β are real numbers (parameters alpha and beta), M_1 and M_2 are matrices (parameters m1 and m2), C is a calling symmetric matrix and v_1 and v_2 are vectors (parameters v1 and v2). First operation for the second version of the function is performed if bTransp passed is false and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rmatrix` and `srsmatrix` Example:

```
using namespace cvm;

double a1[] = {1., 2., 3., 4.};
double a2[] = {1., 2., 3., 4.};
rvector v1(a1,4);
rvector v2(a2,4);
srsmatrix ms(4);
ms.set(1.);
ms.syr2k (2., v1, v2, 1.);
std::cout << ms << std::endl;

rmatrix m1(4,2);
rmatrix m2(4,2);
m1.set(1.);
m2.set(2.);
ms.syr2k (false, 2., m1, m2, 0.);
std::cout << ms << std::endl;
```

```
srsmatrix ms2(2);  
ms2.syr2k (true, 1., m1, m2, 0.);  
std::cout << ms2;
```

prints

```
5 9 13 17  
9 17 25 33  
13 25 37 49  
17 33 49 65
```

```
16 16 16 16  
16 16 16 16  
16 16 16 16  
16 16 16 16
```

```
16 16  
16 16
```

2.13.43 inv

Functions

```
srsmatrix& srsmatrix::inv (const srsmatrix& m) throw (cvmexception);
srsmatrix srsmatrix::inv () const throw (cvmexception);
```

implement symmetric matrix inversion. The first version sets a calling symmetric matrix to be equal to a symmetric matrix `m` inverted and the second one creates an object of type `srsmatrix` as inverted calling matrix. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be inverted is close to singular. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.05};
const srsmatrix m(a, 3);
const srsmatrix mi = m.inv();

std::cout << mi << std::endl;
std::cout << mi * m - eye_real(3);
```

prints

```
1.85000e+002 -2.00000e+000 -6.00000e+001
-2.00000e+000 1.00000e+000 0.00000e+000
-6.00000e+001 0.00000e+000 2.00000e+001

0.00000e+000 0.00000e+000 0.00000e+000
0.00000e+000 0.00000e+000 0.00000e+000
0.00000e+000 0.00000e+000 0.00000e+000
```

2.13.44 exp

Functions

```
srsmatrix& srsmatrix::exp (const srsmatrix& m, TR tol = cvmMachSp())
throw (cvmexception);
```

```
srsmatrix srsmatrix::exp (TR tol = cvmMachSp()) const
throw (cvmexception);
```

compute an exponent of a calling symmetric matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \dots + z^p / p!,$$

where

$$N_{pq}(z) = \sum_{k=0}^p \frac{(p+q-k)! p!}{(p+q)! k! (q-k)!} z^k,$$

$$D_{pq}(z) = \sum_{k=0}^q \frac{(p+q-k)! p!}{(p+q)! k! (q-k)!} (-z)^k$$

along with the matrix normalizing as described in [2], p. 572. The functions use DMEXP (or SMEXP for float version) FORTRAN subroutine implementing the algorithm. The first version sets the calling symmetric matrix to be equal to the exponent of a symmetric matrix m and returns a reference to the matrix changed. The second version creates an object of type `srsmatrix` as the exponent of the calling matrix. The algorithm uses parameter `tol` as $\varepsilon(p, q)$ in order to choose constants p and q so that

$$\varepsilon(p, q) \geq 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the **largest relative spacing** by default. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or when LAPACK subroutine fails. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
```

```
double a[] = {1., 2., 1., 2., 0., -1., 1., -1., 2.};
const srsmatrix m(a, 3);
std::cout << m.exp();
```

```
prints
```

```
9.198262499129184e+000 5.558586002658855e+000 3.852443363622591e+000
5.558586002658857e+000 5.345819135506593e+000 -1.706142639036265e+000
3.852443363622590e+000 -1.706142639036266e+000 1.090440513816545e+001
```

Matlab output:

Columns 1 through 2

```
9.198262499129212e+000 5.558586002658862e+000
5.558586002658865e+000 5.345819135506588e+000
3.852443363622600e+000 -1.706142639036258e+000
```

Column 3

```
3.852443363622601e+000
-1.706142639036260e+000
1.090440513816545e+001
```

2.13.45 polynomial

Functions

```
srsmatrix& srsmatrix::polynom (const srsmatrix& m, const rvector& v)
throw (cvmexception);
```

```
srsmatrix srsmatrix::polynom (const rvector& v) const
throw (cvmexception);
```

compute a symmetric matrix polynomial defined as

$$p(A) = b_0I + b_1A + \cdots + b_qA^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^r B_k(A^s)^k, \quad s = \text{floor}(\sqrt{q}), \quad r = \text{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum_{i=0}^{s-1} b_{sk+i}A^i, & k = 0, 1, \dots, r-1 \\ \sum_{i=0}^{q-sr} b_{sr+i}A^i, & k = r. \end{cases}$$

See also [2], p. 568. The coefficients b_0, b_1, \dots, b_q are passed in the parameter v , where q is equal to $v.size()-1$, so the functions compute matrix polynomial equal to

$$v[1] * I + v[2] * m + \cdots + v[v.size()] * m^{v.size()-1}$$

The first version sets a calling symmetric matrix to be equal to the polynomial of a symmetric matrix m and the second one creates an object of type `srsmatrix` as the polynomial of a calling symmetric matrix. The functions use DPOLY (or SPOLY for float version) FORTRAN subroutine implementing the Horner's algorithm. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);
double a[] = {1., 2., 1., 2., 0., -1., 1., -1., 2.};
double av[] = {2.2, 1.3, 1.1, -0.9, 0.2,
              -0.45, 45, -30, 10, 3, 3.2};
const rvector v(av, 11);
const srsmatrix m(a, 3);

std::cout << m.polynom (v);
```

prints

```
6.2127400e+004 2.3998000e+004 3.4100550e+004
2.3998000e+004 2.8026850e+004 1.0102550e+004
3.4100550e+004 1.0102550e+004 5.2024850e+004
```

Matlab output:

Columns 1 through 2

```
6.2127400000000001e+004 2.399800000000000e+004
2.399800000000000e+004 2.802685000000000e+004
3.410055000000000e+004 1.010255000000000e+004
```

Column 3

```
3.410055000000000e+004
1.010255000000000e+004
5.202485000000000e+004
```

2.13.46 eig

Functions

```
rvector srsmatrix::eig (srmatrix& mEigVect) const throw (cvmexception);
rvector srsmatrix::eig () const throw (cvmexception);
```

solve a **symmetric eigenvalue problem** and return a real vector with eigenvalues of a calling symmetric matrix. The first version sets the output parameter `mEigVect` to be equal to the square matrix containing orthogonal eigenvectors as columns. All the functions throw an exception of type **cvmexception** in case of inappropriate sizes of the operands or in case of convergence error. See also **rvector**, **srmatrix** and **srsmatrix**. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (10);
double a[] = {1., 2., 1., 2., 0., -1., 1., -1., 2.};
const srsmatrix m(a, 3);
srmatrix me(3);
rvector v(3);

v = m.eig(me);
std::cout << v << std::endl;

std::cout << m * me(1) - me(1) * v(1);
std::cout << m * me(2) - me(2) * v(2);
std::cout << m * me(3) - me(3) * v(3);

prints

-2.0489173395e+000 2.3568958679e+000 2.6920214716e+000

4.4408920985e-016 0.0000000000e+000 5.5511151231e-016
-1.1102230246e-016 2.2204460493e-016 2.2204460493e-016
0.0000000000e+000 -1.1102230246e-016 -4.4408920985e-016
```

2.13.47 Cholesky

Function

```
srmatrix srsmatrix::cholesky () const throw (cvmexception);
```

forms the Cholesky factorization of a symmetric positive-definite matrix A defined as

$$A = U^T U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the [LAPACK library](#). The function creates an object of type `srmatrix` as the factorization of a calling matrix. The function throws an exception of type `cvmexception` in case of convergence error. See also `srmatrix` and `srsmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 2., 1., 2., 5., -1., 1., -1., 20.};
    const srsmatrix m(a, 3);

    srmatrix h = m.cholesky();
    std::cout << h << std::endl;
    std::cout << ~h * h - m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 1
0 1 -3
0 0 3.16228

0 0 0
0 0 0
0 0 0
```

2.13.48 Bunch-Kaufman

Function

```
srmatrix srmatrix::bunch_kaufman () throw (cvmexception);
```

forms the Bunch-Kaufman factorization of a calling symmetric matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is the calling matrix, P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D . It utilizes one of ?SYTRF routines of the [LAPACK library](#). The function creates an object of type `srmatrix` as the factorization of a calling matrix. The function throws an exception of type `cvmexception` in case of convergence error. See also [srmatrix](#) and [srmatrix](#). The function is mostly designed to be used for subsequent calls of ?SYTRS, ?SYCON and ?SYTRI routines of the [LAPACK library](#). Currently it's used internally in `srmatrix::det` flow when argument is symmetric but not positive-definite.

2.13.49 identity

Function

```
srsmatrix& srsmatrix::identity();
```

sets a calling symmetric matrix to be equal to identity matrix and returns a reference to the matrix changed. See also [srsmatrix](#). Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (3);  
srsmatrix m(3);  
m.randomize(0.,1.);
```

```
std::cout << m << std::endl;  
std::cout << m.identity();
```

prints

```
1.329e-001 8.527e-001 3.110e-001  
8.527e-001 6.152e-001 3.247e-001  
3.110e-001 3.247e-001 9.145e-001  
  
1.000e+000 0.000e+000 0.000e+000  
0.000e+000 1.000e+000 0.000e+000  
0.000e+000 0.000e+000 1.000e+000
```

2.13.50 vanish

Function

```
srsmatrix& srsmatrix::vanish();
```

sets every element of a calling symmetric matrix to be equal to zero and returns a reference to the matrix changed. See also [srsmatrix](#). Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (3);  
srsmatrix m(3);  
m.randomize(0.,1.);
```

```
std::cout << m << std::endl;  
std::cout << m.vanish();
```

prints

```
1.422e-001 1.477e-001 1.445e-001  
1.477e-001 8.893e-001 1.669e-002  
1.445e-001 1.669e-002 7.766e-001  
  
0.000e+000 0.000e+000 0.000e+000  
0.000e+000 0.000e+000 0.000e+000  
0.000e+000 0.000e+000 0.000e+000
```

2.13.51 `randomize`

Function

```
srsmatrix& srsmatrix::randomize (TR dFrom, TR dTo);
```

fills a calling symmetric matrix with pseudo-random numbers distributed between `dFrom` and `dTo` keeping it to be symmetric. The function returns a reference to the matrix changed. See also `srsmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (7);
```

```
srsmatrix m(3);  
m.randomize(-2.,3.);  
std::cout << m;
```

prints

```
-1.2277291e+000 3.6610004e-001 2.1380047e+000  
3.6610004e-001 1.0336924e+000 -1.8565630e+000  
2.1380047e+000 -1.8565630e+000 1.7774285e+000
```

2.14 schmatrix

This is end-user class encapsulating a hermitian matrix in Euclidean space of complex numbers.

```

template <typename TR, typename TC>
class schmatrix : public scmatrix <TR,TC> {
public:
    schmatrix ();
    explicit schmatrix (int nMN);
    schmatrix (TC* pD, int nMN, TR tol = cvmMachSp());
    schmatrix (const TC* pD, int nMN, TR tol = cvmMachSp());
    schmatrix (const schmatrix& m);
    explicit schmatrix (const cmatrix& m, TR tol = cvmMachSp());
    explicit schmatrix (const rvector& v);
    explicit schmatrix (const srmatrix& m);
    schmatrix (const TR* pRe, const TR* pIm, int nMN,
                TR tol = cvmMachSp());
    schmatrix (const srmatrix& mRe, const srmatrix& mIm,
                TR tol = cvmMachSp());
    schmatrix (schmatrix& m, int nRowCol, int nSize);
    TC operator () (int im, int in) const throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    const cvector diag (int i) const throw (cvmexception);
    const srmatrix real () const;
    const srmatrix imag () const;
    schmatrix& operator = (const schmatrix& m) throw (cvmexception);
    schmatrix& assign (const cvector& v, TR tol = cvmMachSp())
                    throw (cvmexception);
    schmatrix& assign (const TC* pD, TR tol = cvmMachSp())
                    throw (cvmexception);
    schmatrix& assign (int nRowCol, const schmatrix& m)
                    throw (cvmexception);
    schmatrix& set (int nRow, int nCol, TC z);
    schmatrix& set_diag (int i, const cvector& v) throw (cvmexception);
    schmatrix& set_main_diag (const rvector& v) throw (cvmexception);
    schmatrix& assign_real (const srmatrix& m) throw (cvmexception);
    schmatrix& set_real (TR d) throw (cvmexception);
    schmatrix& resize (int nNewMN) throw (cvmexception);
    bool operator == (const schmatrix& v) const;
    bool operator != (const schmatrix& v) const;
    schmatrix& operator << (const schmatrix& m) throw (cvmexception);

```

```

schmatrix operator + (const schmatrix& m) const
    throw (cvmexception);
schmatrix operator - (const schmatrix& m) const
    throw (cvmexception);
schmatrix& sum (const schmatrix& m1,
    const schmatrix& m2) throw (cvmexception);
schmatrix& diff (const schmatrix& m1,
    const schmatrix& m2) throw (cvmexception);
schmatrix& operator += (const schmatrix& m) throw (cvmexception);
schmatrix& operator -= (const schmatrix& m) throw (cvmexception);
schmatrix operator - () const;
schmatrix& operator ++ ();
schmatrix& operator ++ (int);
schmatrix& operator -- ();
schmatrix& operator -- (int);
schmatrix operator * (TR d) const;
schmatrix operator / (TR d) const throw (cvmexception);
scmatrix operator * (TC z) const;
scmatrix operator / (TC z) const throw (cvmexception);
schmatrix& operator *= (TR d);
schmatrix& operator /= (TR d) throw (cvmexception);
schmatrix& normalize ();
schmatrix operator ~ () const;
schmatrix& conj (const schmatrix& m) throw (cvmexception);
schmatrix& conj ();
cvector operator * (const cvector& v) const throw (cvmexception);
cmatrix operator * (const cmatrix& m) const throw (cvmexception);
scmatrix operator * (const scmatrix& m) const throw (cvmexception);
schmatrix& herk (TC alpha,
    const cvector& v, TC beta) throw (cvmexception);
schmatrix& herk (bool bTransp, TC alpha,
    const cmatrix& m, TC beta) throw (cvmexception);
schmatrix& her2k (TC alpha,
    const cvector& v1, const cvector& v2, TC beta)
    throw (cvmexception);
schmatrix& her2k (bool bTransp, TC alpha,
    const cmatrix& m1, const cmatrix& m2, TC beta)
    throw (cvmexception);
schmatrix& inv (const schmatrix& mArg) throw (cvmexception);
schmatrix inv () const throw (cvmexception);
schmatrix& exp (const schmatrix& m,
    TR tol = cvmMachSp ()) throw (cvmexception);
schmatrix exp (TR tol = cvmMachSp ()) const throw (cvmexception);

```

```
schmatrix& polynom (const schmatrix& m, const rvector& v)
    throw (cvmexception);
schmatrix polynom (const rvector& v) const throw (cvmexception);
rvector eig (scmatrix& mEigVect) const throw (cvmexception);
rvector eig () const throw (cvmexception);
scmatrix cholesky () const throw (cvmexception);
scmatrix bunch_kaufman () const throw (cvmexception);
schmatrix& identity ();
schmatrix& vanish ();
schmatrix& randomize_real (TR dFrom, TR dTo);
schmatrix& randomize_imag (TR dFrom, TR dTo);
};
```

2.14.1 schmatrix ()

Constructor

```
schmatrix::schmatrix ();
```

creates an empty schmatrix object. See also [schmatrix](#). Example:

```
using namespace cvm;
```

```
schmatrix m;  
std::cout << m.msize() << " " << m.nsize() << " "  
           << m.size() << std::endl;
```

```
m.resize (3);  
std::cout << m;
```

prints

```
0 0 0  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)
```

2.14.2 schmatrix (int)

Constructor

```
explicit schmatrix::schmatrix (int nMN);
```

creates an $n \times n$ schmatrix object where n is passed in nMN parameter. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
schmatrix m (4);  
std::cout << m.msize() << std::endl  
          << m.nsize() << std::endl  
          << m.size() << std::endl << m;
```

prints

```
4  
4  
16  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (0,0)
```

2.14.3 schmatrix (TC*,int)

Constructor

```
schmatrix::schmatrix (TC* pD, int nMN, TR tol = cvmMachSp());
```

creates an $n \times n$ `schmatrix` object where n is passed in `nMN` parameter. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD`. The constructor throws an exception of type `cvmexception` if the matrix created doesn't appear to be hermitian (tolerance is set by parameter `tol`) or in case of non-positive size passed. If subsequent application flow would change the array passed so it becomes not a hermitian matrix anymore then results are not predictable. See also `schmatrix`, `schmatrix (const TC*,int)`. Example:

```
using namespace cvm;
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
m.set(2,1,std::complex<double>(8.,8.));
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(1,0) (8,-8) (-1,-2)
(8,8) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

```
1 0 8 8 -1 2
```

2.14.4 schmatrix (const TC*,int)

Constructor

```
schmatrix::schmatrix (const TC* pD, int nMN, TR tol = cvmMachSp());
```

creates an $n \times n$ `schmatrix` object where n is passed in `nMN` parameter and copies $n * n$ elements of an array `pD` to it by `columns`. The constructor throws an exception of type `cvmexception` if the matrix created doesn't appear to be hermitian (tolerance is set by parameter `tol`) or in case of non-positive size passed. See also `schmatrix`, `schmatrix (TC*,int)`. Example:

```
using namespace cvm;
const double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
                   0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((const std::complex<double>*)a, 3);
m.set(2,1,std::complex<double>(8.,8.));
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(1,0) (8,-8) (-1,-2)
(8,8) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

```
1 0 2 1 -1 2
```

2.14.5 schmatrix (const schmatrix&)

Copy constructor

`schmatrix::schmatrix (const schmatrix& m)`

creates a `schmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left |
                std::ios::showpos);
```

```
std::cout.precision (1);
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
```

```
schmatrix m ((std::complex<double>*)a, 3);
```

```
schmatrix mc(m);
```

```
m.set(1,2, std::complex<double>(7.7,7.7));
```

```
std::cout << m << std::endl << mc;
```

prints

```
(+1.0e+000,+0.0e+000) (+7.7e+000,+7.7e+000) (-1.0e+000,-2.0e+000)
(+7.7e+000,-7.7e+000) (+2.0e+000,+0.0e+000) (+0.0e+000,-3.0e+000)
(-1.0e+000,+2.0e+000) (+0.0e+000,+3.0e+000) (+3.0e+000,+0.0e+000)
```

```
(+1.0e+000,+0.0e+000) (+2.0e+000,-1.0e+000) (-1.0e+000,-2.0e+000)
(+2.0e+000,+1.0e+000) (+2.0e+000,+0.0e+000) (+0.0e+000,-3.0e+000)
(-1.0e+000,+2.0e+000) (+0.0e+000,+3.0e+000) (+3.0e+000,+0.0e+000)
```

2.14.6 schmatrix (const cmatrix&)

Constructor

```
explicit schmatrix::schmatrix (const cmatrix& m, TR tol = cvmMachSp())
```

creates a schmatrix object as a copy of matrix m. It's assumed that $m \times n$ matrix m must have equal sizes, i.e. $m = n$ is satisfied and it has to be a hermitian one (tolerance is set by parameter tol). The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. See also `schmatrix` and `cmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
cmatrix m ((std::complex<double>*)a, 3, 3);  
schmatrix mch(m);  
std::cout << mch;
```

prints

```
(1,0) (2,-1) (-1,-2)  
(2,1) (2,0) (0,-3)  
(-1,2) (0,3) (3,0)
```

2.14.7 schmatrix (const rvector&)

Constructor

```
explicit schmatrix::schmatrix (const rvector& v);
```

creates a schmatrix object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `schmatrix` and `rvector`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 4., 5.};  
const rvector v (a, 5);  
schmatrix m(v);  
std::cout << m.msize() << " " << m.nsize() << " "  
          << m.size() << std::endl << m;
```

prints

```
5 5 25  
(1,0) (0,0) (0,0) (0,0) (0,0)  
(0,0) (2,0) (0,0) (0,0) (0,0)  
(0,0) (0,0) (3,0) (0,0) (0,0)  
(0,0) (0,0) (0,0) (4,0) (0,0)  
(0,0) (0,0) (0,0) (0,0) (5,0)
```

2.14.8 schmatrix (const srsmatrix&)

Constructor

```
explicit schmatrix::schmatrix (const srsmatrix& m);
```

creates a schmatrix object having the same dimension as real symmetric matrix m and copies the matrix m to its real part. See also [schmatrix](#) and [srsmatrix](#). Example:

```
using namespace cvm;
```

```
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};  
srsmatrix m(a, 3);  
schmatrix mch(m);  
std::cout << mch;
```

prints

```
(1,0) (2,0) (3,0)  
(2,0) (5,0) (6,0)  
(3,0) (6,0) (9,0)
```

2.14.9 schmatrix (const TR*,const TR*,int)

Constructor

```
schmatrix::schmatrix (const TR* pRe, const TR1* pIm, int nMN,  
                     TR tol = cvmMachSp());
```

creates a schmatrix object of size nMN by nMN and copies every element of arrays pointed to by pRe and pIm to a real and imaginary part of the matrix created respectively. Use NULL pointer to fill up appropriate part with zero values. The constructor throws an exception of type `cvexception` if the matrix created doesn't appear to be hermitian (tolerance is et by parameter tol) or in case of memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double re[] = {1., 2., -1., 2., 2., 0., -1., 0., 3.};  
double im[] = {0., 1., 2., -1., 0., 3., -2., -3., 0.};  
schmatrix m(re, im, 3);  
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)  
(2,1) (2,0) (0,-3)  
(-1,2) (0,3) (3,0)
```

2.14.10 schmatrix (const srmatrix&, const srmatrix&)

Constructor

```
schmatrix::schmatrix (const srmatrix& mRe, const srmatrix& mIm,  
                     TR tol = cvmMachSp());
```

creates a schmatrix object of the same size as mRe and mIm has (the constructor throws an exception of type `cvmexception` if mRe and mIm have different sizes) and copies matrices mRe and mIm to a real and imaginary part of the matrix created respectively. The constructor throws an exception of type `cvmexception` if the matrix created doesn't appear to be hermitian (tolerance is et by parameter tol) or in case of memory allocation failure. See also `schmatrix`, `srmatrix`. Example:

```
using namespace cvm;
```

```
double re[] = {1., 2., -1., 2., 2., 0., -1., 0., 3.};  
double im[] = {0., 1., 2., -1., 0., 3., -2., -3., 0.};  
srmatrix mr(re, 3);  
srmatrix mi(im, 3);  
schmatrix m(mr, mi);  
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)  
(2,1) (2,0) (0,-3)  
(-1,2) (0,3) (3,0)
```

2.14.11 submatrix

Submatrix constructor

```
schmatrix::schmatrix (schmatrix& m, int nRowCol, int nSize);
```

creates a `schmatrix` object as a *submatrix* of `m`. It means that the matrix object created shares a memory with some part of `m`. This part is defined by its upper left corner (parameter `nRowCol`, *ℓ-based*) and its size (parameter `nSize`). See also [schmatrix](#). Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
schmatrix m2 (m, 1, 2);
```

```
std::cout << m2 << std::endl;
m2.set_real(7.7);
std::cout << m;
```

prints

```
(1,0) (2,-1)
(2,1) (2,0)
```

```
(7.7,0) (7.7,-1) (-1,-2)
(7.7,1) (7.7,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.12 operator (,)

Indexing operator

```
TC schmatrix::operator () (int im, int in) const throw (cvmexception);
```

returns an element of a hermitian matrix located on *im*-row and *in*-th column. The operator is *l*-based. The operator throws an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
```

```
schmatrix m ((std::complex<double>*)a, 3);
```

```
std::cout << m << std::endl;
```

```
std::cout << m(3,2) << " " << m(1,3) << std::endl;
```

prints

```
(1,0) (2,-1) (-1,-2)
```

```
(2,1) (2,0) (0,-3)
```

```
(-1,2) (0,3) (3,0)
```

```
(0,3) (-1,-2)
```

2.14.13 operator ()

Indexing operator

```
const cvector schmatrix::operator () (int i) const throw (cvmexception);
```

provides access to an i -th column of a hermitian matrix. Unlike `schmatrix::operator ()`, this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is *l-based*. The operator throws an exception of type `cvmexception` if the parameter i is outside of $[1, \text{nsize}()]$ range. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m(1) << std::endl << m(3);
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

```
(1,0) (2,1) (-1,2)
```

```
(-1,-2) (0,-3) (3,0)
```

2.14.14 operator []

Indexing operator

```
const cvector schmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to an *i*-th row of a hermitian matrix. Unlike `schmatrix::operator []`, this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is *l-based*. The operator throws an exception of type `cvmexception` if the parameter *i* is outside of `[1, nsize()]` range. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m[1] << std::endl << m[3];
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

```
(1,0) (2,-1) (-1,-2)
```

```
(-1,2) (0,3) (3,0)
```

2.14.15 diag

Functions

```
const cvector schmatrix::diag (int i) const throw (cvmexception);
```

provide access to an i -th diagonal of a matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. Unlike `cmatrix::diag`, this operator creates only a *copy* of a diagonal and therefore it returns *not an l-value*. The function throws an exception of type `cvmexception` if the parameter i is outside of $[-m\text{size}()+1, n\text{size}()-1]$ range. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m.diag(-2)
          << m.diag(-1)
          << m.diag(0)
          << m.diag(1)
          << m.diag(2);
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(-1,2)
(2,1) (0,3)
(1,0) (2,0) (3,0)
(2,-1) (0,-3)
(-1,-2)
```

2.14.16 real

Function

```
const srsmatrix schmatrix::real () const;
```

creates an object of type `const srsmatrix` as a real part of a calling hermitian matrix. Please note that, unlike `cvector::real`, this function creates new object *not sharing* a memory with a real part of the calling matrix, i.e. the matrix returned is *not an l-value*. See also `srsmatrix`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m.real();
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

```
1 2 -1
2 2 0
-1 0 3
```

2.14.17 imag

Function

```
const srmatrix schmatrix::imag () const;
```

creates an object of type `const srmatrix` as an imaginary part of a calling matrix. Please note that, unlike `cvvector::imag`, this function creates new object *not sharing* a memory with an imaginary part of the calling matrix, i.e. the matrix returned is *not an l-value*. See also `srmatrix`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m ((std::complex<double>*)a, 3);  
std::cout << m << std::endl;  
std::cout << m.imag();
```

prints

```
(1,0) (2,-1) (-1,-2)  
(2,1) (2,0) (0,-3)  
(-1,2) (0,3) (3,0)
```

```
0 -1 -2  
1 0 -3  
2 3 0
```

2.14.18 operator = (const schmatrix&)

Operator

```
schmatrix& schmatrix::operator = (const schmatrix& m)
throw (cvmexception);
```

sets an every element of a calling hermitian matrix to a value of appropriate element of a hermitian matrix *m* and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different matrix sizes. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
                  0., 3., -1., -2., 0., -3., 3., 0.};
    schmatrix m1((std::complex<double>*)a, 3);
    schmatrix m2(3);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
}
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.19 assign (const TC*)

Function

```
schmatrix& schmatrix::assign (const cvector& v, TR tol = cvmMachSp())
throw (cvmexception);
```

```
schmatrix& schmatrix::assign (const TC* pD, TR tol = cvmMachSp())
throw (cvmexception);
```

sets every element of a calling hermitian matrix to a value of appropriate element of a vector *v* or an array pointed to by *pD* and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if the matrix changed doesn't appear to be hermitian (tolerance is set by parameter *tol*). See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
```

```
schmatrix m(3);
m.assign((std::complex<double>*)a);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.20 assign (int, int, const schmatrix&)

Function

```
schmatrix& schmatrix::assign (int nRowCol, const schmatrix& m)
throw (cvmexception);
```

sets main sub-matrix of a calling hermitian matrix beginning with ℓ -based row nRowCol to a hermitian matrix m and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if nRowCol is not positive or matrix m doesn't fit. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
schmatrix m1(5);
schmatrix m2(2);
m2.set_main_diag(rvector(2,2.));
m2.set(1,2,std::complex<double>(2.,2.));
m1.assign(2,m2);
std::cout << m1;
```

prints

```
(0,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (2,0) (2,2) (0,0) (0,0)
(0,0) (2,-2) (2,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (0,0)
```

2.14.21 set (int,int,TC)

Function

```
schmatrix& schmatrix::set (int nRow, int nCol, TC z) throw (cvmexception);
```

sets an element located on nRow's row and nCol's column to a value of parameter z and an element located on nCol's row and nRow's column to a value of parameter z conjugated (keeping a calling matrix to be hermitian). The parameters passed are *l*-based. The function returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if anyone of the parameters passed is outside of [1,msize()] range. It also throws the exception in case of assigning a complex number with non-zero imaginary part to any element of the main diagonal. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
```

```
m.set(1,2,std::complex<double>(7.7,7.7));
m.set(3,3,11.11);
std::cout << m;
```

prints

```
(1,0) (7.7,7.7) (-1,-2)
(7.7,-7.7) (2,0) (0,-3)
(-1,2) (0,3) (11.11,0)
```

2.14.22 set_diag

Function

```
schmatrix& schmatrix::set_diag (int i, const cvector& v)
throw (cvmexception);
```

sets an i -th diagonal of a calling hermitian matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones, to be equal to a complex vector passed in parameter v . The function also sets $-i$ -th diagonal to be equal to the vector v conjugated, thus keeping the matrix to be hermitian. The parameter i passed is ℓ -based. The function returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if the parameter i passed is outside of $[-m\text{size}()+1, n\text{size}()-1]$ range or equal to zero. It also throws the exception if the vector v passed has a size not equal to $m\text{size}()-\text{abs}(i)$. Use `schmatrix::set_main_diag` to set the main diagonal. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
cvector v(2);
v.set(std::complex<double>(7.7,7.7));
m.set_diag(1, v);
std::cout << m;
```

prints

```
(1,0) (7.7,7.7) (-1,-2)
(7.7,-7.7) (2,0) (7.7,7.7)
(-1,2) (7.7,-7.7) (3,0)
```

2.14.23 set_main_diag

Function

```
schmatrix& schmatrix::set_main_diag (const rvector& v)
throw (cvmexception);
```

sets the main diagonal of a calling hermitian matrix to be equal to a real vector passed in parameter *v*. The function returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if the vector *v* passed has a size not equal to `msize()`. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
rvector v(3);
v.set(7.7);
m.set_main_diag(v);
std::cout << m;
```

prints

```
(7.7,0) (2,-1) (-1,-2)
(2,1) (7.7,0) (0,-3)
(-1,2) (0,3) (7.7,0)
```

2.14.24 assign_real

Function

```
schmatrix& schmatrix::assign_real (const srsmatrix& m)
throw (cvmexception);
```

sets real part of a calling hermitian matrix to be equal to a real symmetric matrix *m*. The function returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
srsmatrix ms(3);
ms.set(7.);
m.assign_real(ms);
std::cout << m;
```

prints

```
(7,0) (7,-1) (7,-2)
(7,1) (7,0) (7,-3)
(7,2) (7,3) (7,0)
```

2.14.25 set_real

Function

```
schmatrix& schmatrix::set_real (TR d);
```

sets every element of a real part of a calling hermitian matrix to be equal to a real number d. The function returns a reference to the matrix changed. See also [schmatrix](#). Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m((std::complex<double>*)a,3);  
m.set_real(7.);  
std::cout << m;
```

prints

```
(7,0) (7,-1) (7,-2)  
(7,1) (7,0) (7,-3)  
(7,2) (7,3) (7,0)
```

2.14.26 resize

Function

```
schmatrix& schmatrix::resize (int nNewMN) throw (cvmexception);
```

changes a size of a calling hermitian matrix to nNewMN by nNewMN and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m << std::endl;
m.resize(4);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

```
(1,0) (2,-1) (-1,-2) (0,0)
(2,1) (2,0) (0,-3) (0,0)
(-1,2) (0,3) (3,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.14.27 operator ==

Operator

```
bool schmatrix::operator == (const schmatrix& m) const;
```

compares a calling hermitian matrix with a hermitian matrix `m` and returns true if they have the same sizes and their appropriate elements differ by not more than the **smallest normalized positive number**. Returns false otherwise. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m1((std::complex<double>*)a,3);  
schmatrix m2(3);  
m2.assign((std::complex<double>*)a);  
std::cout << (m1 == m2) << std::endl;
```

prints

1

2.14.28 operator !=

Operator

```
bool schmatrix::operator != (const schmatrix& m) const;
```

compares a calling hermitian matrix with a hermitian matrix `m` and returns true if they have different sizes or at least one of their appropriate elements differs by more than the **smallest normalized positive number**. Returns false otherwise. See also `schmatrix`.

Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m1((std::complex<double>*)a,3);  
schmatrix m2(3);  
m2.assign((std::complex<double>*)a);  
m2.set(2,1,std::complex<double>(2.,1.000001));  
std::cout << (m1 != m2) << std::endl;
```

prints

```
1
```

2.14.29 operator <<

Operator

```
schmatrix& schmatrix::operator << (const schmatrix& m)
throw (cvmexception);
```

destroys a calling hermitian matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
schmatrix mc(1);
std::cout << m << std::endl;
std::cout << mc << std::endl;
mc << m;
std::cout << mc;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

```
(0,0)
```

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.30 operator +

Operator

```
schmatrix schmatrix::operator + (const schmatrix& m) const  
throw (cvmexception);
```

creates an object of type `schmatrix` as a sum of a calling hermitian matrix and a hermitian matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `schmatrix::sum`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,  
              1., 1., 1., -1., 1., -1., 1., 0.};  
schmatrix m1((std::complex<double>*)a,3);  
schmatrix m2((std::complex<double>*)b,3);  
std::cout << m1 + m2;
```

prints

```
(2,0) (3,-2) (0,-3)  
(3,2) (3,0) (1,-4)  
(0,3) (1,4) (4,0)
```

2.14.31 operator -

Operator

```
schmatrix schmatrix::operator - (const schmatrix& m) const  
throw (cvmexception);
```

creates an object of type `schmatrix` as a difference of a calling hermitian matrix and a hermitian matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `schmatrix::diff`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,  
              1., 1., 1., -1., 1., -1., 1., 0.};  
schmatrix m1((std::complex<double>*)a,3);  
schmatrix m2((std::complex<double>*)b,3);  
std::cout << m1 - m2;
```

prints

```
(0,0) (1,0) (-2,-1)  
(1,0) (1,0) (-1,-2)  
(-2,1) (-1,2) (2,0)
```

2.14.32 sum

Function

```
schmatrix& schmatrix::sum (const schmatrix& m1, const schmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of hermitian matrices m1 and m2 to a calling hermitian matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `schmatrix::operator +`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
schmatrix m(3);
std::cout << m.sum(m1, m2);
```

prints

```
(2,0) (3,-2) (0,-3)
(3,2) (3,0) (1,-4)
(0,3) (1,4) (4,0)
```

2.14.33 diff

Function

```
schmatrix& schmatrix::diff (const schmatrix& m1, const schmatrix& m2)  
throw (cvmexception);
```

assigns a result of subtraction of hermitian matrices m1 and m2 to a calling hermitian matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `schmatrix::operator -`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,  
              1., 1., 1., -1., 1., -1., 1., 0.};  
schmatrix m1((std::complex<double>*)a,3);  
schmatrix m2((std::complex<double>*)b,3);  
schmatrix m(3);  
std::cout << m.diff(m1, m2);
```

prints

```
(0,0) (1,0) (-2,-1)  
(1,0) (1,0) (-1,-2)  
(-2,1) (-1,2) (2,0)
```

2.14.34 operator +=

Operator

```
schmatrix& schmatrix::operator += (const schmatrix& m)
throw (cvmexception);
```

adds a hermitian matrix *m* to a calling hermitian matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `schmatrix::operator +`, `schmatrix::sum`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
m1 += m2;
m2 += m2;
std::cout << m1 << std::endl;
std::cout << m2;
```

prints

```
(2,0) (3,-2) (0,-3)
(3,2) (3,0) (1,-4)
(0,3) (1,4) (4,0)
```

```
(2,0) (2,-2) (2,-2)
(2,2) (2,0) (2,-2)
(2,2) (2,2) (2,0)
```

2.14.35 operator -=

Operator

```
schmatrix& schmatrix::operator -= (const schmatrix& m)
throw (cvmexception);
```

subtracts a hermitian matrix `m` from a calling hermitian matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `schmatrix::operator -`, `schmatrix::diff`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
m1 -= m2;
m2 -= m2;
std::cout << m1 << std::endl;
std::cout << m2;
```

prints

```
(0,0) (1,0) (-2,-1)
(1,0) (1,0) (-1,-2)
(-2,1) (-1,2) (2,0)
```

```
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.36 operator - ()

Operator

```
schmatrix schmatrix::operator - () const throw (cvmexception);
```

creates an object of type `schmatrix` as a calling hermitian matrix multiplied by -1 . See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m((std::complex<double>*)a,3);  
std::cout << -m;
```

prints

```
(-1,0) (-2,1) (1,2)  
(-2,-1) (-2,0) (0,3)  
(1,-2) (0,-3) (-3,0)
```

2.14.37 operator ++

Operator

```
schmatrix& schmatrix::operator ++ ();
schmatrix& schmatrix::operator ++ (int);
```

adds identity matrix to a calling hermitian matrix and returns a reference to the matrix changed. See also [schmatrix](#). Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
```

```
schmatrix m((std::complex<double>*)a,3);
```

```
m++;
```

```
std::cout << m << std::endl;
```

```
std::cout << ++m;
```

prints

```
(2,0) (2,-1) (-1,-2)
```

```
(2,1) (3,0) (0,-3)
```

```
(-1,2) (0,3) (4,0)
```

```
(3,0) (2,-1) (-1,-2)
```

```
(2,1) (4,0) (0,-3)
```

```
(-1,2) (0,3) (5,0)
```

2.14.38 operator --

Operator

```
schmatrix& schmatrix::operator -- ();  
schmatrix& schmatrix::operator -- (int);
```

subtracts identity matrix from a calling hermitian matrix and returns a reference to the matrix changed. See also [schmatrix](#). Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m((std::complex<double>*)a,3);  
m--;  
std::cout << m << std::endl;  
std::cout << --m;
```

prints

```
(0,0) (2,-1) (-1,-2)  
(2,1) (1,0) (0,-3)  
(-1,2) (0,3) (2,0)  
  
(-1,0) (2,-1) (-1,-2)  
(2,1) (0,0) (0,-3)  
(-1,2) (0,3) (1,0)
```

2.14.39 operator * (TR)

Operator

```
schmatrix schmatrix::operator * (TR d) const;
```

creates an object of type `schmatrix` as a product of a calling hermitian matrix and a real number `d`. See also `schmatrix::operator *=`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m((std::complex<double>*)a,3);  
std::cout << m * 5.;
```

prints

```
(5,0) (10,-5) (-5,-10)  
(10,5) (10,0) (0,-15)  
(-5,10) (0,15) (15,0)
```

2.14.40 operator / (TR)

Operator

```
schmatrix schmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `schmatrix` as a quotient of a calling hermitian matrix and a real number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the **smallest normalized positive number**. See also `schmatrix::operator /=`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
             0., 3., -1., -2., 0., -3., 3., 0.};
```

```
schmatrix m((std::complex<double>*)a,3);
```

```
std::cout << m / 2.;
```

prints

```
(0.5,0) (1,-0.5) (-0.5,-1)
```

```
(1,0.5) (1,0) (0,-1.5)
```

```
(-0.5,1) (0,1.5) (1.5,0)
```

2.14.41 operator * (TC)

Operator

```
schmatrix schmatrix::operator * (TC z) const;
```

creates an object of type `schmatrix` as a product of a calling hermitian matrix and a complex number `z`. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m((std::complex<double>*)a,3);  
std::cout << m * std::complex<double>(1.,1.);
```

prints

```
(1,1) (3,1) (1,-3)  
(1,3) (2,2) (3,-3)  
(-3,1) (-3,3) (3,3)
```

2.14.42 operator / (TC)

Operator

```
schmatrix schmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `schmatrix` as a quotient of a calling hermitian matrix and a complex number `z`. It throws an exception of type `cvmexception` if `z` has an absolute value equal or less than the `smallest normalized positive number`. See also `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m((std::complex<double>*)a,3);  
std::cout << m / std::complex<double>(1.,1.);
```

prints

```
(0.5,-0.5) (0.5,-1.5) (-1.5,-0.5)  
(1.5,-0.5) (1,-1) (-1.5,-1.5)  
(0.5,1.5) (1.5,1.5) (1.5,-1.5)
```

2.14.43 operator *= (TR)

Operator

```
schmatrix& schmatrix::operator *= (TR d);
```

multiplies a calling hermitian matrix by a real number d and returns a reference to the matrix changed. See also `schmatrix::operator *`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m((std::complex<double>*)a,3);  
m *= 5.;  
std::cout << m;
```

prints

```
(5,0) (10,-5) (-5,-10)  
(10,5) (10,0) (0,-15)  
(-5,10) (0,15) (15,0)
```

2.14.44 operator /= (TR)

Operator

```
schmatrix& schmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling hermitian matrix by a real number d and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if d has an absolute value equal or less than the `smallest normalized positive number`. See also `schmatrix::operator /`, `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};
```

```
schmatrix m((std::complex<double>*)a,3);
```

```
m /= 2.;
```

```
std::cout << m;
```

prints

```
(0.5,0) (1,-0.5) (-0.5,-1)
```

```
(1,0.5) (1,0) (0,-1.5)
```

```
(-0.5,1) (0,1.5) (1.5,0)
```

2.14.45 normalize

Function

```
schmatrix& schmatrix::normalize ();
```

normalizes a calling hermitian matrix so its **Euclidean norm** becomes equal to 1 if it was greater than the **smallest normalized positive number** before the call (otherwise the function does nothing). See also **schmatrix**. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);  
std::cout.precision (2);  
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m((std::complex<double>*)a,3);
```

```
m.normalize();  
std::cout << m << m.norm() << std::endl;
```

prints

```
(1.39e-001,0.00e+000) (2.77e-001,-1.39e-001) (-1.39e-001,-2.77e-001)  
(2.77e-001,1.39e-001) (2.77e-001,0.00e+000) (0.00e+000,-4.16e-001)  
(-1.39e-001,2.77e-001) (0.00e+000,4.16e-001) (4.16e-001,0.00e+000)  
1.00e+000
```

2.14.46 conjugation

Operator and functions

```
schmatrix schmatrix::operator ~ () const throw (cvmexception);  
schmatrix& schmatrix::conj (const schmatrix& m) throw (cvmexception);  
schmatrix& schmatrix::conj () throw (cvmexception);
```

do nothing since a matrix calling is hermitian. They are provided to override the same member functions and operator of the class `schmatrix`. See also `schmatrix`. Example:

```
using namespace cvm;  
  
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,  
              0., 3., -1., -2., 0., -3., 3., 0.};  
schmatrix m((std::complex<double>*)a,3);  
  
std::cout << m - ~m;  
  
prints  
  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)
```

2.14.47 operator * (const cvector&)

Operator

```
cvector schmatrix::operator * (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling hermitian matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `cvector::mult` in order to get rid of a new object creation. See also `schmatrix` and `cvector`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
try {
    schmatrix m((std::complex<double>*)a,3);
    cvector v(3);
    v.set(std::complex<double>(1.,1.));

    std::cout << m * v;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(5,-1) (6,2) (-3,7)
```

2.14.48 operator * (const cmatrix&)

Operator

```
cmatrix schmatrix::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as a product of a calling hermitian matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `cmatrix::mult` in order to get rid of a new object creation. See also `cmatrix` and `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
try {
    schmatrix ms((std::complex<double>*)a,3);
    cmatrix m(3,2);
    m.set(std::complex<double>(1.,1.));

    std::cout << ms * m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(5,-1) (5,-1)
(6,2) (6,2)
(-3,7) (-3,7)
```

2.14.49 operator * (const schmatrix&)

Operator

```
schmatrix schmatrix::operator * (const schmatrix& m) const
throw (cvmexception);
```

creates an object of type `schmatrix` as a product of a calling hermitian matrix and a matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `cmatrix::mult` in order to get rid of a new object creation. See also `cmatrix` and `schmatrix`. Example:

```
using namespace cvm;
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
```

```
try {
    schmatrix ms((std::complex<double>*)a,3);
    schmatrix m(3);
    m.set(std::complex<double>(1.,1.));

    std::cout << ms * m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(5,-1) (5,-1) (5,-1)
(6,2) (6,2) (6,2)
(-3,7) (-3,7) (-3,7)
```

2.14.50 herk

Functions

```
schmatrix&
schmatrix::herk (TC alpha, const cvector& v, TC beta)
throw (cvmexception);
```

```
schmatrix&
schmatrix::herk (bool bTransp, TC alpha, const cmatrix& m, TC beta)
throw (cvmexception);
```

call one of ?HERK routines of the [BLAS library](#) performing a matrix-vector operation defined for the first version as rank-1 update operation

$$C = \alpha v \cdot v' + \beta C,$$

and for the second version as

$$C = \alpha M \cdot M^H + \beta C \quad \text{or} \quad C = \alpha M^H \cdot M + \beta C.$$

Here α and β are complex numbers (parameters alpha and beta), M is a complex matrix (parameter m), C is a calling hermitian matrix and v is a complex vector (parameter v). First operation for the second version of the function is performed if bTransp passed is false and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cmatrix` and `schmatrix` Example:

```
using namespace cvm;

double a[] = {1., -1., 2., 2., 3., -3.};
cvector v((std::complex<double>*)a, 3);
schmatrix mh(3);
mh.set_real(1.);
mh.herk (std::complex<double>(2.,-2.), v,
        std::complex<double>(1.,1.));
std::cout << mh << std::endl;

cmatrix m(3,2);
m(1) = v;
m(2).set(std::complex<double>(-1.,1.));
mh.herk (false, std::complex<double>(2.,-1.), m, 0.);
std::cout << mh << std::endl;

schmatrix mh2(2);
```

```
mh2.herk (true, std::complex<double>(1.,1.), m,  
         std::complex<double>(0.,1.));  
std::cout << mh2;
```

prints

```
(5,0) (1,-8) (13,0)  
(1,8) (17,0) (1,24)  
(13,0) (1,-24) (37,0)
```

```
(8,0) (4,-8) (16,0)  
(4,8) (20,0) (4,24)  
(16,0) (4,-24) (40,0)
```

```
(28,0) (-8,4)  
(-8,-4) (6,0)
```

2.14.51 her2k

Functions

```
schmatrix&
schmatrix::her2k (TC alpha, const cvector& v1,
                 const cvector& v2, TC beta) throw (cvmexception);
```

```
schmatrix&
schmatrix::her2k (bool bTransp, TC alpha, const cmatrix& m1,
                 const cmatrix& m2, TC beta) throw (cvmexception);
```

call one of ?HER2K routines of the [BLAS library](#) performing a matrix-vector operation defined for the first version as rank-1 update operation

$$C = \alpha v_1 \cdot v_2' + \alpha v_2 \cdot v_1' + \beta C,$$

and for the second version as

$$C = \alpha M_1 \cdot M_2^H + \alpha M_2 \cdot M_1^H + \beta C \quad \text{or} \quad C = \alpha M_1^H \cdot M_2 + \alpha M_2^H \cdot M_1 + \beta C.$$

Here α and β are complex numbers (parameters alpha and beta), M_1 and M_2 are complex matrices (parameters m1 and m2), C is a calling hermitian matrix and v_1 and v_2 are vectors (parameters v1 and v2). First operation for the second version of the function is performed if bTransp passed is false and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cmatrix` and `schmatrix` Example:

```
using namespace cvm;
```

```
double a1[] = {1., -1., 2., 2., 3., -3.};
double a2[] = {-2., 1., 1., -2., 1., -3.};
cvector v1((std::complex<double>*)a1, 3);
cvector v2((std::complex<double>*)a2, 3);
schmatrix mh(3);
mh.set_real(1.); mh.set(1,3,std::complex<double>(4.,1.));
mh.her2k (std::complex<double>(2.,-1.), v1, v2,
         std::complex<double>(-1.,-1.));
std::cout << mh << std::endl;
```

```
cmatrix m1(3,2);
cmatrix m2(3,2);
m1.set_real(2.); m1.set_imag(-1.);
m2.set_real(-3.); m2.set_imag(1.);
mh.her2k (false, std::complex<double>(2.,1.), m1, m2,
```

```
        std::complex<double>(3.,-2.));
std::cout << mh << std::endl;

schmatrix mh2(2);
mh2.her2k (true, std::complex<double>(1.,1.), m1, m2,
          std::complex<double>(2.,-3.));
std::cout << mh2;
```

prints

```
(-11,0) (-4,9) (-9,-16)
(-4,-9) (3,0) (20,23)
(-9,16) (20,-23) (59,0)

(-93,0) (-72,27) (-87,-48)
(-72,-27) (-51,0) (0,69)
(-87,48) (0,-69) (117,0)

(-36,0) (-36,0)
(-36,0) (-36,0)
```

2.14.52 inv

Functions

```
schmatrix& schmatrix::inv (const schmatrix& m) throw (cvmexception);
schmatrix schmatrix::inv () const throw (cvmexception);
```

implement hermitian matrix inversion. The first version sets a calling hermitian matrix to be equal to a hermitian matrix *m* inverted and the second one creates an object of type *schmatrix* as inverted calling matrix. The functions throw exception of type *cvmexception* in case of inappropriate sizes of the operands or when the matrix to be inverted is close to singular. See also *schmatrix*. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
```

```
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
const schmatrix mi = m.inv();
```

```
std::cout << mi << std::endl;
std::cout << mi * m - eye_complex(3);
```

prints

```
(-1.50e+000,0.00e+000) (-1.67e-016,8.33e-017) (-5.00e-001,-1.00e+000)
(-1.67e-016,-8.33e-017) (-1.00e+000,0.00e+000) (0.00e+000,-1.00e+000)
(-5.00e-001,1.00e+000) (0.00e+000,1.00e+000) (-1.50e+000,0.00e+000)

(2.22e-016,0.00e+000) (1.11e-016,0.00e+000) (2.78e-017,0.00e+000)
(2.78e-016,2.22e-016) (4.44e-016,0.00e+000) (0.00e+000,-4.44e-016)
(2.22e-016,-4.44e-016) (0.00e+000,-8.88e-016) (-2.22e-016,0.00e+000)
```

2.14.53 exp

Functions

```
schmatrix& schmatrix::exp (const schmatrix& m, TR tol = cvmMachSp ())
throw (cvmexception);
```

```
schmatrix schmatrix::exp (TR tol = cvmMachSp ()) const
throw (cvmexception);
```

compute an exponent of a calling hermitian matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \dots + z^p / p!,$$

where

$$N_{pq}(z) = \sum_{k=0}^p \frac{(p+q-k)! p!}{(p+q)! k! (q-k)!} z^k,$$

$$D_{pq}(z) = \sum_{k=0}^q \frac{(p+q-k)! p!}{(p+q)! k! (q-k)!} (-z)^k$$

along with the matrix normalizing as described in [2], p. 572. The functions use ZMEXP (or CMEXP for float version) FORTRAN subroutine implementing the algorithm. The first version sets the calling hermitian matrix to be equal to the exponent of a hermitian matrix *m* and returns a reference to the matrix changed. The second version creates an object of type *schmatrix* as the exponent of the calling matrix. The algorithm uses parameter *tol* as $\varepsilon(p, q)$ in order to choose constants *p* and *q* so that

$$\varepsilon(p, q) \geq 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the **largest relative spacing** by default. The functions throw an exception of type **cvmexception** in case of inappropriate sizes of the operands or when LAPACK subroutine fails. See also **schmatrix**. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left |
                std::ios::showpos);
std::cout.precision (15);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
schmatrix me(3);
me.exp(m);
```

```

std::cout << "Column 1" << std::endl
<< me(1,1) << std::endl << me(2,1) << std::endl << me(3,1) << std::endl
    << "Column 2" << std::endl
<< me(1,2) << std::endl << me(2,2) << std::endl << me(3,2) << std::endl
    << "Column 3" << std::endl
<< me(1,3) << std::endl << me(2,3) << std::endl << me(3,3) << std::endl;

```

prints

```

Column 1
(+2.673228708372002e+002,+1.091141066389412e-014)
(+3.071187567026803e+002,+1.535593783513402e+002)
(-1.749365628720766e+002,+3.498731257441531e+002)
Column 2
(+3.071187567026803e+002,-1.535593783513401e+002)
(+4.422594337092766e+002,+1.919736460939478e-015)
(-9.600094996571151e-015,+5.034325040954932e+002)
Column 3
(-1.749365628720765e+002,-3.498731257441531e+002)
(+6.184072406183948e-015,-5.034325040954932e+002)
(+5.744416275398805e+002,+1.540673883337074e-014)

```

Matlab output:

Column 1

```

2.673228708371998e+002 -7.105427357601002e-015i
3.071187567026802e+002 +1.535593783513401e+002i
-1.749365628720764e+002 +3.498731257441527e+002i

```

Column 2

```

3.071187567026802e+002 -1.535593783513401e+002i
4.422594337092769e+002 -5.489286670342458e-016i
3.549798266275454e-015 +5.034325040954932e+002i

```

Column 3

```

-1.749365628720763e+002 -3.498731257441526e+002i
-1.776065298147746e-014 -5.034325040954931e+002i
5.744416275398801e+002 -2.096383162906490e-014i

```

2.14.54 polynomial

Functions

```
schmatrix& schmatrix::polynom (const schmatrix& m, const rvector& v)
throw (cvmexception);
```

```
schmatrix schmatrix::polynom (const rvector& v) const
throw (cvmexception);
```

compute a hermitian matrix polynomial defined as

$$p(A) = b_0 I + b_1 A + \dots + b_q A^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^r B_k (A^s)^k, \quad s = \text{floor}(\sqrt{q}), \quad r = \text{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum_{i=0}^{s-1} b_{sk+i} A^i, & k = 0, 1, \dots, r-1 \\ \sum_{i=0}^{q-sr} b_{sr+i} A^i, & k = r. \end{cases}$$

See also [2], p. 568. The *real* coefficients b_0, b_1, \dots, b_q are passed in the parameter v , where q is equal to $v.size() - 1$, so the functions compute matrix polynomial equal to

$$v[1] * I + v[2] * m + \dots + v[v.size()] * m^{v.size()-1}$$

The first version sets a calling hermitian matrix to be equal to the polynomial of a hermitian matrix m and the second one creates an object of type `schmatrix` as the polynomial of a calling symmetric matrix. The functions use ZPOLY (or CPOLY for float version) FORTRAN subroutine implementing the Horner's algorithm. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `schmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left |
                std::ios::showpos);
std::cout.precision (10);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
double re[]={2.2,1.3,1.1,-0.9,0.2,-0.45,45.,-30.,10.,3.,1.13};
const rvector vr(re, 11);
```

```

schmatrix mp(3);

mp.polynom (m, vr);

std::cout << "Column 1" << std::endl
<< mp(1,1) << std::endl << mp(2,1) << std::endl << mp(3,1) << std::endl
    << "Column 2" << std::endl
<< mp(1,2) << std::endl << mp(2,2) << std::endl << mp(3,2) << std::endl
    << "Column 3" << std::endl
<< mp(1,3) << std::endl << mp(2,3) << std::endl << mp(3,3) << std::endl;

```

prints

```

Column 1
(+1.2319548758e+008,+0.0000000000e+000)
(+1.4179323916e+008,+7.0896619580e+007)
(-8.0802738460e+007,+1.6160547692e+008)
Column 2
(+1.4179323916e+008,-7.0896619580e+007)
(+2.0399822604e+008,+0.0000000000e+000)
(+0.0000000000e+000,+2.3250209650e+008)
Column 3
(-8.0802738460e+007,-1.6160547692e+008)
(+0.0000000000e+000,-2.3250209650e+008)
(+2.6498872674e+008,+0.0000000000e+000)

```

Matlab output:

```

Column 1

1.231954875800000e+008
1.417932391600000e+008 +7.089661958000000e+007i
-8.080273845999999e+007 +1.616054769200000e+008i

Column 2

1.417932391600000e+008 -7.089661958000000e+007i
2.039982260400000e+008
0 +2.325020965000000e+008i

Column 3

-8.080273845999999e+007 -1.616054769200000e+008i
0 -2.325020965000000e+008i
2.649887267400000e+008

```

2.14.55 eig

Functions

```
rvector schmatrix::eig (scmatrix& mEigVect) const throw (cvmexception);
rvector schmatrix::eig () const throw (cvmexception);
```

solve a **symmetric eigenvalue problem** and return a real vector with eigenvalues of a calling hermitian matrix. The first version sets the output parameter `mEigVect` to be equal to the square matrix containing orthogonal eigenvectors as columns. All the functions throw an exception of type **cvmexception** in case of inappropriate sizes of the operands or in case of convergence error. See also **rvector**, **scmatrix** and **schmatrix**. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
scmatrix me(3);
rvector v(3);
```

```
v = m.eig(me);
std::cout << v << std::endl;
cvector vc(v);
```

```
std::cout << m * me(1) - me(1) * vc(1);
std::cout << m * me(2) - me(2) * vc(2);
std::cout << m * me(3) - me(3) * vc(3);
// orthogonality check:
std::cout << me(1) % me(2) << std::endl;
std::cout << me(2) % me(3) << std::endl;
std::cout << me(1) % me(3) << std::endl;
```

prints

```
-8.13e-001 -3.44e-001 7.16e+000
```

```
(1.39e-016,2.22e-016) (5.25e-017,-1.11e-016) (1.94e-016,1.67e-016)
(4.86e-016,4.44e-016) (7.63e-017,0.00e+000) (3.33e-016,2.22e-016)
(-2.22e-016,-8.88e-016) (-5.55e-017,-8.88e-016) (8.88e-016,-5.55e-017)
(-5.17e-017,-9.74e-017)
(-5.81e-017,-5.40e-017)
(2.37e-018,-3.56e-017)
```

2.14.56 Cholesky

Function

```
schmatrix schmatrix::cholesky () const throw (cvmexception);
```

forms the Cholesky factorization of a hermitian positive-definite matrix A defined as

$$A = U^H U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the [LAPACK library](#). The function creates an object of type `schmatrix` as the factorization of a calling matrix. The function throws an exception of type `cvmexception` in case of convergence error. See also `schmatrix` and `schmatrix`. Example:

```
using namespace cvm;
```

```
try {
    double a[] = {3., 0., 2., 1., -1., 2., 2., -1., 3., 0.,
                 0., 3., -1., -2., 0., -3., 5., 0.};
    const schmatrix m((std::complex<double>*)a,3);

    schmatrix h = m.cholesky();
    std::cout << h << std::endl;
    std::cout << ~h * h - m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1.73205,0) (1.1547,-0.57735) (-0.57735,-1.1547)
(0,0) (1.1547,0) (0,-1.1547)
(0,0) (0,0) (1.41421,0)

(-4.44089e-016,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.57 Bunch-Kaufman

Function

```
scmatrix schmatrix::bunch_kaufman () throw (cvmexception);
```

forms the Bunch-Kaufman factorization of a calling hermitian matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is the calling matrix, P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D . It utilizes one of ?HETRF routines of the [LAPACK library](#). The function creates an object of type `scmatrix` as the factorization of a calling matrix. The function throws an exception of type `cvmexception` in case of convergence error. See also `scmatrix` and `schmatrix`. The function is mostly designed to be used for subsequent calls of ?HETRS, ?HECON and ?HETRI routines of the [LAPACK library](#). Currently it's used internally in `scmatrix::det` flow when argument is hermitian but not positive-definite matrix.

2.14.58 identity

Function

```
schmatrix& schmatrix::identity();
```

sets a calling hermitian matrix to be equal to identity matrix and returns a reference to the matrix changed. See also [schmatrix](#). Example:

```
using namespace cvm;
```

```
schmatrix m(3);  
m.randomize_real(0.,3.);  
m.randomize_imag(-1.,2.);
```

```
std::cout << m << std::endl;  
std::cout << m.identity();
```

prints

```
(1.93548,0) (1.84027,1.08353) (0.429579,-0.614093)  
(1.84027,-1.08353) (1.76922,0) (1.71364,1.82788)  
(0.429579,0.614093) (1.71364,-1.82788) (0.824915,0)
```

```
(1,0) (0,0) (0,0)  
(0,0) (1,0) (0,0)  
(0,0) (0,0) (1,0)
```

2.14.59 vanish

Function

```
schmatrix& schmatrix::vanish();
```

sets every element of a calling hermitian matrix to be equal to zero and returns a reference to the matrix changed. See also [schmatrix](#). Example:

```
using namespace cvm;
```

```
schmatrix m(3);  
m.randomize_real(0.,3.);  
m.randomize_imag(-1.,2.);
```

```
std::cout << m << std::endl;  
std::cout << m.vanish();
```

prints

```
(1.95499,0) (1.03925,0.830378) (0.951628,0.563677)  
(1.03925,-0.830378) (0.150426,0) (2.29365,-0.580218)  
(0.951628,-0.563677) (2.29365,0.580218) (0.0841395,0)
```

```
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)  
(0,0) (0,0) (0,0)
```

2.14.60 `randomize_real`

Function

```
schmatrix& schmatrix::randomize_real (TR dFrom, TR dTo);
```

fills a real part of a calling hermitian matrix with pseudo-random numbers distributed between `dFrom` and `dTo` keeping it to be hermitian. The function returns a reference to the matrix changed. See also [schmatrix](#). Example:

```
using namespace cvm;
```

```
schmatrix m(3);  
m.randomize_real(0.,3.);  
std::cout << m << std::endl;
```

prints

```
(1.98245,0) (2.72103,0) (0.167272,0)  
(2.72103,0) (0.0285653,0) (1.63765,0)  
(0.167272,0) (1.63765,0) (1.15882,0)
```

2.14.61 randomize_imag

Function

```
schmatrix& schmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills an imaginary part of a calling hermitian matrix with pseudo-random numbers distributed between dFrom and dTo keeping it to be hermitian. The function returns a reference to the matrix changed. See also [schmatrix](#). Example:

```
using namespace cvm;
```

```
schmatrix m(3);  
m.randomize_imag(0.,3.);  
std::cout << m << std::endl;
```

prints

```
(0,0) (0,0.13834) (0,2.39903)  
(0,-0.13834) (0,0) (0,0.609577)  
(0,-2.39903) (0,-0.609577) (0,0)
```

2.15 **cvmexception**

The CVM library exceptions class.

```
class cvmexception : public std::exception
{
public:
    explicit cvmexception (int nCause, ...);
    int cause () const;
    virtual const char* what () const;
    static int getNextCause ();
    static bool add (int nNewCause, const char* szNewMessage);
};
```

2.15.1 **cause**

Function

```
int cvmexception::cause () const;
```

returns a numeric code of an exception thrown. Possible codes can be found in `cvm.h` file. See also [cvmexception](#). Example:

```
using namespace cvm;
```

```
try {  
    rvector v(10);  
    v[11] = 1.;  
}  
catch (cvmexception& e) {  
    std::cout << "Exception " << e.cause () << ": "  
              << e.what () << std::endl;  
}
```

prints

```
Exception 2: Out of range
```

2.15.2 **what**

Function

```
virtual const char* cvmexception::what () const throw();
```

returns a string describing an exception happened. This function overrides `std::exception::what()`. This allows you to catch just one type of exception in your application. See also [cvmexception](#). Example:

```
using namespace cvm;
```

```
try {  
    double a[] = {1., 2., 1., 2.};  
    const srsmatrix m(a, 2);  
    std::cout << m;  
}  
catch (std::exception& e) {  
    std::cout << "Exception: " << e.what () << std::endl;  
}
```

prints

```
Exception: The matrix passed doesn't appear to be symmetric
```

2.15.3 Customization

Constructor and functions

```
explicit cvmexception (int nCause, ...);
static bool cvmexception::add (int nNewCause, const char* szNewMessage);
static int cvmexception::getNextCause ();
```

allow to add and use customized exception codes and messages. See also [cvmexception](#).
Example:

```
using namespace cvm;

const int nNextCause = cvmexception::getNextCause();
cvmexception::add (nNextCause,
                  "My first exception with %d parameter");
cvmexception::add (nNextCause + 1,
                  "My second exception with %s parameter");

try {
    throw cvmexception (nNextCause, 1234);
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}

try {
    throw cvmexception (nNextCause + 1, "Hi!");
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}

prints
```

```
Exception: My first exception with 1234 parameter
Exception: My second exception with Hi! parameter
```

2.16 Utilities

These functions have `cvm` namespace scope and can be used for different purposes.

```
template <typename T>
T* cvmMalloc (size_t nEls) throw (cvmexception);
template <typename T>
T* cvmAddRef (const T* pD);
template <typename T>
int cvmFree (T*& pD);
void cvmExit ();
treal cvmMachMin ();
treal cvmMachSp ();
srmatrix eye_real (int nM);
scmatrix eye_complex (int nM);
operator * (,);
```

2.16.1 cvmMalloc

Since version 6.0 if CVM_USE_POOL_MANAGER macro is not defined (this is the default) this function simply calls `::new T[nEls]` operator.

```
template <typename T>
T* cvmMalloc (size_t nEls) throw (cvmexception);
```

allocates nEls units of type T from the CVM library's **memory pool** and returns a pointer to the memory allocated. See also **cvmAddRef** and **cvmFree**. Example:

```
using namespace cvm;
```

```
double* p = cvmMalloc<double> (10);
p[0] = 1.;
p[1] = 2.;
p[2] = 3.;
```

```
rvector v(3);
v.assign(p);
std::cout << v;
```

```
cvmFree (p);
```

prints

```
1 2 3
```

2.16.2 `cvmAddRef`

Since version 6.0 it's available only if `CVM_USE_POOL_MANAGER` is defined.

```
template <typename T>
void cvmAddRef (const T* pD);
```

increments a reference counter for a memory block pointed to by `pD` if this block was allocated from the CVM library's memory pool (using `cvmMalloc` function). If `pD` points to a foreign memory block then the function does nothing. See also `cvmAddRef`. Example:

```
using namespace cvm;
```

```
double* p = cvmMalloc<double> (10);
p[0] = 1.;
p[1] = 2.;
p[2] = 3.;
```

```
cvmAddRef (p);
cvmFree (p);    // this call doesn't deallocate a memory
cvmFree (p);    // this one does
```

2.16.3 cvmFree

Since version 6.0 if CVM_USE_POOL_MANAGER macro is not defined (this is the default) then it simply calls `::delete[] pD` operator if `pD!=NULL` and returns zero.

```
template <typename T>
int cvmFree (T*& pD);
```

decrements a reference counter for a memory block pointed to by `pD` if this block was allocated from the CVM library's memory pool (using `cvmMalloc` function) and returns the reference counter it changed. If the function returns zero then it sets the pointer `pD` to be equal to `NULL` and "frees" the memory, i.e. returns the memory block to a list of free ones (see [CVM memory management](#) for details). If `pD` points to a foreign memory block then the function does nothing and returns `-1`. See also `cvmAddRef`. Example:

```
using namespace cvm;

double* pf = new double[10];
double* p = cvmMalloc<double> (10);

cvmAddRef (p);

std::cout << cvmFree (p) << " ";
std::cout << p << std::endl;

std::cout << cvmFree (p) << " ";
std::cout << p << std::endl;

std::cout << cvmFree (pf) << " ";
std::cout << pf << std::endl;

delete[] pf;

prints

1 003C66B0
0 00000000
-1 003C7A40
```

2.16.4 cvmExit

Since version 6.0 it's available only if `CVM_USE_POOL_MANAGER` is defined.

```
void cvmExit ();
```

destroys the CVM library's **memory pool** if `CVM_USE_POOL_MANAGER` is defined. Otherwise does nothing. All objects created using this pool are not accessible after calling of this function. Call this function in the last expression only if you have problems with memory deallocation while finishing execution of your program (I was not able to experience such problems, this function is provided just in case). See also `cvmMalloc`. Example:

```
using namespace cvm;
```

```
int main(int argc, char* argv[])
    try {
        rvector v(3);
        v[1] = v[2] = v[3] = 1.;
        std::cout << v.norm2() << std::endl;
    }
    catch (cvmexception& e) {
        std::cout << "Exception " << e.cause ()
            << ": " << e.what () << std::endl;
    }

    cvmExit ();
    return 1;
}
```

prints

1.73205

2.16.5 `cvmMachMin`

Function

```
treal cvmMachMin ();
```

returns the smallest normalized positive number or, i.e. `numeric_limits<treal>::min()` where `treal` is typedef'ed as `double` by default or as `float` for float version of the library. See also `cvmMachSp`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::showpos);  
std::cout.precision (15);  
std::cout << cvmMachMin() << std::endl;
```

for Intel Pentium® III machine prints

```
+2.225073858507201e-308
```

2.16.6 `cvmMachSp`

Function

```
treal cvmMachSp ();
```

returns the largest relative spacing or, in other words, the difference between 1 and the least value greater than 1 that is representable, i.e. `numeric_limits<treal>::epsilon()` where `treal` is typedef'ed as `double` by default or as `float` for float version of the library. See also `cvmMachMin`. Example:

```
using namespace cvm;
```

```
std::cout.setf (std::ios::scientific | std::ios::showpos);  
std::cout.precision (15);  
std::cout << cvmMachSp() << std::endl;
```

for Intel Pentium® III machine prints

```
+2.220446049250313e-016
```

2.16.7 eye_real

Function

```
srmatrix eye_real (int nM);
```

creates a nM by nM object of type srmatrix equal to identity matrix. See also [srmatrix](#).

Example:

```
using namespace cvm;
```

```
std::cout << eye_real (4);
```

prints

```
1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
```

2.16.8 `eye_complex`

Function

```
scmatrix eye_complex (int nM);
```

creates a nM by nM object of type `scmatrix` equal to identity matrix. See also `scmatrix`.

Example:

```
using namespace cvm;
```

```
std::cout << eye_complex (4);
```

prints

```
(1,0) (0,0) (0,0) (0,0)
(0,0) (1,0) (0,0) (0,0)
(0,0) (0,0) (1,0) (0,0)
(0,0) (0,0) (0,0) (1,0)
```

2.16.9 operator *

Operators

```

inline rvector operator * (TR d, const rvector& v);
inline rmatrix operator * (TR d, const rmatrix& m);
inline srmatrix operator * (TR d, const srmatrix& m);
inline srbmatrix operator * (TR d, const srbmatrix& m);
inline srsrmatrix operator * (TR d, const srsrmatrix& m);
inline cvector operator * (TR d, const cvector& v);
inline cmatrix operator * (TR d, const cmatrix& m);
inline scmatrix operator * (TR d, const scmatrix& m);
inline scbmatrix operator * (TR d, const scbmatrix& m);
inline schmatrix operator * (TR d, const schmatrix& m);
inline cvector operator * (std::complex<TR> c, const cvector& v);
inline cmatrix operator * (std::complex<TR> c, const cmatrix& m);
inline scmatrix operator * (std::complex<TR> c, const scmatrix& m);
inline scbmatrix operator * (std::complex<TR> c, const scbmatrix& m);
inline schmatrix operator * (std::complex<TR> c, const schmatrix& m);
inline rvector operator * (CVM_LONGEST_INT d, const rvector& v);
inline rmatrix operator * (CVM_LONGEST_INT d, const rmatrix& m);
inline srmatrix operator * (CVM_LONGEST_INT d, const srmatrix& m);
inline srbmatrix operator * (CVM_LONGEST_INT d, const srbmatrix& m);
inline srsrmatrix operator * (CVM_LONGEST_INT d, const srsrmatrix& m);
inline cvector operator * (CVM_LONGEST_INT d, const cvector& v);
inline cmatrix operator * (CVM_LONGEST_INT d, const cmatrix& m);
inline scmatrix operator * (CVM_LONGEST_INT d, const scmatrix& m);
inline scbmatrix operator * (CVM_LONGEST_INT d, const scbmatrix& m);
inline schmatrix operator * (CVM_LONGEST_INT d, const schmatrix& m);

```

provide an ability to make left-sided multiplication of numbers by different CVM objects.
Example:

```

using namespace cvm;

const schmatrix scm = eye_complex (4);
std::cout << std::complex<double>(2.,1.) * scm << std::endl;

rvector v(3);
v(1) = 1.;
v(2) = 2.;
v(3) = 3.;

std::cout << 3. * v;

```

prints

```
(2,1) (0,0) (0,0) (0,0)
(0,0) (2,1) (0,0) (0,0)
(0,0) (0,0) (2,1) (0,0)
(0,0) (0,0) (0,0) (2,1)
```

3 6 9

2.17 Pool Manager

The memory management mechanism described below is no longer supported by default. It was a good solution more than 10 years ago when memory allocation operator was relatively expensive, but now standard allocator does this job much faster. However, the algorithm described below may be useful as an error detection helper. You will need to rebuild the library in debug mode with CVM_USE_POOL_MANAGER defined in order to use it.

Since its third release the CVM library implements nontrivial memory management which should be described in detail. Earlier a memory was being allocated using operator `new` in every constructor, and freed with help of `delete` in every destructor. Let us consider an operation of multiplying of vector `a` by matrix `A` and assignment of result to vector `b`:

```
b = a * A;
```

This harmless code calls two constructors (a constructor allocating memory for output and a copy constructor, returning output to a calling function), as well as two destructors deleting those temporary objects¹¹. If sizes of the objects are relatively small, your processor will be longer allocating memory than multiplying¹². The idea of nontrivial memory management came from Jeff Alger's book [1]. Author suggests some approaches to memory allocation (overloading of operators `new` and `delete` or implementation of a class controlling a pool), and also some ways of a pool compaction (Baker's algorithm and in-place compaction) and references counting (using master or "genius" pointers). I decided to implement a class controlling a pool in the CVM library.

Pool is controlled by `MemoryPool` class. It allocates a memory by blocks (so-called "outer blocks"). The memory block concept is encapsulated in a class `MemoryBlocks`. Size of an outer block equals to the nearest degree of 2 of a requested number of bytes multiplied by two. This can be illustrated on the following example. Let us suppose that it's required to allocate a memory for storage of a vector consisting of 1000 units:

```
rvector v(1000);
```

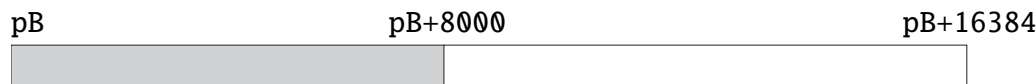
If at the moment of execution of this statement there is no free block of size 8000 or greater in a list of free memory blocks, then the following (simplified) code will be executed:

```
const int nUpBytes = up_value (nBytes);
const int nRest    = nUpBytes - nBytes;
try {
    pB = ::new tbyte[nUpBytes];
}
catch (const std::bad_alloc&) {
    throw (cvmexception (CVM_OUTOFMEMORY));
}
m_lOutBlocks.push_back (pB);
m_blocks.AddPair (pB, nBytes, nRest);
```

¹¹ In order to avoid those memory allocations you can use `b.mult(a,A)`;

¹² At least under Win32

where variable `nBytes` has value of 8000 ($1000 * \text{sizeof}(treal)$), and function `up_value` returns the nearest degree of two multiplied by two, i.e. 16384. Thus, the allocated outer block can be represented as follows:

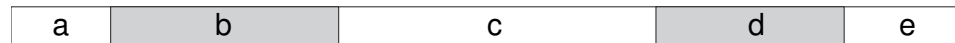


The block used for storage 1000 units of a vector `v` (its start address here is stored in the pointer `pB`) is filled. The remaining block (named as “free block”) consists of `nRest=8384` bytes. Further, if application needs one more block of the same size (it happens in most cases while execution of a copy constructor), the memory will be allocated from this free block without calling of operator `new`. The result will be the following:



Remaining free block of 384 bytes will be utilized for memory allocation of small objects. In case of creation of an object of size greater than 384 bytes one more outer block will be created, etc. While using of this scheme sooner or later memory begins to be like a sieve of free and occupied blocks. To avoid this chaos, I have applied the logic of blocks releasing, which acts like an algorithm of free space compaction. Difference of this logic from the algorithms of compaction described in [1] is that occupied blocks are never being moved.

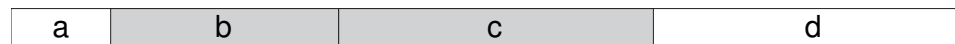
Let's say one of outer blocks looks like



And the block `d` is going to be released. In this case neighbor blocks `c` and `e` will be checked and if one of them (or both, as in this case) will appear as free, then it will be joined with the block being released. The result of release of the block `d` will be the following:



And if the following object to be created will not find a room in the block `a`, but find a room in the block `c`, the result of memory allocation will be the following:



So, there is no any chaos.

References

- [1] *Jeff Alger*. C++ for Real Programmers, Morgan Kaufmann Publishers, 1998, 388 p., ISBN 0120499428
- [2] *Gene H. Golub, Charles F. Van Loan*. Matrix Computations, The Johns Hopkins University Press, 1996, 694 p., ISBN 0-8018-5413-X
- [3] *Peter Lancaster*. Theory of Matrices, Academic Press, New York, 1969
- [4] *Scott Meyers*. More Effective C++: 35 new ways to improve your programs and designs, Addison-Wesley, 1996, 320 p., ISBN 0-201-63371-X
- [5] *Roger A. Horn, Charles R. Johnson*. Matrix Analysis, Cambridge University Press, 1990, 561 p., ISBN 0-521-38632-2
- [6] Ф.Р. Гантмахер Теория Матриц, Москва, Наука, 1988, 522 стр., ISBN 5-02-013722-7

Contents

1	Preface	1
1.1	Brief history	1
1.2	Libraries Supported	1
1.3	Object Model	2
1.4	Installation	3
1.4.1	Directory Structure	3
1.4.2	Usage Notes	4
1.4.3	Installation – Win32	4
1.4.4	Installation – Unix and MinGW	4
1.5	Storage	5
1.6	Indexing	6
1.7	Polymorphism	7
1.8	Multi-threading	7
1.9	Regression test utility	7
2	CVM Class Library Reference	8
2.1	basic_array	8
2.1.1	basic_array()	10
2.1.2	basic_array(int, bool)	11
2.1.3	basic_array(const T*, int)	12
2.1.4	basic_array(const T*, const T*)	13
2.1.5	basic_array(const basic_array&)	14
2.1.6	size()	15
2.1.7	get(), operator T*()	16
2.1.8	Indexing operators	17
2.1.9	operator = (const basic_array&)	18
2.1.10	assign(const T*)	19
2.1.11	set(T)	20
2.1.12	resize	21
2.1.13	STL-specific type definitions	22
2.1.14	STL-specific functions: begin(), end(), rbegin(), rend(), max_size(), capacity(), empty(), front(), back(), assign(), resize(), clear(), swap()	24
2.1.15	at()	26
2.1.16	push_back(const T&), pop_back()	27
2.1.17	insert (iterator, const T&), erase (iterator)	28
2.1.18	operator >> <> (std::istream& is, basic_array<T>& aIn)	29
2.1.19	operator << <> (std::ostream& os, const basic_array<T>& aOut)	30
2.2	Array	31
2.2.1	incr	32

2.2.2	indofmax	33
2.2.3	indofmin	34
2.2.4	norm	35
2.2.5	norminf	36
2.2.6	norm1	37
2.2.7	norm2	38
2.2.8	operator >> <> (std::istream& is, Array<TR,TC>& aIn)	39
2.2.9	operator << <> (std::ostream& os, const Array<TR,TC>& aOut)	40
2.3	rvector	41
2.3.1	rvector ()	43
2.3.2	rvector (int)	44
2.3.3	rvector (int, TR)	45
2.3.4	rvector (TR*,int,int)	46
2.3.5	rvector (const TR*,int,int)	47
2.3.6	rvector (const rvector&)	48
2.3.7	operator = (const rvector&)	49
2.3.8	assign(const TR*, int)	50
2.3.9	assign(int, const TR*, int)	51
2.3.10	assign (int, const rvector&)	52
2.3.11	set(TR)	53
2.3.12	resize	54
2.3.13	operator ==	55
2.3.14	operator !=	56
2.3.15	operator <<	57
2.3.16	operator +	58
2.3.17	operator -	59
2.3.18	sum	60
2.3.19	diff	61
2.3.20	operator +=	62
2.3.21	operator -=	63
2.3.22	operator - ()	64
2.3.23	operator * (TR)	65
2.3.24	operator / (TR)	66
2.3.25	operator *=	67
2.3.26	operator /=	68
2.3.27	normalize	69
2.3.28	operator * (const rvector&)	70
2.3.29	operator * (const rmatrix&)	71
2.3.30	mult (const rvector&, const rmatrix&)	72
2.3.31	mult (const rmatrix&, const rvector&)	73
2.3.32	rank1update	74
2.3.33	solve	75
2.3.34	solve_lu	76

2.3.35	gels	78
2.3.36	gelsy	79
2.3.37	gelss	80
2.3.38	gelsd	81
2.3.39	svd	82
2.3.40	eig	84
2.3.41	gemv	86
2.3.42	gbmv	87
2.3.43	randomize	88
2.4	cvector	89
2.4.1	cvector ()	92
2.4.2	cvector (int)	93
2.4.3	cvector (int, TC)	94
2.4.4	cvector (TC*,int,int)	95
2.4.5	cvector (const TC*,int,int)	96
2.4.6	cvector (const cvector&)	97
2.4.7	cvector (const TR*,const TR*,int,int)	98
2.4.8	cvector (const rvector&, const rvector&)	99
2.4.9	cvector (const TR*,int,bool,int)	100
2.4.10	cvector (const rvector&,bool)	101
2.4.11	real	102
2.4.12	imag	103
2.4.13	operator = (const cvector&)	104
2.4.14	assign(const TC*, int)	105
2.4.15	assign(int, const TC*, int)	106
2.4.16	assign (int, const cvector&)	107
2.4.17	set(TC)	108
2.4.18	assign_real	109
2.4.19	assign_imag	110
2.4.20	set_real	111
2.4.21	set_imag	112
2.4.22	resize	113
2.4.23	operator ==	114
2.4.24	operator !=	115
2.4.25	operator <<	116
2.4.26	operator +	117
2.4.27	operator -	118
2.4.28	sum	119
2.4.29	diff	120
2.4.30	operator +=	121
2.4.31	operator -=	122
2.4.32	operator - ()	123
2.4.33	operator * (TR)	124

2.4.34	operator / (TR)	125
2.4.35	operator * (TC)	126
2.4.36	operator / (TC)	127
2.4.37	operator *= (TR)	128
2.4.38	operator /= (TR)	129
2.4.39	operator *= (TC)	130
2.4.40	operator /= (TC)	131
2.4.41	normalize	132
2.4.42	conjugation	133
2.4.43	operator * (const cvector&)	134
2.4.44	operator %	135
2.4.45	operator * (const cmatrix&)	136
2.4.46	mult (const cvector&, const cmatrix&)	137
2.4.47	mult (const cmatrix&, const cvector&)	138
2.4.48	rank1update_u	139
2.4.49	rank1update_c	140
2.4.50	solve	141
2.4.51	solve_lu	142
2.4.52	gels	144
2.4.53	gelsy	146
2.4.54	gelss	147
2.4.55	gelsd	148
2.4.56	eig	149
2.4.57	gemv	151
2.4.58	gbmv	152
2.4.59	randomize_real	153
2.4.60	randomize_imag	154
2.5	Matrix	155
2.5.1	msize	156
2.5.2	nsize	157
2.5.3	ld	158
2.5.4	rowofmax	159
2.5.5	rowofmin	160
2.5.6	colofmax	161
2.5.7	colofmin	162
2.5.8	norm1	163
2.5.9	operator << <> (std::ostream& os, const Matrix<TR,TC>& aOut)	164
2.6	rmatrix	165
2.6.1	rmatrix ()	169
2.6.2	rmatrix (int,int)	170
2.6.3	rmatrix (TR*,int,int)	171
2.6.4	rmatrix (const TR*,int,int)	172
2.6.5	rmatrix (const rmatrix&)	173

2.6.6	<code>rmatrix (const rvector&,bool)</code>	174
2.6.7	<code>submatrix</code>	175
2.6.8	<code>operator (,)</code>	176
2.6.9	<code>operator ()</code>	177
2.6.10	<code>operator []</code>	178
2.6.11	<code>diag</code>	179
2.6.12	<code>operator = (const rmatrix&)</code>	180
2.6.13	<code>assign (const TR*)</code>	181
2.6.14	<code>assign (int, int, const rmatrix&)</code>	182
2.6.15	<code>set (TR)</code>	183
2.6.16	<code>resize</code>	184
2.6.17	<code>operator ==</code>	185
2.6.18	<code>operator !=</code>	186
2.6.19	<code>operator <<</code>	187
2.6.20	<code>operator +</code>	188
2.6.21	<code>operator -</code>	189
2.6.22	<code>sum</code>	190
2.6.23	<code>diff</code>	191
2.6.24	<code>operator +=</code>	192
2.6.25	<code>operator -=</code>	193
2.6.26	<code>operator - ()</code>	194
2.6.27	<code>operator * (TR)</code>	195
2.6.28	<code>operator / (TR)</code>	196
2.6.29	<code>operator *= (TR)</code>	197
2.6.30	<code>operator /= (TR)</code>	198
2.6.31	<code>normalize</code>	199
2.6.32	<code>transposition</code>	200
2.6.33	<code>operator * (const rvector&)</code>	202
2.6.34	<code>operator * (const rmatrix&)</code>	203
2.6.35	<code>mult</code>	204
2.6.36	<code>rank1update</code>	205
2.6.37	<code>swap_rows</code>	206
2.6.38	<code>swap_cols</code>	207
2.6.39	<code>solve</code>	208
2.6.40	<code>solve_lu</code>	210
2.6.41	<code>svd</code>	212
2.6.42	<code>pinv</code>	214
2.6.43	<code>gels</code>	216
2.6.44	<code>gelsy</code>	218
2.6.45	<code>gelss</code>	220
2.6.46	<code>gelsd</code>	222
2.6.47	<code>rank</code>	224
2.6.48	<code>ger</code>	225

2.6.49	gemm	226
2.6.50	symm	227
2.6.51	qr	229
2.6.52	lq	230
2.6.53	rq	231
2.6.54	ql	232
2.6.55	vanish	233
2.6.56	randomize	234
2.7	cmatrix	235
2.7.1	cmatrix ()	239
2.7.2	cmatrix (int,int)	240
2.7.3	cmatrix (TC*,int,int)	241
2.7.4	cmatrix (const TC*,int,int)	242
2.7.5	cmatrix (const cmatrix&)	243
2.7.6	cmatrix (const cvector&,bool)	244
2.7.7	cmatrix (const rmatrix&,bool)	245
2.7.8	cmatrix (const TR*,const TR*,int,int)	246
2.7.9	cmatrix (const rmatrix&, const rmatrix&)	247
2.7.10	submatrix	248
2.7.11	operator (,)	249
2.7.12	operator ()	250
2.7.13	operator []	251
2.7.14	diag	252
2.7.15	real	253
2.7.16	imag	254
2.7.17	operator = (const cmatrix&)	255
2.7.18	assign (const TC*)	256
2.7.19	assign (int, int, const cmatrix&)	257
2.7.20	set (TC)	258
2.7.21	assign_real	259
2.7.22	assign_imag	260
2.7.23	set_real	261
2.7.24	set_imag	262
2.7.25	resize	263
2.7.26	operator ==	264
2.7.27	operator !=	265
2.7.28	operator <<	266
2.7.29	operator +	267
2.7.30	operator -	268
2.7.31	sum	269
2.7.32	diff	270
2.7.33	operator +=	271
2.7.34	operator -=	272

2.7.35	operator - ()	273
2.7.36	operator * (TR)	274
2.7.37	operator / (TR)	275
2.7.38	operator * (TC)	276
2.7.39	operator / (TC)	277
2.7.40	operator *= (TR)	278
2.7.41	operator /= (TR)	279
2.7.42	operator *= (TC)	280
2.7.43	operator /= (TC)	281
2.7.44	normalize	282
2.7.45	conjugation	283
2.7.46	operator * (const cvector&)	284
2.7.47	operator * (const cmatrix&)	285
2.7.48	mult	286
2.7.49	rank1update_u	287
2.7.50	rank1update_c	288
2.7.51	swap_rows	289
2.7.52	swap_cols	290
2.7.53	solve	291
2.7.54	solve_lu	292
2.7.55	svd	294
2.7.56	pinv	296
2.7.57	gels	298
2.7.58	gelsy	300
2.7.59	gelss	302
2.7.60	gelzd	304
2.7.61	rank	306
2.7.62	qr	307
2.7.63	lq	308
2.7.64	rq	309
2.7.65	ql	310
2.7.66	vanish	311
2.7.67	geru	312
2.7.68	gerc	313
2.7.69	gemm	314
2.7.70	hemm	316
2.7.71	randomize_real	318
2.7.72	randomize_imag	319
2.8	srmatrix	320
2.8.1	srmatrix ()	323
2.8.2	srmatrix (int)	324
2.8.3	srmatrix (TR*,int)	325
2.8.4	srmatrix (const TR*,int)	326

2.8.5	srmatrix (const srmatrix&)	327
2.8.6	srmatrix (const rmatrix&)	328
2.8.7	srmatrix (const rvector&)	329
2.8.8	submatrix	330
2.8.9	operator (,)	331
2.8.10	operator ()	332
2.8.11	operator []	333
2.8.12	operator = (const srmatrix&)	334
2.8.13	assign (const TR*)	335
2.8.14	assign (int, int, const rmatrix&)	336
2.8.15	set (TR)	337
2.8.16	resize	338
2.8.17	operator <<	339
2.8.18	operator +	340
2.8.19	operator -	341
2.8.20	sum	342
2.8.21	diff	343
2.8.22	operator +=	344
2.8.23	operator -=	345
2.8.24	operator - ()	346
2.8.25	operator ++	347
2.8.26	operator --	348
2.8.27	operator * (TR)	349
2.8.28	operator / (TR)	350
2.8.29	operator *= (TR)	351
2.8.30	operator /= (TR)	352
2.8.31	normalize	353
2.8.32	transposition	354
2.8.33	operator * (const rvector&)	355
2.8.34	operator * (const rmatrix&)	356
2.8.35	operator * (const srmatrix&)	357
2.8.36	operator *= (const srmatrix&)	358
2.8.37	swap_rows	359
2.8.38	swap_cols	360
2.8.39	solve	361
2.8.40	solve_lu	363
2.8.41	det	366
2.8.42	low_up	367
2.8.43	cond	369
2.8.44	inv	370
2.8.45	exp	371
2.8.46	polynomial	373
2.8.47	eig	375

2.8.48	Cholesky	377
2.8.49	Bunch-Kaufman	378
2.8.50	qr	379
2.8.51	lq	380
2.8.52	ql	381
2.8.53	rq	382
2.8.54	identity	383
2.8.55	vanish	384
2.8.56	randomize	385
2.9	scmatrix	386
2.9.1	scmatrix ()	389
2.9.2	scmatrix (int)	390
2.9.3	scmatrix (TC*,int)	391
2.9.4	scmatrix (const TC*,int)	392
2.9.5	scmatrix (const scmatrix&)	393
2.9.6	scmatrix (const cmatrix&)	394
2.9.7	scmatrix (const cvector&)	395
2.9.8	scmatrix (const srmatrix&,bool)	396
2.9.9	scmatrix (const TR*,const TR*,int)	397
2.9.10	scmatrix (const srmatrix&, const srmatrix&)	398
2.9.11	submatrix	399
2.9.12	operator (,)	400
2.9.13	operator ()	401
2.9.14	operator []	402
2.9.15	real	403
2.9.16	imag	404
2.9.17	operator = (const scmatrix&)	405
2.9.18	assign (const TC*)	406
2.9.19	assign (int, int, const cmatrix&)	407
2.9.20	set (TC)	408
2.9.21	assign_real	409
2.9.22	assign_imag	410
2.9.23	set_real	411
2.9.24	set_imag	412
2.9.25	resize	413
2.9.26	operator <<	414
2.9.27	operator +	415
2.9.28	operator -	416
2.9.29	sum	417
2.9.30	diff	418
2.9.31	operator +=	419
2.9.32	operator -=	420
2.9.33	operator - ()	421

2.9.34	operator ++	422
2.9.35	operator --	423
2.9.36	operator * (TR)	424
2.9.37	operator / (TR)	425
2.9.38	operator * (TC)	426
2.9.39	operator / (TC)	427
2.9.40	operator *= (TR)	428
2.9.41	operator /= (TR)	429
2.9.42	operator *= (TC)	430
2.9.43	operator /= (TC)	431
2.9.44	normalize	432
2.9.45	conjugation	433
2.9.46	operator * (const cvector&)	434
2.9.47	operator * (const cmatrix&)	435
2.9.48	operator * (const scmatrix&)	436
2.9.49	operator *= (const scmatrix&)	437
2.9.50	swap_rows	438
2.9.51	swap_cols	439
2.9.52	solve	440
2.9.53	solve_lu	442
2.9.54	det	444
2.9.55	low_up	445
2.9.56	cond	447
2.9.57	inv	448
2.9.58	exp	449
2.9.59	polynomial	451
2.9.60	eig	453
2.9.61	Cholesky	454
2.9.62	Bunch-Kaufman	455
2.9.63	qr	456
2.9.64	lq	457
2.9.65	rq	458
2.9.66	ql	459
2.9.67	identity	460
2.9.68	vanish	461
2.9.69	randomize_real	462
2.9.70	randomize_imag	463
2.10	BandMatrix	464
2.10.1	lsize	465
2.10.2	usize	466
2.11	srbmatrix	467
2.11.1	srbmatrix ()	469
2.11.2	srbmatrix (int)	470

2.11.3	srbmatrix (int,int,int)	471
2.11.4	srbmatrix (TR*,int,int,int)	472
2.11.5	srbmatrix (const TR*,int,int,int)	473
2.11.6	srbmatrix (const srbmatrix&)	474
2.11.7	srbmatrix (const rmatrix&,int,int)	475
2.11.8	srbmatrix (const rvector&)	476
2.11.9	operator (,)	477
2.11.10	operator ()	478
2.11.11	operator []	479
2.11.12	operator = (const srbmatrix&)	480
2.11.13	assign (const TR*)	481
2.11.14	set (TR)	482
2.11.15	resize	483
2.11.16	resize_lu	484
2.11.17	operator ==	485
2.11.18	operator !=	486
2.11.19	operator <<	487
2.11.20	operator +	488
2.11.21	operator -	489
2.11.22	sum	490
2.11.23	diff	491
2.11.24	operator +=	492
2.11.25	operator -=	493
2.11.26	operator - ()	494
2.11.27	operator ++	495
2.11.28	operator --	496
2.11.29	operator * (TR)	497
2.11.30	operator / (TR)	498
2.11.31	operator *= (TR)	499
2.11.32	operator /= (TR)	500
2.11.33	normalize	501
2.11.34	transposition	502
2.11.35	operator * (const rvector&)	503
2.11.36	operator * (const rmatrix&)	504
2.11.37	operator * (const srmatrix&)	505
2.11.38	operator * (const srbmatrix&)	506
2.11.39	low_up	507
2.11.40	identity	509
2.11.41	vanish	510
2.11.42	randomize	511
2.12	scbmatrix	512
2.12.1	scbmatrix ()	514
2.12.2	scbmatrix (int)	515

2.12.3	scbmatrix (int,int,int)	516
2.12.4	scbmatrix (TC*,int,int,int)	517
2.12.5	scbmatrix (const TC*,int,int,int)	518
2.12.6	scbmatrix (const scbmatrix&)	519
2.12.7	scbmatrix (const cmatrix&,int,int)	520
2.12.8	scbmatrix (const cvector&)	521
2.12.9	scbmatrix (const srbmatrix&,bool)	522
2.12.10	scbmatrix (const srbmatrix&, const srbmatrix&)	523
2.12.11	operator (,)	524
2.12.12	operator ()	525
2.12.13	operator []	526
2.12.14	real	527
2.12.15	imag	528
2.12.16	operator = (const scbmatrix&)	529
2.12.17	assign (const TC*)	530
2.12.18	set (TC)	531
2.12.19	assign_real	532
2.12.20	assign_imag	533
2.12.21	set_real	534
2.12.22	set_imag	535
2.12.23	resize	536
2.12.24	resize_lu	537
2.12.25	operator ==	538
2.12.26	operator !=	539
2.12.27	operator <<	540
2.12.28	operator +	541
2.12.29	operator -	542
2.12.30	sum	543
2.12.31	diff	544
2.12.32	operator +=	545
2.12.33	operator -=	546
2.12.34	operator - ()	547
2.12.35	operator ++	548
2.12.36	operator --	549
2.12.37	operator * (TR)	550
2.12.38	operator / (TR)	551
2.12.39	operator * (TC)	552
2.12.40	operator / (TC)	553
2.12.41	operator *= (TR)	554
2.12.42	operator /= (TR)	555
2.12.43	operator *= (TC)	556
2.12.44	operator /= (TC)	557
2.12.45	normalize	558

2.12.46	conjugation	559
2.12.47	operator * (const cvector&)	561
2.12.48	operator * (const cmatrix&)	562
2.12.49	operator * (const scmatrix&)	563
2.12.50	operator * (const scbmatrix&)	564
2.12.51	low_up	565
2.12.52	identity	567
2.12.53	vanish	568
2.12.54	randomize_real	569
2.12.55	randomize_imag	570
2.13	srsmatrix	571
2.13.1	srsmatrix ()	573
2.13.2	srsmatrix (int)	574
2.13.3	srsmatrix (TR*,int)	575
2.13.4	srsmatrix (const TR*,int)	576
2.13.5	srsmatrix (const srsmatrix&)	577
2.13.6	srsmatrix (const rmatrix&)	578
2.13.7	srsmatrix (const rvector&)	579
2.13.8	submatrix	580
2.13.9	operator (,)	581
2.13.10	operator ()	582
2.13.11	operator []	583
2.13.12	diag	584
2.13.13	operator = (const srsmatrix&)	585
2.13.14	assign (const TR*)	586
2.13.15	assign (int, int, const srsmatrix&)	587
2.13.16	set (TR)	588
2.13.17	set (int,int,TR)	589
2.13.18	set_diag (int,rvector)	590
2.13.19	resize	591
2.13.20	operator ==	592
2.13.21	operator !=	593
2.13.22	operator <<	594
2.13.23	operator +	595
2.13.24	operator -	596
2.13.25	sum	597
2.13.26	diff	598
2.13.27	operator +=	599
2.13.28	operator -=	600
2.13.29	operator - ()	601
2.13.30	operator ++	602
2.13.31	operator --	603
2.13.32	operator * (TR)	604

2.13.33	operator / (TR)	605
2.13.34	operator *= (TR)	606
2.13.35	operator /= (TR)	607
2.13.36	normalize	608
2.13.37	transposition	609
2.13.38	operator * (const rvector&)	610
2.13.39	operator * (const rmatrix&)	611
2.13.40	operator * (const srmatrix&)	612
2.13.41	syrk	613
2.13.42	syr2k	615
2.13.43	inv	617
2.13.44	exp	618
2.13.45	polynomial	620
2.13.46	eig	622
2.13.47	Cholesky	623
2.13.48	Bunch-Kaufman	624
2.13.49	identity	625
2.13.50	vanish	626
2.13.51	randomize	627
2.14	schmatrix	628
2.14.1	schmatrix ()	631
2.14.2	schmatrix (int)	632
2.14.3	schmatrix (TC*,int)	633
2.14.4	schmatrix (const TC*,int)	634
2.14.5	schmatrix (const schmatrix&)	635
2.14.6	schmatrix (const cmatrix&)	636
2.14.7	schmatrix (const rvector&)	637
2.14.8	schmatrix (const srmatrix&)	638
2.14.9	schmatrix (const TR*,const TR*,int)	639
2.14.10	schmatrix (const srmatrix&, const srmatrix&)	640
2.14.11	submatrix	641
2.14.12	operator (,)	642
2.14.13	operator ()	643
2.14.14	operator []	644
2.14.15	diag	645
2.14.16	real	646
2.14.17	imag	647
2.14.18	operator = (const schmatrix&)	648
2.14.19	assign (const TC*)	649
2.14.20	assign (int, int, const schmatrix&)	650
2.14.21	set (int,int,TC)	651
2.14.22	set_diag	652
2.14.23	set_main_diag	653

2.14.24	assign_real	654
2.14.25	set_real	655
2.14.26	resize	656
2.14.27	operator ==	657
2.14.28	operator !=	658
2.14.29	operator <<	659
2.14.30	operator +	660
2.14.31	operator -	661
2.14.32	sum	662
2.14.33	diff	663
2.14.34	operator +=	664
2.14.35	operator -=	665
2.14.36	operator - ()	666
2.14.37	operator ++	667
2.14.38	operator --	668
2.14.39	operator * (TR)	669
2.14.40	operator / (TR)	670
2.14.41	operator * (TC)	671
2.14.42	operator / (TC)	672
2.14.43	operator *= (TR)	673
2.14.44	operator /= (TR)	674
2.14.45	normalize	675
2.14.46	conjugation	676
2.14.47	operator * (const cvector&)	677
2.14.48	operator * (const cmatrix&)	678
2.14.49	operator * (const scmatrix&)	679
2.14.50	herk	680
2.14.51	her2k	682
2.14.52	inv	684
2.14.53	exp	685
2.14.54	polynomial	687
2.14.55	eig	689
2.14.56	Cholesky	690
2.14.57	Bunch-Kaufman	691
2.14.58	identity	692
2.14.59	vanish	693
2.14.60	randomize_real	694
2.14.61	randomize_imag	695
2.15	cvmexception	696
2.15.1	cause	697
2.15.2	what	698
2.15.3	Customization	699
2.16	Utilities	700

2.16.1	cvmMalloc	701
2.16.2	cvmAddRef	702
2.16.3	cvmFree	703
2.16.4	cvmExit	704
2.16.5	cvmMachMin	705
2.16.6	cvmMachSp	706
2.16.7	eye_real	707
2.16.8	eye_complex	708
2.16.9	operator *	709
2.17	Pool Manager	711